

Aspects Regarding a New Method for the Optimal Control Law's Synthesis of Aircrafts' Move

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Abstract: - The paper presents a new algorithm for optimal control law's synthesis in rapport with state vector of the aircraft's move. Starting from state equation of the movement, the system is brought to Jordan canonic form and Riccati algebraic matricidal equation (whose solution is the gain matrix of the control law) is brought to an equivalent form with transformation relation. The new gain matrix is partitioned conform to equations (35)÷(40). Using ALGLX algorithm, one studies elastic no deformed (no dimensional description) longitudinal and lateral move of an aircraft and dimensional elastic deformed move. One obtained Matlab/Simulink models and numerical programs and with them time characteristics expressing state variables' dynamic are obtained.

Key-Words: - algorithm, optimal, control law, longitudinal and lateral move, synthesis.

1 Introduction

To project optimal control systems one must take into account to minimize physic variables' deviation from their imposed values [1],[2]. The measurement of this fact is made using a performance indicator (a function) [3], [4]. The project's requests refer not only to system's performances but to assure physic reliability and imposed command restraints.

The chose of most appropriate performance indicator for a system is a difficult job, especially in the case of complex systems. Determination of the performance indicator must me done not only from the mathematic point of view, but also from practical issues point of view. Generally one must make a compromise between physical performances evaluation and solve of the mathematical problem.

Solution for optimal problem exists if the system

is controllable and observable.

The performance indicator determines the system's configuration. A system that is optimal from the point of view of one performance indicator isn't optimal from the point of view of other performance indicator. Hardware project of an optimal control law may be difficult and expensive. An example of optimal problem with analytical solve is the one of linear system based on quality quadratic indicator.

2 Description of aircraft's move and of the optimal control law

The move of an aircraft is described by state equation [5]

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad (1)$$

with $u(t)$ – command vector ($m \times 1$), $x(t)$ – state vector ($n \times 1$), $A(t)$ and $B(t)$ – matrices ($n \times n$) and respectively ($n \times m$). Control law has the form

$$u(t) = -K(t)x(t), \quad (2)$$

with $K(t)$ – gain matrix ($m \times n$) [6]

For evaluation of the system's performances one chooses performance indicator

$$J = \int_{t_0}^{t_1} \mathcal{L}\{x(t), u(t)\} dt; \quad (3)$$

J has a determined numerical value which differs from one trajectory to another. The array of admitted trajectories \mathcal{T} is generated by commands array \mathcal{U} . If J admits on array \mathcal{U} one or multiple extreme points one may express correctly the optimisation problem as a problem of command's determination $u(t) \in \mathcal{U}$ which leads indicator J to the extreme value.

Optimal synthesis consists of control law's determination which leads the system from initial state x_0 in the final state x_1 , in a time interval $[t_0, t_1]$ so that quadratic criterion

$$J = \frac{1}{2} x^T(t_1) M x(t_1) + \frac{1}{2} \int_{t_0}^{t_1} [x^T(t) Q(t) x(t) + u^T(t) R(t) u(t)] dt \quad (4)$$

takes minimum possible value; M and $Q(n \times n)$ are symmetric, positive defined and non-singular matrices; $R(t)$ – matrix of command variables' weights.

Setting that state vector tends to zero, $x(t_1 \rightarrow \infty) = 0$, choosing initial moment $t_0 = 0$, performance indicator (4) becomes

$$J = \frac{1}{2} \int_0^{\infty} [x^T(t) Q(t) x(t) + u^T(t) R(t) u(t)] dt. \quad (5)$$

The Hamiltonian for the system is [7]

$$H = \frac{1}{2} x^T Q x + \frac{1}{2} u^T R u + \psi^T \dot{x} = \frac{1}{2} x^T Q x + \frac{1}{2} u^T R u + \psi^T (A x + B u), \quad (6)$$

where ψ is the adjunct vector

Differential equation of the adjunct vector is

$$\dot{\psi} = -\frac{\partial H}{\partial x}. \quad (7)$$

or, taking into account equation (6) and setting the minimum of H condition in rapport with

$$u \left(\frac{\partial H}{\partial u} = 0 \right), \quad u = -R^{-1} B^T \psi. \quad (8)$$

The dependence between vectors x and ψ may

have, for example, the form [8]

$$\psi(t) = P(t)x(t), \quad (9)$$

where $P(n \times n)$ is a symmetric positive defined matrix. Matrix $P(t)$ is the solution of Riccati algebraic matricidal equation (41).

Taking into account the slow variation of the flight parameters (dynamic regimes are shorter than variation time of parameters), all the matrices will be considered, from this moment, constant and matrix P will be obtained by solving EMAR equation (41).

For control law's synthesis the system (1) is brought to Jordan canonic form

$$\dot{\bar{x}} = \bar{A} \bar{x} + \bar{B} u, u = -\bar{K} \bar{x}, \bar{K} = \bar{R}^{-1} \bar{B}^T \bar{P}, \quad (10)$$

where \bar{x} is the new state vector which verifies equation

$$x = T \bar{x}, \bar{A} = T^{-1} A T, \bar{B} = T^{-1} B = [I_m \quad ; \quad 0]^T; \quad (11)$$

T is a non singular transformation ($n \times n$), I_m – unity matrix ($m \times m$).

Transformation matrix T is chosen so that [9]

$$T = [B \quad ; \quad \tilde{T}], \quad (12)$$

where \tilde{T} is an arbitrary matrix so that $\text{rang } T = n$.

In paper [10],

$$S_{\lambda_i} = [\lambda_i I_n - \bar{A} \quad ; \quad B], R_{\lambda_i} = [N_{\lambda_i} \quad ; \quad M_{\lambda_i}], \quad (13)$$

where I_n – unity matrix ($n \times n$), λ_i – eigenvalues of the closed loop system and $N_{\lambda_i}, M_{\lambda_i}$ – sub-matrices (columns of R_{λ_i}) form a base for S_{λ_i} , columns of N_{λ_i} are linear independent; one calculates gain matrix

$$\bar{K} = W(\Phi)^{-1}, \quad (14)$$

where

$$W = [W_1 \quad W_2 \quad \dots \quad W_i \quad \dots \quad W_n], \quad (15)$$

$$W_i = M_{\lambda_i} \bar{p}_i, \Phi = \Phi_a \bar{V},$$

$$\bar{V} = \text{diag}[\bar{p}_1 \quad \bar{p}_2 \quad \dots \quad \bar{p}_i \quad \dots \quad \bar{p}_n], \bar{p}_i = (\Omega_i)^+ n_k, \quad (16)$$

where $(\Omega_i)^+$ – pseudo inverse of matrix Ω_i, n_k is the column of unity matrix corresponding of the k^{th} sub matrix of matrix $\Phi_d^{-1} \Phi_a, \Phi_d = \Phi_a \bar{V},$

$$[\Omega_1 \quad \Omega_2 \quad \dots \quad \Omega_i \quad \dots \quad \Omega_n] = \Phi_d^{-1} \Phi_a, \quad (17)$$

$$\Phi_a = [N_{\lambda_1} \quad N_{\lambda_2} \quad \dots \quad N_{\lambda_i} \quad \dots \quad N_{\lambda_n}]. \quad (18)$$

