Synthesis Algorithm of the Optimal Control Law for Flying Objects' Longitudinal Move

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Abstract: - The paper presents a new algorithm for optimal control law's synthesis in rapport with state vector of the aircraft's longitudinal move. Starting from state equation of the movement, the system is brought to Jordan canonic form and Riccati algebraic matriceal equation (whose solution is the gain matrix of the control law) is brought to an equivalent form with transformation relation. The new gain matrix is partitioned conform to equations (7) $\div$ (12). Using ALGLX algorithm, one studies elastic no deformed (no dimensional description) longitudinal move of an aircraft and dimensional elastic deformed move. One obtained Matlab/Simulink model and numerical program and with them time characteristics expressing state variables' dynamic are obtained.

Key-Words: algorithm, optimal, control law, longitudinal move

1 Description of flying object's movement and of the optimal control law

Longitudinal move of an aircraft is described by state equation

$$\dot{x} = Ax + Bu, \quad (1)$$

with $u-$ command vector $(m \times 1)$, $x-$ state vector $(n \times 1)$, A and $B-$ matrices $(n \times n)$ and respectively $(n \times m)$. Control law has the form

$$u = -Kx, \quad (2)$$

with $K-$ gain matrix $(m \times n)$.

For control law's synthesis the system (1) is brought to Jordan canonic form

$$\dot{\overline{x}} = \overline{A}\overline{x} + \overline{B}u, u = -\overline{K}\overline{x},\ \overline{K} = \overline{R}^{-1}\overline{B}^{T}\overline{P}, \quad (3)$$

where $\overline{x}$ is the new state vector which verifies equation

$$x = T\overline{x}, \overline{A} = T^{-1}AT, \overline{B} = T^{-1}B = \begin{bmatrix} I_m & \vdots & 0 \end{bmatrix}^T; \quad (4)$$

T is a non singular transformation $(n \times n)$, I_m – unity matrix $(m \times m)$.

Transformation matrix T is chosen so that [1]

$$T = \begin{bmatrix} B & \vdots & \widetilde{T} \end{bmatrix}, \quad (5)$$

where $\widetilde{T}$ is an arbitrary matrix so that $\operatorname{rang} T = n$.

Matrix $\overline{P}$ is EMAR's solution for the system (3)

$$\overline{P}\overline{A} + \overline{A}^T\overline{P} - \overline{P}\overline{B}\overline{K} + \overline{Q} = 0; \quad (6)$$

matrices $\overline{P}$ and $\overline{K}$ may be partitioned as below [2]

$$\overline{P} = \begin{bmatrix} \overline{P}_{11} & \overline{P}_{12} \\ \overline{P}_{21} & \overline{P}_{22} \end{bmatrix}, \overline{K} = \begin{bmatrix} \overline{K}_1 & \vdots & \overline{K}_2 \end{bmatrix}, \overline{P}_{12} = \overline{P}_{21}^T, \overline{P}_{22} = \overline{P}_{22}^T, \quad (7)$$

where $\overline{P}_{11}$ and $\overline{K}_1$ are sub matrices with dimension $(m \times m)$, $\overline{P}_{22}$ doesn't interfere in calculus for obtaining of $\overline{K}$ if matrix $\overline{P}$ is diagonalizable; in this case $\overline{P}_{11}$ must be diagonalizable also; it must be chosen in rapport with matrix R; by replacing $\overline{B}$ with form (4) in equation $\overline{K} = R^{-1}\overline{B}^T\overline{P}$, one obtains

$$\begin{aligned} \begin{bmatrix} \overline{K}_1 & \vdots & \overline{K}_2 \end{bmatrix} &= R^{-1}\begin{bmatrix} I_m & \vdots & 0 \end{bmatrix}\begin{bmatrix} \overline{P}_{11} & \overline{P}_{12} \\ \overline{P}_{21} & \overline{P}_{22} \end{bmatrix} = \\ \begin{bmatrix} R^{-1} & \vdots & 0 \end{bmatrix}\begin{bmatrix} \overline{P}_{11} & \overline{P}_{12} \\ \overline{P}_{21} & \overline{P}_{22} \end{bmatrix} &= \begin{bmatrix} R^{-1}\overline{P}_{11} & \vdots & R^{-1}\overline{P}_{12} \end{bmatrix}; \end{aligned} \quad (8)$$