Solution of the Riccati equation may be obtained using eigenvalues and eigenvectors of matrix N . Columns of modal matrix U are eigenvectors of matrix $N; U$ verifies equation

$$N U = U \Lambda, \quad (19)$$

where

$$\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}, \Lambda_1 = \text{diag} \lambda_i, \Lambda_2 = \text{diag} \lambda_j, \quad (20)$$

with $\lambda_i, i = 1, n$, eigenvalues with real part negative,

$\lambda_j, j = \overline{n+1, 2n}$, eigenvalues with real part positive.

Setting $z(t)$ of form [8]

$$z(t) = Ue^{\Lambda t} z(0), \tag{21}$$

equivalent with equation

$$\begin{bmatrix} x(t) \\ \psi(t) \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \cdot \begin{bmatrix} e^{\Lambda_1 t} & 0 \\ 0 & e^{\Lambda_2 t} \end{bmatrix} \cdot \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \cdot \begin{bmatrix} x(0) \\ \psi(0) \end{bmatrix}, \tag{22}$$

or equivalent with equations system

$$\begin{aligned} x(t) &= U_{11}e^{\Lambda_1 t} [V_{11}x(0) + V_{12}\psi(0)] + \\ &+ U_{12}e^{\Lambda_2 t} [V_{21}x(0) + V_{22}\psi(0)], \\ \psi(t) &= U_{21}e^{\Lambda_1 t} [V_{11}x(0) + V_{12}\psi(0)] + \\ &+ U_{22}e^{\Lambda_2 t} [V_{21}x(0) + V_{22}\psi(0)] \end{aligned} \tag{23}$$

and taking into account equation $\psi(0) = Px(0)$, one obtains

$$P = U_{21}U_{11}^{-1}. \tag{24}$$

Other forms of command law, based on eigenvalues and eigenvectors' use are presented in [3], [11]. For the system described by equation (1), with $y = Cx$, one chooses command law $u = Ky$, with K - gain matrix. With these, equation (1) becomes

$$K = B^+(U\Lambda - AU)(CU)^{-1}, \tag{25}$$

where B^+ is the pseudo inverse of B .

In [3] one has presented the algorithm for calculus of matrices Q, R and K .

Another algorithm for calculus of P uses sign matrix [12]; this is now short presented.

For a matrix $A(n \times n)$, having eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_i, \dots$, with multiplicity orders $\mu_1, \mu_2, \dots, \mu_i, \dots$, there is a non singular matrix T (modal matrix) which brings matrix A to the canonical Jordan form

$$A_j = \begin{bmatrix} \lambda_i & 1 & 0 & \dots & 0 \\ 0 & \lambda_i & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_i & 1 \\ 0 & 0 & \dots & 0 & \lambda_i \end{bmatrix}; \tag{26}$$

transformation relation is

$$A = T A_j T^{-1}. \tag{27}$$

If A is a dichotomic matrix ($\text{Re} \lambda_i \neq 0$), one may define sign matrix of the Jordan cell J_i ;

$$\text{sgn } J_i = \begin{bmatrix} \text{sgn } \lambda_i & 0 & 0 & \dots & 0 \\ 0 & \text{sgn } \lambda_i & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \text{sgn } \lambda_i \end{bmatrix} = (\text{sgn } \lambda_i) I_{\mu_i}, \tag{28}$$

where

$$\text{sgn } \lambda_i = \begin{cases} +1, \text{Re } \lambda_i > 0, \\ -1, \text{Re } \lambda_i < 0. \end{cases} \tag{29}$$

Sign matrix associated to matrix A may be

expressed as follows

$$\text{sgn } A = T \begin{bmatrix} \text{sgn } J_1 & 0 & 0 & \dots & 0 \\ 0 & \text{sgn } J_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \text{sgn } J_n \end{bmatrix} T^{-1}. \tag{30}$$

Choosing matrix T so that first Jordan cells correspond to eigenvalues with negative real parts and the following ones to eigenvalues with positive real parts, then

$$\text{sgn } A = T \begin{bmatrix} I_- & 0 \\ 0 & I_+ \end{bmatrix} T^{-1}, \tag{31}$$

where

$$\begin{aligned} I_- &= \begin{bmatrix} -1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -1 \end{bmatrix}_{q \times q}, \\ I_+ &= \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}_{(n-q) \times (n-q)}. \end{aligned} \tag{32}$$

With notation

$$M = \text{sgn } A, M_+ = \frac{1}{2}(I_n + M), M_- = \frac{1}{2}(I_n - M), \tag{33}$$

it results

$$M_+ = T \begin{bmatrix} 0 & 0 \\ 0 & I_+ \end{bmatrix} T^{-1}, M_- = T \begin{bmatrix} I_+ & 0 \\ 0 & 0 \end{bmatrix} T^{-1}. \tag{34}$$

Matrix \bar{P} is EMAR's solution for the system (10)

$$\bar{P}A + \bar{A}^T \bar{P} - \bar{P}B\bar{K} + \bar{Q} = 0; \tag{35}$$

matrices \bar{P} and \bar{K} may be partitioned as below [3]

$$\bar{P} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}, \bar{K} = [\bar{K}_1 \quad \bar{K}_2], \tag{36}$$

$$\bar{P}_{12} = \bar{P}_{21}^T, \bar{P}_{22} = \bar{P}_{22}^T,$$

where \bar{P}_{11} and \bar{K}_1 are sub matrices with dimension $(m \times m)$, \bar{P}_{22} doesn't interfere in calculus for obtaining of \bar{K} if matrix \bar{P} is diagonalizable; in this case \bar{P}_{11} must be diagonalizable also; it must be chosen in rapport with matrix R ; by replacing \bar{B} with form (4) in equation $\bar{K} = R^{-1} \bar{B}^T \bar{P}$, one obtains

$$\begin{aligned} [\bar{K}_1 \quad \bar{K}_2] &= R^{-1} [I_m \quad 0] \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix} = \\ &= [R^{-1} \quad 0] \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix} = [R^{-1} \bar{P}_{11} \quad R^{-1} \bar{P}_{12}]; \end{aligned} \tag{37}$$

one results

$$\bar{P}_{11} = R\bar{K}_1, \bar{P}_{12} = R\bar{K}_2; \tag{38}$$

$$\bar{P} = \begin{bmatrix} R\bar{K}_1 & R\bar{K}_2 \\ (R\bar{K}_2)^T & I_{n-m} \end{bmatrix}, \tag{39}$$

where, for simplicity, one has chosen $\bar{P}_{22} = I_{n-m}$ unity matrix; \bar{P}_{22} may have any form because it doesn't interfere in calculus of K .

For $m = 1$ and $n = 4$ (the case of longitudinal move of the aircraft) matrices have the following dimensions:

$$\begin{aligned} &A(4 \times 4), T(4 \times 4), \bar{A}(4 \times 4), B(4 \times 1), \bar{B}(4 \times 1), P(4 \times 4), \\ &\bar{K}_1(1 \times 1), \bar{K}_2(1 \times 3), \bar{P}(4 \times 4), K(1 \times 4), \bar{K}(1 \times 4), \\ &\bar{P}_{11}(1 \times 1), \bar{P}_{12} = \bar{P}_{21}^T(1 \times 3), P_{22}(3 \times 3); \\ &\bar{P}_{11} = [\bar{p}_{11}], \bar{P}_{12} = \bar{P}_{21}^T = [\bar{p}_{11} \quad \bar{p}_{12} \quad \bar{p}_{13}], \bar{P}_{22} = I_3, R = [1], \\ &\bar{K} = [\bar{K}_1 \quad \bar{K}_2] = [k_1 \quad k_{21} \quad k_{22} \quad k_{23}]; \bar{K}_1 = [k_1], \quad (40) \\ &\bar{K}_2 = [k_{21} \quad k_{22} \quad k_{23}] \end{aligned}$$

Replacing \bar{P}_{11}, R and \bar{K}_1 in (38) and (39) one results

$$\begin{aligned} \bar{P}_{11} &= [k_1], \bar{P}_{12} = \bar{P}_{21}^T = [k_{21} \quad k_{22} \quad k_{23}], \\ \bar{P} &= \begin{bmatrix} k_1 & \vdots & k_{21} & k_{22} & k_{23} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ k_{21} & \vdots & 1 & 0 & 0 \\ k_{22} & \vdots & 0 & 1 & 0 \\ k_{23} & \vdots & 0 & 0 & 1 \end{bmatrix}. \quad (41) \end{aligned}$$

For $m = 2$ (the case of lateral move of the aircraft),

$$\bar{P}_{11} = \begin{bmatrix} \bar{p}_{11} & \bar{p}_{12} \\ \bar{p}_{21} & \bar{p}_{22} \end{bmatrix}, R = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}, \bar{K}_1 = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}, \quad (42)$$

which, replaced in first equation (38) leads to

$$\begin{bmatrix} \bar{p}_{11} & \bar{p}_{12} \\ \bar{p}_{21} & \bar{p}_{22} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}; \quad (43)$$

for $r_{11} = r_{22} = 1$, expressing \bar{p}_{12} and \bar{p}_{21} from this one obtains

$$r = r_{12} = r_{21} = \frac{k_{12} - k_{21}}{k_{11} - k_{22}}; R = \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}. \quad (44)$$

The relations for calculus of P, \bar{P}, K and \bar{K}, Q, \bar{Q} are obtained. These verify Riccati algebraic equations afferent to system (1) and (10)

$$PA + A^T P - PBK + Q = 0, \quad (45)$$

$$\bar{P}\bar{A} + \bar{A}^T \bar{P} - \bar{P}\bar{B}\bar{K} + \bar{Q} = 0. \quad (46)$$

For mentioned relations' obtaining equation (46) is left multiplied with $(T^{-1})^T$ and right multiplied with T^{-1} . By terms' identification of the equation (45) one obtained the desired equations. First of them is

$$Q = (T^{-1})^T \bar{Q} T^{-1}. \quad (47)$$

Taking into account that $\bar{A} = T^{-1}AT$ and $(T^{-1})^T T^T = (TT^{-1})^T = I^T = I$, one results

$$P = (T^{-1})^T \bar{P} T^{-1}. \quad (48)$$

By equaling the third term of (46) with the third term of equation obtained by left multiplied with

$(T^{-1})^T$ and right multiplied with (T^{-1}) of (48) and taking into account that $\bar{B} = T^{-1}B$, one yields

$$K = \bar{K}T^{-1}. \quad (49)$$

3 Synthesis algorithm (ALGLX) of the optimal control law using state vector

Step 1: one brings the system described by pair $(A, B), A(n \times n), B(n \times m)$, to Jordan canonical form (\bar{A}, \bar{B}) , using transformation $x = T\bar{x}$; here T is a non singular linear transformation;

$$\bar{A} = T^{-1}AT, \bar{B} = T^{-1}B = [I_m \quad 0], \quad (50)$$

where T has the form $T = [B \quad \tilde{T}]$, with \tilde{T} random matrix $(n \times (n - m))$ so that $\text{rang} T = n$ [9].

Step 2: gain matrix \bar{K} for the optimal control of system (\bar{A}, \bar{B}) is obtained so that closed loop system with matrix $\bar{G} = \bar{A} - \bar{B}\bar{K}$ has imposed stable eigenvalues.