one results

$$\overline{P}_{11} = R\overline{K}_1, \overline{P}_{12} = R\overline{K}_2; \quad (9)$$

$$\overline{P} = \begin{bmatrix} R\overline{K}_1 & R\overline{K}_2 \\ \left(R\overline{K}_2\right)^T & I_{n-m} \end{bmatrix}, \quad (10)$$

where, for simplicity, one has chosen $\overline{P}_{22} = I_{n-m}$ unity matrix; $\overline{P}_{22}$ may have any form because it doesn't interfere in calculus of K.

For $m = 1$ and $n = 4$ (the case of longitudinal move of the aircraft) matrices have the following dimensions:

$A(4\times4)$, $T(4\times4)$, $\overline{A}(4\times4), B(4\times1), \overline{B}(4\times1)$, $P(4\times4)$, $\overline{K}_1(1\times1), \overline{K}_2(1\times3)$, $\overline{P}(4\times4), K(1\times4)$, $\overline{K}(1\times4)$, $\overline{P}_{11}(1\times1), \overline{P}_{12} = \overline{P}_{21}^T(1\times3), P_{22}(3\times3)$;

$$\begin{aligned} &\overline{P}_{11} = [\overline{p}_1], \overline{P}_{12} = \overline{P}_{21}^T = \begin{bmatrix} \overline{p}_{11} & \overline{p}_{12} & \overline{p}_{13} \end{bmatrix}, \overline{P}_{22} = I_3, R = [1], \\ &\overline{K} = \begin{bmatrix} \overline{K}_1 & \vdots & \overline{K}_2 \end{bmatrix} = \begin{bmatrix} k_1 & \vdots & k_{21} & k_{22} & k_{23} \end{bmatrix}; \overline{K}_1 = [k_1], \\ &\overline{K}_2 = \begin{bmatrix} k_{21} & k_{22} & k_{23} \end{bmatrix}. \end{aligned} \quad (11)$$

Replacing $\overline{P}_{11}, R$ and $\overline{K}_1$ in (9) and (10) one results

$$\begin{aligned} &\overline{P}_{11} = [k_1], \overline{P}_{12} = \overline{P}_{21}^T = \begin{bmatrix} k_{21} & k_{22} & k_{23} \end{bmatrix}, \\ &\overline{P} = \begin{bmatrix} k_1 & \vdots & k_{21} & k_{22} & k_{23} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ k_{21} & \vdots & 1 & 0 & 0 \\ k_{22} & \vdots & 0 & 1 & 0 \\ k_{23} & \vdots & 0 & 0 & 1 \end{bmatrix}. \end{aligned} \quad (12)$$

The relations for calculus of $P, \overline{P}, K$ and $\overline{K}, Q, \overline{Q}$ are obtained. These verify Riccati algebraic equations afferent to system (1) and (3)

$$PA + A^TP - PBK + Q = 0, \quad (13)$$

$$\overline{P}\overline{A} + \overline{A}^T\overline{P} - \overline{P}\overline{B}\overline{K} + \overline{Q} = 0. \quad (14)$$

For mentioned relations' obtaining equation (14) is left multiplied with $\left(T^{-1}\right)^T$ and right multiplied with T^{-1}. By terms' identification of the equation (13) one obtained the desired equations. First of them is

$$Q = \left(T^{-1}\right)^T\overline{Q}T^{-1}. \quad (15)$$

Taking into account that $\overline{A} = T^{-1}AT$ and $\left(T^{-1}\right)^T T^T = \left(TT^{-1}\right)^T = I^T = I$, one results

$$P = \left(T^{-1}\right)^T\overline{P}T^{-1}. \quad (16)$$

By equaling the third term of (13) with the third term of equation obtained by left multiplied with $\left(T^{-1}\right)^T$ and right multiplied with $\left(T^{-1}\right)$ of (14) and taking into account that $\overline{B} = T^{-1}B$, one yields