Step 3: matrices \bar{K} and \bar{P} are partitioned as follows

$$\bar{K} = [\bar{K}_1 \quad \bar{K}_2], \bar{P} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}, \quad (51)$$

$$\bar{P}_{12} = \bar{P}_{21}^T, \bar{P}_{22} = \bar{P}_{22}^T;$$

\bar{K}_1 and \bar{P}_{11} are sub matrices $(m \times m)$; sub matrices $\bar{P}_{11}, \bar{P}_{12}, \bar{P}_{22}$ and \bar{R} are calculated in rapport with sub matrices of matrix \bar{K} and with weight matrix $\bar{R} = R$

$$\begin{aligned} \bar{P}_{11} &= R\bar{K}_1, \\ \bar{P}_{12} &= \bar{P}_{21}^T = R\bar{K}_2, \\ \bar{P}_{22} &= I_{n-m}, \end{aligned} \quad (52)$$

where I_{n-m} is the unity matrix $(n - m) \times (n - m)$; for $m = 1$

$$\begin{aligned} \bar{K} &= [k_1 \quad k_{21} \quad k_{22} \quad k_{23}], \\ R &= [1]. \end{aligned} \quad (53)$$

and for $m = 2$

$$\bar{P} = \begin{bmatrix} R\bar{K}_1 & R\bar{K}_2 \\ (R\bar{K}_2)^T & I_2 \end{bmatrix}, \quad (54)$$

where \bar{K}_1 and \bar{K}_2 have forms

$$\bar{K} = [\bar{K}_1 \quad \bar{K}_2], \bar{K}_1 = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}, \bar{K}_2 = \begin{bmatrix} k'_{11} & k'_{12} \\ k'_{21} & k'_{22} \end{bmatrix}, \quad (55)$$

$$R = \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}, r = \frac{k_{12} - k_{21}}{k_{11} - k_{22}}.$$

Step 4:

Variant 1: matrices \bar{Q} and Q are calculated

$$\bar{Q} = -[\bar{P}A + \bar{A}^T \bar{P} - \bar{P}B\bar{K}], \quad (56)$$

$$Q = (T^{-1})^T \bar{Q} T^{-1}; \quad (57)$$

then, knowing matrices A, B, Q and R , one solves EMAR and obtains P

$$PA + A^T P - PBR^{-1}B^T P + Q = 0; \quad (58)$$

one calculates gain matrix with equation

$$K = R^{-1}B^T P. \quad (59)$$

Variant 2

Using form (41) of \bar{P} , matrices P and K are obtained with equation $P = (T^{-1})^T \bar{P} T^{-1}, K = R^{-1}B^T P$.

Step 5: one calculates the eigenvalues of matrix $G = (A - BK)$; if these are placed in left complex semi plane (matrix G is stable), then gain matrix is the one already obtained; otherwise one returns to step 1 and chooses another matrix \tilde{T} , of course another matrix T and the calculus conform to algorithm's steps is again achieved.

Comparatively with algorithm presented in [3], calculus of matrix \bar{K} (step 2) and of matrix T (step 5) differs; in [3] another formula for calculus of \bar{K} is used and K is calculated using formula $K = \bar{K}T^{-1}$. Usually, gain matrix is calculated directly by EMAR's solution using matrices Q and R random chosen or calculated with other methods.

4 Numeric examples

Examples 4.1 (aircraft longitudinal move)

One considers the case of longitudinal move of an aircraft described by equation (1) with no dimensional variables,

$$\begin{bmatrix} \dot{\hat{V}} \\ \dot{\hat{\alpha}} \\ \dot{\hat{\theta}} \\ \dot{\hat{\omega}}_y \end{bmatrix} = \begin{bmatrix} -0.026 & 0.025 & -0.1 & 0 \\ -0.36 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.4212 & -38.49 & 0 & -3.67 \end{bmatrix} \begin{bmatrix} \hat{V} \\ \hat{\alpha} \\ \hat{\theta} \\ \hat{\omega}_y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \delta_p, \quad (60)$$

where

$$\hat{V} = \frac{\Delta V}{V^*}, \hat{t} = \frac{t}{\tau_a}, \hat{\omega}_y = \frac{\bar{b}}{V^*} \omega_y, \quad (61)$$

$$\hat{\theta} = \Delta\theta, \hat{\alpha} = \Delta\alpha;$$

$\tau_a = 2,1s$ – aerodynamic time constant, V – flight velocity, α – attack angle, θ – pitch angle, $\omega_y = \dot{\theta}$ and $u = \delta_p$ – elevator deflection. For move's study one goes over algorithm's steps 15 times till the condition from step 5 is certified; one imposes, for example, the following eigenvalues for matrix $(\bar{A} - \bar{B}\bar{K})$

$$-3.4 \pm 6.21j, -0.33, -0.1. \quad (62)$$

One yields the following matrices

$$\begin{aligned} T &= \begin{bmatrix} 0 & -0.691 & -1.441 & 0.815 \\ 0 & 0.858 & 0.571 & 0.711 \\ 0 & 1.254 & -0.399 & 1.290 \\ 1 & -1.593 & 0.690 & 0.668 \end{bmatrix}, \\ \bar{A} &= \begin{bmatrix} -4.775 & -15.972 & -21.143 & -20.010 \\ -0.032 & 2.946 & 1.354 & 3.036 \\ 0.573 & -4.456 & -1.402 & -3.326 \\ 0.984 & -5.479 & -1.216 & -3.464 \end{bmatrix}, \\ \bar{B}^T &= [1 \ 0 \ 0 \ 0], \\ \bar{K} &= [1.539 \ 3.102 \ -6.042 \ 6.133], \\ R &= [1], \\ \bar{P} &= \begin{bmatrix} 0.539 & 3.102 & -6.042 & 6.133 \\ 3.102 & 1 & 0 & 0 \\ -6.042 & 0 & 1 & 0 \\ 6.133 & 0 & 0 & 1 \end{bmatrix}, \\ \bar{Q} &= \begin{bmatrix} 0.500 & 22.689 & -26.496 & 34.147 \\ 22.689 & 102.850 & -46.560 & 181.529 \\ -26.496 & -46.560 & -216.207 & -23.758 \\ 34.147 & 181.529 & -23.758 & 290.023 \end{bmatrix}. \end{aligned} \quad (63)$$

Using both variant 1 and variant 2 one obtains the same matrices

$$\begin{aligned} P &= \begin{bmatrix} -2.860 & 5.927 & -10.454 & 0.547 \\ 5.927 & 38.528 & -40.554 & -5.020 \\ -10.454 & -40.554 & 39.066 & 6.897 \\ 0.547 & -5.020 & 6.897 & 0.539 \end{bmatrix}, \quad (64) \\ K &= [0.547 \ -5.020 \ 6.897 \ 0.539]. \end{aligned}$$

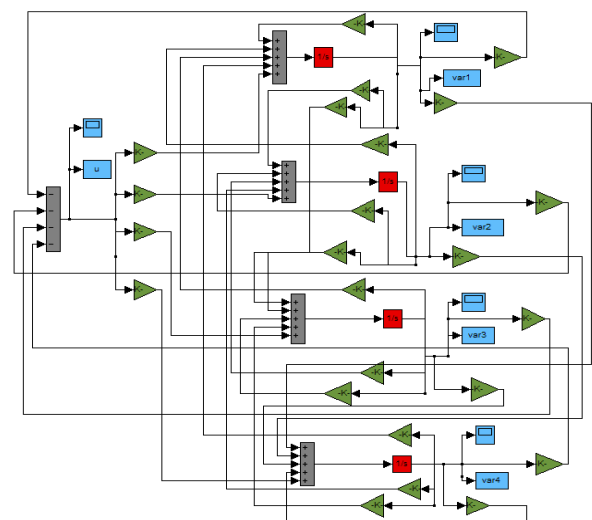


Fig.1 Matlab/Simulink model associated to the longitudinal move of an aircraft

Calculus program for the algorithm's validation is presented Section 5; one uses instruction PLACE [13] for calculus of matrix \bar{K} and instruction LQR for calculus of matrix K using the first variant of

the algorithm.

For obtaining the characteristics $\hat{V}(\hat{t}), \hat{\alpha}(\hat{t}), \hat{\theta}(\hat{t}), \hat{\omega}_y(\hat{t}), \hat{\delta}_p(\hat{t})$ one uses Matlab/Simulink model from fig.1; the obtained characteristics are presented in fig.2; initial values of state variables are

$$\hat{V}(0) = 0.1, \hat{\alpha}(0) = 0.08, \hat{\theta}(0) = 0.5, \hat{\omega}_y(0) = 0.08.$$

The obtained characteristics by the two methods are the same.

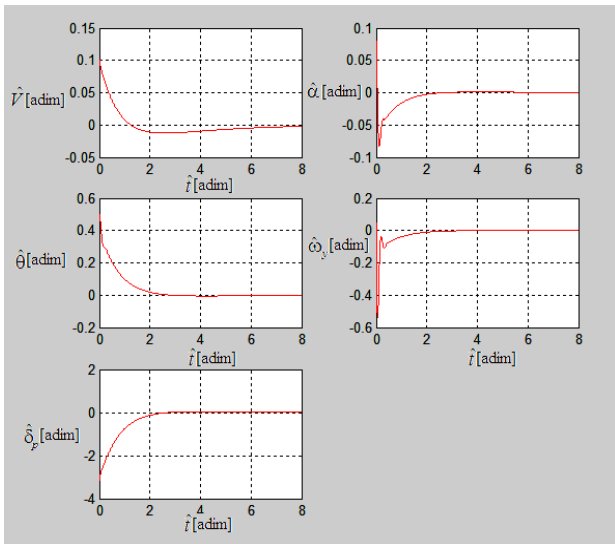


Fig.2 Characteristics $\hat{V}(\hat{t}), \hat{\alpha}(\hat{t}), \hat{\theta}(\hat{t}), \hat{\omega}_y(\hat{t}), \hat{\delta}_p(\hat{t})$

Let's consider now the case of longitudinal move of an aircraft whose wing is affected by elastic deformations (bend); in this case state vector and input vector are [8]

$$x^T = [\Delta\alpha \quad \Delta\omega_y \quad \lambda_1 \quad \lambda_1 \quad \lambda_5 \quad \lambda_5 \quad \lambda_7 \quad \lambda_7 \quad \lambda_8 \quad \lambda_8 \quad \lambda_{12} \quad \lambda_{12}], \quad (65)$$

$$u^T = [\delta_p \quad \delta_m];$$

$$A = \begin{bmatrix} -1.6 & 1 & -0.03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6.9 & -2.24 & -0.039 & 0.03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -283.3 & -17.52 & -56.81 & -5.53 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -53.1 & 9.71 & 0 & 0 & -231.39 & 0.09 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -82.4 & 9.52 & 0 & 0 & 0 & 0 & -384.87 & -2.68 & -10.71 & -0.52 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 12.1 & 2.2 & 0 & 0 & 0 & 0 & 1.24 & -0.176 & -390.1 & -0.474 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -147.1 & 5.24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1466.1 & -1.75 \end{bmatrix}$$

$$B^T = \begin{bmatrix} -0.07 & 3.74 & 0 & 22.52 & -18.3 & 0 & -22.93 & 0 & -4.41 & 0 & 36.57 \\ -0.006 & -0.276 & 0 & 0.765 & -2.14 & 0 & -2.1 & 0 & -1.37 & 0 & 3.993 \end{bmatrix}$$

where $\lambda_1, \lambda_5, \lambda_7, \lambda_8, \lambda_{12}$ are wing local bend angles. The algorithm steps are gone over 10 times. The imposed eigenvalues of matrix $(\bar{A} - \bar{B}\bar{K})$ are [14]

$$\begin{aligned} & -37.2, -10.81 \pm 32j, -0.54 \pm 19.66j, \\ & -4.01 \pm 13.73j, -2.26 \pm 16.73j, -8.04, -6.88, -1.01 \end{aligned} \quad (66)$$

Initial state vector is

$$x_0^T = [1 \quad 0 \quad 2 \quad 1 \quad 0 \quad 0.5 \quad -1 \quad -1 \quad 2 \quad 0 \quad 0 \quad 10].$$

The obtained time characteristics $x_i(t), i = \overline{1,12}$, are presented in fig.3. The obtained characteristics by the two methods are the same.

$$P = \begin{bmatrix} -271.46 & 170.21 & -160.34 & 113.89 & -265.67 & 322.89 & -13.4 & -91.01 & 266.99 & -53.21 & 74.12 & 10.93 \\ 170.21 & 60.90 & -22.81 & 9.05 & 8.02 & 45.43 & -19.25 & -11.52 & 72.12 & -6.98 & 48.12 & 3.38 \\ -160.34 & -22.81 & -2.89 & 7.51 & -39.22 & 7.13 & 13.56 & 5.14 & 5.97 & 4.62 & -27.66 & -1.21 \\ 113.89 & 9.05 & 7.51 & -7.53 & 33.46 & -15.02 & -9.68 & 5.19 & 6.02 & 4.66 & 16.65 & 0.45 \\ -265.67 & 8.02 & -39.22 & 33.46 & -104.52 & 76.82 & 14.22 & -23.54 & 30.51 & -14.05 & -20.45 & 1.02 \\ 322.89 & 45.43 & 7.13 & -15.02 & 76.82 & -13.34 & -25.73 & 5.72 & 39.75 & 3.43 & 54.34 & 2.45 \\ -13.4 & -19.25 & 13.56 & -9.68 & 14.22 & -25.73 & 6.72 & 6.37 & -30.89 & 0.53 & -16.34 & -1.29 \\ -91.01 & -11.52 & 5.14 & 5.19 & -23.54 & 5.72 & 6.37 & -1.78 & -8.78 & -0.23 & -13.43 & -0.50 \\ 266.99 & 72.12 & 5.97 & 6.02 & 30.51 & 39.75 & -30.89 & -8.78 & 89.20 & 0.32 & 71.98 & 4.06 \\ -53.21 & -6.98 & 4.62 & 4.66 & -14.05 & 3.43 & 0.53 & -0.23 & 0.32 & 4.39 & -3.01 & -0.10 \\ 74.12 & 48.12 & -27.66 & 16.65 & -20.45 & 54.34 & -16.34 & -13.43 & 71.98 & -3.01 & 41.99 & 3.35 \\ 10.93 & 3.38 & -1.21 & 0.45 & 1.02 & 2.45 & -1.29 & -0.50 & 4.06 & -0.10 & 3.35 & 0.42 \end{bmatrix}$$