$$K = \overline{K}T^{-1}. \quad (17)$$

2 Synthesis algorithm of the optimal control law using state vector (ALGLX)

Step 1: one brings the system described by pair $(A, B), A(n \times n), B(n \times m)$, to Jordan canonical form $\left(\overline{A}, \overline{B}\right)$, using transformation $x = T\overline{x}$; here T is a non singular linear transformation;

$$\overline{A} = T^{-1}AT, \overline{B} = T^{-1}B = \begin{bmatrix} I_m & \vdots & 0 \end{bmatrix}, \quad (18)$$

where T has the form $T = \begin{bmatrix} B & \vdots & \widetilde{T} \end{bmatrix}$, with $\widetilde{T}$ random matrix $(n \times (n - m))$ so that $\operatorname{rang} T = n$ [1].

Step 2: gain matrix $\overline{K}$ for the optimal control of system $\left(\overline{A}, \overline{B}\right)$ is obtained so that closed loop system with matrix $\overline{G} = \overline{A} - \overline{B}\overline{K}$ has imposed stable eigenvalues.

Step 3: matrices $\overline{K}$ and $\overline{P}$ are partitioned as follows

$$\begin{aligned} &\overline{K} = \begin{bmatrix} \overline{K}_1 & \vdots & \overline{K}_2 \end{bmatrix}, \overline{P} = \begin{bmatrix} \overline{P}_{11} & \overline{P}_{12} \\ \overline{P}_{21} & \overline{P}_{22} \end{bmatrix}, \\ &\overline{P}_{12} = \overline{P}_{21}^T, \overline{P}_{22} = \overline{P}_{22}^T; \end{aligned} \quad (19)$$

$\overline{K}_1$ and $\overline{P}_{11}$ are sub matrices $(m \times m)$; sub matrices $\overline{P}_{11}, \overline{P}_{12}, \overline{P}_{22}$ and $\overline{R}$ are calculated in rapport with sub matrices of matrix $\overline{K}$ and with weight matrix $\overline{R} = R$

$$\overline{P}_{11} = R\overline{K}_1, \quad \overline{P}_{12} = \overline{P}_{21}^T = R\overline{K}_2, \quad \overline{P}_{22} = I_{n-m}, \tag{20}$$

where I_{n-m} is the unity matrix $(n-m)\times(n-m)$; for $m = 1$

$$\overline{K} = [k_1 \ \vdots \ k_{21} \quad k_{22} \quad k_{23}], \quad R = [1]. \tag{21}$$

Step 4:

Variant 1: matrices $\overline{Q}$ and Q are calculated

$$\overline{Q} = -[\overline{PA} + \overline{A}^T\overline{P} - \overline{PBK}], \tag{22}$$

$$Q = (T^{-1})^T \overline{Q} T^{-1}; \tag{23}$$

then, knowing matrices A, B, Q and R, one solves EMAR and obtains P

$$PA + A^TP - PBR^{-1}B^TP + Q = 0; \tag{24}$$

one calculates gain matrix with equation

$$K = R^{-1}B^TP. \tag{25}$$

Variant 2

Using form (12) of $\overline{P}$, matrices P and K are obtained with equation $P = (T^{-1})^T \overline{P} T^{-1}, K = R^{-1}B^TP.$

Step 5: one calculates the eigenvalues of matrix $G = (A - BK)$; if these are placed in left complex semi plane (matrix G is stable), then gain matrix is the one already obtained; otherwise one returns to step 1 and chooses another matrix $\widetilde{T}$, of course another matrix T and the calculus conform to algorithm's steps is again achieved.

Comparatively with algorithm presented in [2], calculus of matrix $\overline{K}$ (step 2) and of matrix T (step 5) differs; in [2] another formula for calculus of $\overline{K}$ is used and K is calculated using formula $K = \overline{K}T^{-1}$. Usually, gain matrix is calculated directly by EMAR's solution using matrices Q and R random chosen or calculated with other methods.

3 Numeric examples

One considers the case of longitudinal move of an aircraft described by equation (1) with no dimensional variables,

$$\begin{bmatrix} \dot{\hat{V}} \\ \dot{\hat{\alpha}} \\ \dot{\hat{\theta}} \\ \dot{\hat{\omega}}_y \end{bmatrix} = \begin{bmatrix} -0.026 & 0.025 & -0.1 & 0 \\ -0.36 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.4212 & -38.49 & 0 & -3.67 \end{bmatrix} \begin{bmatrix} \hat{V} \\ \hat{\alpha} \\ \hat{\theta} \\ \hat{\omega}_y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \delta_p, \tag{26}$$

where

$$\hat{V} = \frac{\Delta V}{V^*}, \hat{t} = \frac{t}{\tau_a}, \hat{\omega}_y = \frac{\overline{b}}{V^*}\omega_y, \quad \hat{\theta} = \Delta\theta, \hat{\alpha} = \Delta\alpha; \tag{27}$$

$\tau_a = 2{,}1\,\mathrm{s}$ – aerodynamic time constant, V – flight velocity, α – attack angle, θ – pitch angle, $\omega_y = \dot{\theta}$ and $u = \delta_p$ – elevator deflection.

For move's study one goes over algorithm's steps 15 times till the condition from step 5 is certified; one imposes, for example, the following eigenvalues for matrix $(\overline{A} - \overline{BK})$

$$-3.4 \pm 6.21j, -0.33, -0.1. \tag{28}$$

One yields the following matrices

$$\mathrm{T} = \begin{bmatrix} 0 & -0.691 & -1.441 & 0.815 \\ 0 & 0.858 & 0.571 & 0.711 \\ 0 & 1.254 & -0.399 & 1.290 \\ 1 & -1.593 & 0.690 & 0.668 \end{bmatrix},$$

$$\overline{A} = \begin{bmatrix} -4.775 & -15.972 & -21.143 & -20.010 \\ -0.032 & 2.946 & 1.354 & 3.036 \\ 0.573 & -4.456 & -1.402 & -3.326 \\ 0.984 & -5.479 & -1.216 & -3.464 \end{bmatrix},$$

$$\overline{\mathrm{B}}^{\mathrm{T}} = [1 \quad 0 \quad 0 \quad 0],$$

$$\overline{K} = [1.539 \quad 3.102 \quad -6.042 \quad 6.133],$$

$$R = [1],$$

$$\overline{P} = \begin{bmatrix} 0.539 & 3.102 & -6.042 & 6.133 \\ 3.102 & 1 & 0 & 0 \\ -6.042 & 0 & 1 & 0 \\ 6.133 & 0 & 0 & 1 \end{bmatrix},$$

$$\overline{Q} = \begin{bmatrix} 0.500 & 22.689 & -26.496 & 34.147 \\ 22.689 & 102.850 & -46.560 & 181.529 \\ -26.496 & -46.560 & -216.207 & -23.758 \\ 34.147 & 181.529 & -23.758 & 290.023 \end{bmatrix}. \tag{29}$$

Using both variant 1 and variant 2 one obtains the same matrices

$$P = \begin{bmatrix} -2.860 & 5.927 & -10.454 & 0.547 \\ 5.927 & 38.528 & -40.554 & -5.020 \\ -10.454 & -40.554 & 39.066 & 6.897 \\ 0.547 & -5.020 & 6.897 & 0.539 \end{bmatrix}, \tag{30}$$

$$K = [0.547 \quad -5.020 \quad 6.897 \quad 0.539].$$

Calculus program for presented algorithm's validation is presented in appendix; one uses instruction PLACE [3] for calculus of matrix $\overline{K}$ and instruction LQR for calculus of matrix K using the first variant of the algorithm.

For obtaining the characteristics $\hat{V}(\hat{t}), \hat{\alpha}(\hat{t}),$

$\hat{\theta}(\hat{t}), \hat{\omega}_y(\hat{t}), \hat{\delta}_p(\hat{t})$ one uses Matlab/Simulink model from fig.1; the obtained characteristics are presented in fig.2; initial values of state variables are

$$\hat{V}(0) = 0.1, \hat{\alpha}(0) = 0.08, \hat{\theta}(0) = 0.5,\ \hat{\omega}_y(0) = 0.08.$$

The obtained characteristics by the two methods are the same.