$$K = \begin{bmatrix} 94.55 & -0.82 & 22.15 & 0.15 & -7.30 & 0.01 & -0.04 & -7.74 & -0.22 & 1.32 & -343.98 \\ -60.55 & 2.09 & 6.34 & -67.21 & -4.27 & 34.45 & -2.34 & -69.12 & -7.11 & 1.06 & -2.22 \end{bmatrix}$$

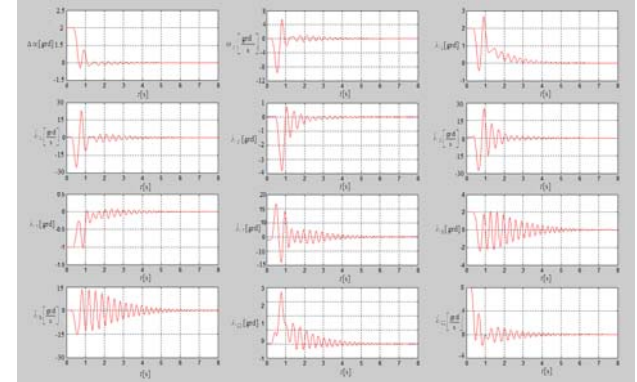


Fig.3 Characteristics $x_i(t), i = \overline{1,12}$,

Examples 4.2 (aircraft lateral move)

One considers now the case of lateral move of a Boeing 744 [15], which flies with $M = 0.8$, and $H = 4 \cdot 10^3$ ft; the lateral move's state equation is

$$\begin{bmatrix} \Delta\beta \\ \Delta\omega_z \\ \Delta\omega_x \\ \Delta\phi \end{bmatrix} = \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.598 & -0.115 & -0.0318 & 0 \\ 0.305 & 0.388 & -0.465 & 0 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta\omega_z \\ \Delta\omega_x \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0.0073 & 0 \\ -0.475 & 0.123 \\ 0.153 & 1.063 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_d \\ \delta_e \end{bmatrix} \quad (67)$$

Algorithm steps are gone over 7 times. Imposed eigenvalues of matrix $(\bar{A} - \bar{B}\bar{K})$ are

$$-13.25, -1.43 \pm 1.598j, -0.318. \quad (68)$$

Using both variant 1 and variant 2 one obtains the same matrices

$$P = \begin{bmatrix} -3.956 & -9.447 & 8.344 & -64.110 \\ -9.447 & -2.656 & 2.725 & -55.942 \\ 8.344 & 2.725 & 3.406 & 47.703 \\ -64.110 & -55.942 & 47.703 & -583.535 \end{bmatrix}, \quad (69)$$

$$K = \begin{bmatrix} -2.545 & -1.880 & -13.786 & -11.773 \\ 9.848 & 4.151 & 15.548 & 53.727 \end{bmatrix}.$$

For obtaining of the characteristics $\Delta\beta(t), \Delta\omega_x(t), \Delta\phi(t), \delta_d(t)$ and $\delta_e(t)$ one used Matlab/Simulink model from fig.4. Initial values of the state variables are

$$\begin{aligned} \Delta\beta(0) &= 4.583\text{grd}, \Delta\omega_z(0) = 1.145\text{grd/s}, \\ \Delta\omega_x(0) &= 1.718\text{grd/s}, \Delta\phi(0) = 17.188\text{grd}. \end{aligned} \quad (70)$$

One obtained the following matrices

$$\begin{aligned}
 T &= \begin{bmatrix} 0.007 & 0 & -0.432 & -1.146 \\ -0.475 & 0.123 & -1.665 & 1.190 \\ 0.153 & 1.063 & 0.125 & 1.189 \\ 0 & 0 & 0.287 & -0.037 \end{bmatrix}, \\
 \bar{A} &= \begin{bmatrix} 2.480 & -15.096 & -2.354 & -15.012 \\ 0.709 & 2.902 & 1.168 & 2.174 \\ 0.325 & 3.546 & -0.216 & 4.356 \\ -0.561 & -1.401 & -1.422 & -0.841 \end{bmatrix}, \\
 \bar{B}^T &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, R = \begin{bmatrix} 1 & 0.84 \\ 0.84 & 1 \end{bmatrix}, \\
 \bar{K} &= \begin{bmatrix} -1.234 & -14.886 & -0.881 & -15.272 \\ 0.478 & 17.039 & 6.230 & 10.121 \end{bmatrix}, \\
 \bar{P} &= \begin{bmatrix} -0.831 & -0.559 & 4.357 & -6.762 \\ -0.559 & 4.521 & 5.489 & -2.720 \\ 4.357 & 5.489 & 1 & 0 \\ -6.762 & -2.720 & 0 & 1 \end{bmatrix}, \\
 \bar{Q} &= \begin{bmatrix} -13.010 & -40.917 & -6.140 & -43.184 \\ -40.917 & -4.321 & 65.684 & -82.912 \\ -6.140 & 65.684 & 38.483 & 26.814 \\ -43.184 & -82.912 & 26.814 & -113.789 \end{bmatrix} \quad (71)
 \end{aligned}$$