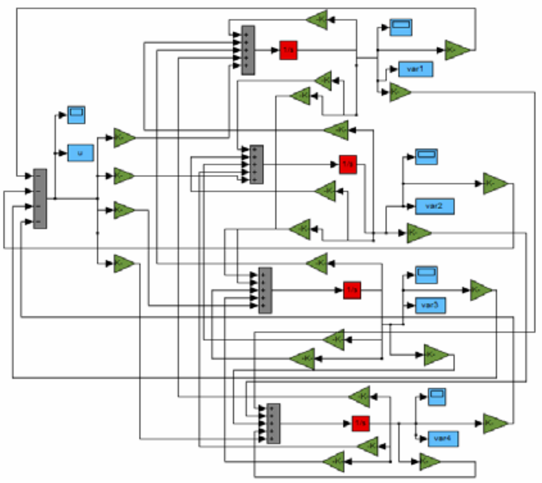

Fig.1

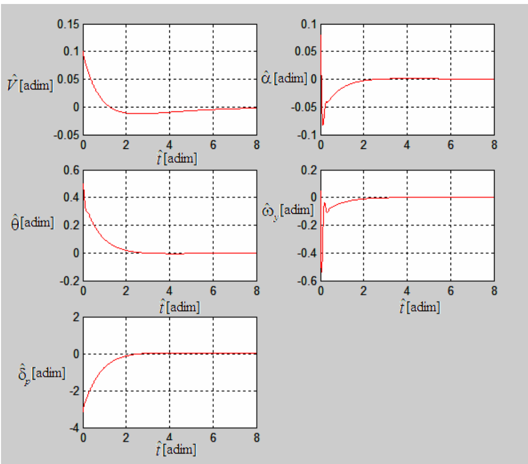

Fig.2

Let's consider now the case of longitudinal move of an aircraft whose wing is affected by elastic deformations (bend); in this case state vector and input vector are [4]

$$x^T = [\Delta\alpha \quad \Delta\omega_y \quad \lambda_1 \quad \dot{\lambda}_1 \quad \lambda_5 \quad \dot{\lambda}_5 \quad \lambda_7 \quad \dot{\lambda}_7 \quad \lambda_8 \quad \dot{\lambda}_8 \quad \lambda_{12} \quad \dot{\lambda}_{12}], \quad (31)$$

$$u^T = [\delta_p \quad \delta_m];$$

$$A = \begin{bmatrix} -1.6 & 1 & -0.03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6.9 & -2.24 & -0.039 & 0.03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -283.3 & -17.52 & -56.81 & -5.53 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -53.1 & 9.71 & 0 & 0 & -231.39 & 0.09 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -82.4 & 9.52 & 0 & 0 & 0 & 0 & -384.87 & -2.68 & -10.71 & -0.52 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 12.1 & 2.2 & 0 & 0 & 0 & 0 & 1.24 & -0.176 & -390.1 & -0.474 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -147.1 & 5.24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1466.1 & -1.75 \end{bmatrix},$$

$$B^T = \begin{bmatrix} -0.07 & 3.74 & 0 & 22.52 & 0 & -18.3 & 0 & -22.93 & 0 & -4.41 & 0 & 36.57 \\ -0.006 & -0.276 & 0 & 0.765 & 0 & -2.14 & 0 & -2.1 & 0 & -1.37 & 0 & 3.993 \end{bmatrix}.$$

where $\lambda_1, \lambda_5, \lambda_7, \lambda_8, \lambda_{12}$ are wing local bend angles.

The algorithm steps are gone over 10 times. The imposed eigenvalues of matrix $(\overline{A} - \overline{B}\overline{K})$ are [5]

$$\begin{aligned} &-37.2, -10.81 \pm 32j, -0.54 \pm 19.66j, \\ &-4.01 \pm 13.73j, -2.26 \pm 16.73j, \\ &-8.04, -6.88, -1.01 \end{aligned} \quad (32)$$

Initial state vector is

$$x_0^T = [1 \quad 0 \quad 2 \quad 1 \quad 0 \quad 0.5 \quad -1 \quad -1 \quad 2 \quad 0 \quad 0 \quad 10].$$

The obtained time characteristics $x_i(t), i = \overline{1,12}$, are presented in fig.3. The obtained characteristics by the two methods are the same.

$$P = \begin{bmatrix} -271.46 & 170.21 & -160.34 & 113.89 & -265.67 & 322.89 & -13.4 & -91.01 & 266.99 & -53.21 & 74.12 & 10.93 \\ 170.21 & 60.90 & -22.81 & 9.05 & 8.02 & 45.43 & -19.25 & -11.52 & 72.12 & -6.98 & 48.12 & 3.38 \\ -160.34 & -22.81 & -2.89 & 7.51 & -39.22 & 7.13 & 13.56 & 5.14 & 5.97 & 4.62 & -27.66 & -1.21 \\ 113.89 & 9.05 & 7.51 & -7.53 & 33.46 & -15.02 & -9.68 & 5.19 & 6.02 & 4.66 & 16.65 & 0.45 \\ -265.67 & 8.02 & -39.22 & 33.46 & -104.52 & 76.82 & 14.22 & -23.54 & 30.51 & -14.05 & -20.45 & 1.02 \\ 322.89 & 45.43 & 7.13 & -15.02 & 76.82 & -13.34 & -25.73 & 5.72 & 39.75 & 3.43 & 54.34 & 2.45 \\ -13.4 & -19.25 & 13.56 & -9.68 & 14.22 & -25.73 & 6.72 & 6.37 & -30.89 & 0.53 & -16.34 & -1.29 \\ -91.01 & -11.52 & 5.14 & 5.19 & -23.54 & 5.72 & 6.37 & -1.78 & -8.78 & -0.23 & -13.43 & -0.50 \\ 266.99 & 72.12 & 5.97 & 6.02 & 30.51 & 39.75 & -30.89 & -8.78 & 89.20 & 0.32 & 71.98 & 4.06 \\ -53.21 & -6.98 & 4.62 & 4.66 & -14.05 & 3.43 & 0.53 & -0.23 & 0.32 & 4.39 & -3.01 & -0.10 \\ 74.12 & 48.12 & -27.66 & 16.65 & -20.45 & 54.34 & -16.34 & -13.43 & 71.98 & -3.01 & 41.99 & 3.35 \\ 10.93 & 3.38 & -1.21 & 0.45 & 1.02 & 2.45 & -1.29 & -0.50 & 4.06 & -0.10 & 3.35 & 0.42 \end{bmatrix},$$

$$K = \begin{bmatrix} 94.55 & -0.82 & 22.15 & 0.15 & -7.30 & 0.01 & -0.04 & -7.74 & -0.22 & 1.32 & -343.98 \\ -60.55 & 2.09 & 6.34 & -67.21 & -4.27 & 34.45 & -2.34 & -69.12 & -7.11 & 1.06 & -2.22 \end{bmatrix}.$$

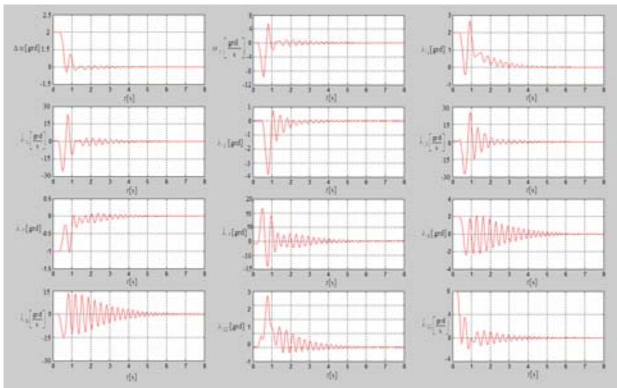

Fig.3

4 Appendix

```
close all;
a=[-0.026 0.025 -0.1 0;
   -0.36 -3 0 1;
    0 0 0 1;
    0.4212 -38.42 0 -3.67];
b=[0;0;0;1];
q=[10 0 0 0;
    0 10 0 0;
    0 0 100 0;
    0 0 0 1];
r=[2];
[k,p,e] = lqr(a,b,q,r);
i2=[1 0;0 1];
n3=randn(4,3);
contor=1;
t(:,1)=b(:,1);
for i=1:4
```