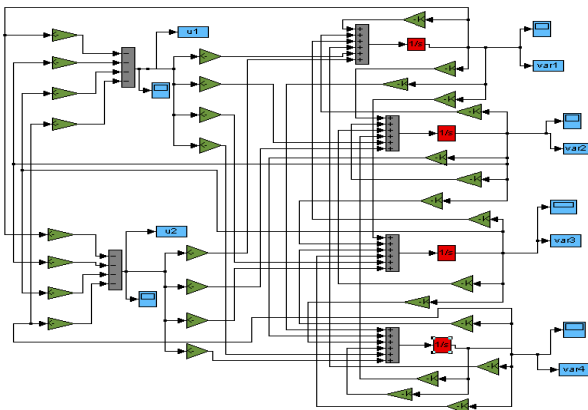


Fig.4 Matlab/Simulink model associated to the lateral move of an aircraft

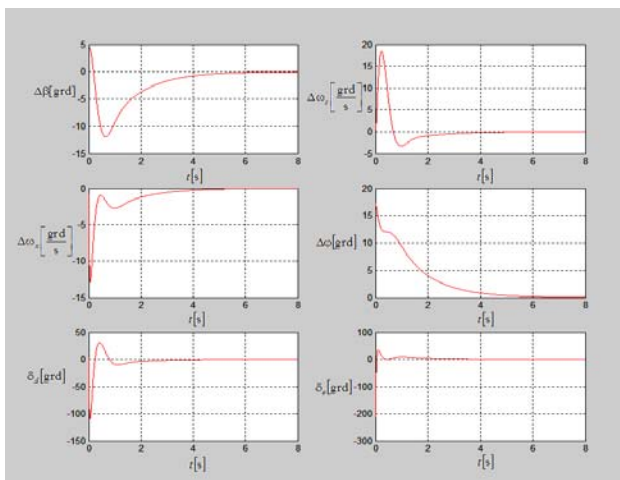


Fig.5 Time variation of the state variables and input variables for the lateral move

The obtained characteristics are presented in fig.5 (the characteristics are obtained with the two variants of the algorithm).

Let's consider now the case of lateral move of an aircraft whose wing is affected by elastic deformations (torsion); in this case state vector and input vector are [8]

$$\begin{aligned}
 x^T &= [\Delta\beta \ \Delta\omega_x \ \Delta\omega_z \ \Delta\varphi \ \Delta\psi \ \gamma_1 \ \dot{\gamma}_1 \ \gamma_2 \ \dot{\gamma}_2 \ \gamma_3 \ \dot{\gamma}_3 \ \gamma_9 \ \dot{\gamma}_9 \ \gamma_{10} \ \dot{\gamma}_{10}]; \\
 u^T &= [\delta_e \ \delta_d];
 \end{aligned}$$

one considers matrices A and B

$$\begin{aligned}
 A &= \begin{bmatrix} -0.18 & 4.9 & 6725 & 385 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -103.6 & 5.5 & 0 & 0 \\ -0.0004 & -2.26 & -0.343 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.864 & 0.046 & 0 & 0 \\ -0.00037 & 0.06 & 0.413 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.437 & 0.001 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.01 & -7.98 & -10.63 & 0 & 0 & -97.67 & -1.89 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.003 & -32.27 & -1.61 & 0 & 0 & 0 & 0 & -151 & -3.57 & 150.1 & 3.73 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0043 & 6.73 & 2.35 & 0 & 0 & 0 & 0 & 8.72 & 0.6 & -160 & 1.54 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -0.027 & -4.35 & -4.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -532.64 & -1.72 & 3.25 & -0.36 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -0.018 & -2.27 & -14.29 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.54 & 0.92 & -990.3 & -2.3 & 0 \end{bmatrix} \\
 B^T &= \begin{bmatrix} 15.78 & -0.79 & 0.052 & 0 & 0 & 0 & 2.14 & 0 & 0.4 & 0 & 4.716 & 0 & -0.411 & 0 & 0.712 \\ -274.47 & -0.312 & -4.32 & 0 & 0 & 0 & -25.68 & 0 & -4.32 & 0 & 7.132 & 0 & 6.46 & 0 & -24.28 \end{bmatrix}
 \end{aligned}$$

The algorithm steps are gone over 10 times. The imposed eigenvalues of matrix $(\bar{A} - \bar{B}\bar{K})$ are

$$\begin{aligned}
 &-2.527, -1.1439, -0.0245, -0.012 \pm 0.304i, \\
 &-0.008 \pm 0.23i, -0.0319 \pm 0.15i, -0.0307 \pm 0.0973i, \\
 &-0.0098 \pm 0.0995i, -0.0035 \pm 0.0024i.
 \end{aligned} \quad (72)$$

The obtained time characteristics are the ones from fig.6.

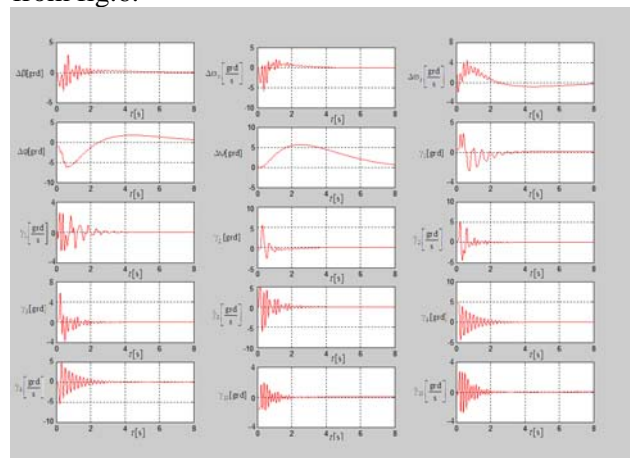


Fig.6 State variables of the optimal command system for lateral move using ALGLX algorithm

5 Matlab programs

One presents below the programs associated to the algorithm (ALGLX) for optimal control law's synthesis in rapport with state vector of the aircraft's longitudinal and lateral move. In these programs the Matlab/Simulink models presented in fig.1 and fig.4 are simulated.

The programs for the longitudinal and lateral move of the aircraft whose wing is affected by

elastic deformations (bend or torsion) are similar.

Algorithm ALGLX – longitudinal move

```

close all;
A=[-0.026 0.025 -0.1 0;-0.36 -3 0 1;
    0 0 0 1; 0.4212 -38.42 0 -3.67];
B=[0;0;0;1];
Q=[10 0 0 0;0 10 0 0; 0 0 100 0;0 0 0 1];
R=[2];
[K,P,E] = LQR(A,B,Q,R);
I2=[1 0;0 1];
% T presentation
N3=randn(4,3);
contor=1;
T(:,1)=B(:,1);
for i=1:4
    for j=1:3
        T(i,j+1)=N3(i,j);
    end
end
Ab=(inv(T))*A*T;
Bb=(inv(T)*B);
Kb=place(Ab,Bb,E);
e=eig(Ab-Bb*Kb);
k1=Kb(1);k21=Kb(2);k22=Kb(3);
k23=Kb(4);r1=1;Rb=[r1];R=Rb;
Pb=r1*[k1 k21 k22 k23;k21 1 0 0;
    k22 0 1 0;k23 0 0 1];
ee=eig(Rb);
Qb=-(Pb*Ab+(transpose(Ab))*Pb-Pb*Bb*Kb);
% Variant 2
PPP=transpose(inv(T))*Pb*inv(T);
KKK=inv(R)*transpose(B)*PPP;
EEE=eig(A-B*KKK);m=rank(T);
while real(EEE(1))>0 | real(EEE(2))>0 |
real(EEE(3))>0 | real(EEE(4))>0 | m<4
% T presentation
N3=randn(4,3);
contor=contor+1;
T(:,1)=B(:,1);
for i=1:4
    for j=1:3
        T(i,j+1)=N3(i,j);
    end
end
Ab=(inv(T))*A*T;
Bb=(inv(T)*B);
Kb=place(Ab,Bb,E);
e=eig(Ab-b*Kb);
k1=Kb(1);k21=Kb(2);k22=Kb(3); k23=Kb(4);
r1=5;Rb=[r1]; R=Rb;
Pb=r1*[k1 k21 k22 k23;k21 1 0 0;
    k22 0 1 0;k23 0 0 1];
ee=eig(Rb);
Qb=-(Pb*Ab+(transpose(Ab))*Pb-Pb*Bb*Kb);

```

% Variant 2

```

PPP=transpose(inv(T))*Pb*inv(T);
KKK=inv(R)*transpose(B)*PPP;
EEE=eig(A-B*KKK);m=rank(T);
end
contor
% Variant 1
Q=transpose(inv(T))*Qb*inv(T);
R=Rb;
[KK,PP,EE] = LQR(A,B,Q,R);
K=KK;
sim('schprog1');
subplot(321);plot(t,var1);grid;hold on;
subplot(322);plot(t,var2);grid;hold on;
subplot(323);plot(t,var3);grid;hold
on;subplot(324);plot(t,var4);grid; hold on;
subplot(325);plot(t,u);grid;hold on;
K=KKK;
sim('schprog1');
subplot(321);plot(t,var1,'r');
subplot(322);plot(t,var2,'r');
subplot(323);plot(t,var3,'r');
subplot(324);plot(t,var4,'r');
subplot(325);plot(t,u,'r');

```

Algorithm ALGLX – lateral move

```

clear all;close all;
A=[-0.0558 -0.9968 0.0802 0.04415;
    0.598 -0.115 -0.0318 0;
    0.305 0.388 -0.465 0;
    0 0.0805 1 0];
B=[0.0073 0;-0.475 0.123; 0.153 1.063;0 0];
Q=[100 0 0 0;0 1 0 0;0 0 100 0;0 0 0 10];
R=[1 -0.5;-0.5 1];
[K,P,E] = LQR(A,B,Q,R);
I2=[1 0;0 1];
% T presentation
N3=randn(4,2);
contor=1;
for i=1:4
    for j=1:2
        T(i,j)=B(i,j);
        T(i,j+2)=N3(i,j);
    end
end
Ab=(inv(T))*A*T;
Bb=(inv(T)*B);
Kb=place(Ab,Bb,E);
e=eig(Ab-Bb*Kb);
for i=1:2
    for j=1:2
        K10(i,j)=Kb(i,j);
        K20(i,j)=Kb(i,j+2);
    end
end

```



```

k11=K10(1,1);k12=K10(1,2);
k21=K10(2,1);k22=K10(2,2);
Rb=[1 (k12-k21)/(k11-k22);
(k12-k21)/(k11-k22) 1];
ee=eig(Rb);
N1=Rb*K20;
N2=transpose(N1);
Pb=[Rb*K10 Rb*K20;N2 I2];
R=Rb;
Qb=-(Pb*Ab+(transpose(Ab))*Pb-Pb*Bb*Kb);
% Variant 2
PPP=transpose(inv(T))*Pb*inv(T);
KKK=inv(R)*transpose(B)*PPP;
EEE=eig(A-B*KKK);m=rank(T);
while real(ee(1))<0 | real(ee(2))<0 | real(EEE(1))>0 |
real(EEE(2))>0 | real(EEE(3))>0 | real(EEE(4))>0 |
m<4
% T presentation
N3=randn(4,2);
contor=contor+1;
for i=1:4
    for j=1:2
        T(i,j)=B(i,j);
        T(i,j+2)=N3(i,j);
    end
end
% Aflarea Ab,Bb,Kb,Rb
Ab=(inv(T))*A*T;
Bb=(inv(T)*B);
Kb=place(Ab,Bb,E);
e=eig(Ab-Bb*Kb);
for i=1:2
    for j=1:2
        K10(i,j)=Kb(i,j);
        K20(i,j)=Kb(i,j+2);
    end
end
k11=K10(1,1);k12=K10(1,2);
k21=K10(2,1);k22=K10(2,2);
Rb=[1 (k12-k21)/(k11-k22);(k12-k21)/(k11-k22) 1];
ee=eig(Rb);
N1=Rb*K20;
N2=transpose(N1);
Pb=[Rb*K10 Rb*K20;N2 I2];
R=Rb;
Qb=-(Pb*Ab+(transpose(Ab))*Pb-Pb*Bb*Kb);
% Variant 2
PPP=transpose(inv(T))*Pb*inv(T);
KKK=inv(R)*transpose(B)*PPP;
EEE=eig(A-B*KKK);
m=rank(T);
end
% Variant 1
Q=transpose(inv(T))*Qb*inv(T);
R=Rb;[KK,PP,EE] = LQR(A,B,Q,R);

```

```

K=KK;
sim('schprog3');
subplot(321);plot(t,var1);grid;hold on;
subplot(322);plot(t,var2);grid;hold on;
subplot(323);plot(t,var3);grid;hold on;
subplot(324);plot(t,var4);grid;hold on;
subplot(325);plot(t,u1);grid;hold on;
subplot(326);plot(t,u2);grid;hold on;
K=KKK;
sim('schprog3');
subplot(321);plot(t,var1,'r');
subplot(322);plot(t,var2,'r');
subplot(323);plot(t,var3,'r');
subplot(324);plot(t,var4,'r');
subplot(325);plot(t,u1,'r');
subplot(326);plot(t,u2,'r');

```

6 Conclusion

The paper presents a new algorithm for optimal control law's synthesis in rapport with state vector of the aircraft's longitudinal and lateral move. The presented algorithm (ALGLX) is illustrated for models of aircrafts' longitudinal move, no dimensional description (elastic no deformable) and dimensional description (elastic deformable).

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