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```
   for j=1:3
      t(i,j+1)=n3(i,j);
   end
end
ab=(inv(t))*a*t;
bb=(inv(t)*b);
kb=place(ab,bb,e);
e=eig(ab-bb*kb);
k1=kb(1);
k21=kb(2);
k22=kb(3);
k23=kb(4);
r1=1;
rb=[r1];
r=rb;
pb=r1*[k1 k21 k22 k23];
k21 1 0 0;
k22 0 1 0;
k23 0 0 1];
ee=eig(rb);
qb=-(pb*ab+(transpose(ab))*pb-pb*bb*kb);
% Variant 2
ppp=transpose(inv(t))*pb*inv(t);
kkk=inv(r)*transpose(b)*ppp;
eee=eig(a-b*kkk);
m=rank(t);
while real(eee(1))>0 | real(eee(2))>0 | real(eee(3))>0
| real(eee(4))>0 | m<4
n3=randn(4,3);
contor=contor+1;
t(:,1)=b(:,1);
for i=1:4
   for j=1:3
      t(i,j+1)=n3(i,j);
   end
end
ab=(inv(t))*a*t;
bb=(inv(t)*b);
kb=place(ab,bb,e);
e=eig(ab-bb*kb);
k1=kb(1);
k21=kb(2);
k22=kb(3);
k23=kb(4);
r1=5;
rb=[r1];
r=rb;
pb=r1*[k1 k21 k22 k23;
        k21 1 0 0;
        k22 0 1 0;
        k23 0 0 1];
ee=eig(rb);
qb=-(pb*ab+(transpose(ab))*pb-pb*bb*kb);
% Variant 2
ppp=transpose(inv(t))*pb*inv(t);
kkk=inv(r)*transpose(b)*ppp;
eee=eig(a-b*kkk);
m=rank(t);
end
% end while
contor
% Variant 1
q=transpose(inv(t))*qb*inv(t);
r=rb;
[kk,pp,ee] = lqr(a,b,q,r);
k=kk;
sim('schprog1');
subplot(321);
plot(t,var1);
grid;
hold on;
subplot(322);
plot(t,var2);
grid;
hold on;
subplot(323);
plot(t,var3);
grid;
hold on;
subplot(324);
plot(t,var4);
grid;
hold on;
subplot(325);
plot(t,u);
grid;
hold on;
k=kkk;
sim('schprog1');
subplot(321);
plot(t,var1,'r');
subplot(322);
plot(t,var2,'r');
subplot(323);
plot(t,var3,'r');
subplot(324);
plot(t,var4,'r');
subplot(325);
plot(t,u,'r');
```

5 Conclusion

The paper presents a new algorithm for optimal control law's synthesis in rapport with state vector of the aircraft's longitudinal move. The presented algorithm (ALGLX) is illustrated for models of aircrafts' longitudinal move, no dimensional description (elastic no deformable) and dimensional description (elastic deformable).

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Optimal Commnad of Aircrafts' Move Using a Reduced Order Observer

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Abstract: - In this paper one presents an algorithm for project of a reduced order observer (ALGLOOR) with applications to aircraft's longitudinal and lateral move. The command law is an optimal one in rapport with state vector of the observer. Numeric calculus examples are also presented; using Matlab/Simulink model of closed loop system one has made numeric calculus program and with it one obtained different time characteristics, which expresses state variables' dynamics of the aircraft dynamic models.

Key-Words: algorithm, optimal, control law, observer.

1 Dynamic models of aircraft and reduced order observer

Aircraft dynamic may be described by equations

$$\dot{x} = Ax + Bu + Eu_p, \quad (1)$$

$$y = Cx, \quad (2)$$

where x is the state vector $(n \times 1)$ of aircraft model, $y-$ output vector $(p \times 1)$, $u-$ the vector containing knowable inputs $(m_1 \times 1)$, u_p- vector $(m_2 \times 1)$ of unknown inputs (disturbances or nonlinear functions of non modeled systems),

$A-$ matrix $(n \times n)$, $B-$ matrix $(n \times m_1)$, $E-$ matrix $(n \times m_2)$, $C-$ matrix $(p \times n)$; matrices A, B, C are known.

Let's consider the reduced order observer described by equations [1]

$$M\dot{z} = Fz + Gu + Hy, \quad (3)$$

$$\hat{x} = Pz + Qy, \quad (4)$$

with

$$z(r \times 1), \hat{x}(n \times 1), M(r \times r), F(r \times r), \det M = 0$$