

Prediction State Observer for the Longitudinal and Lateral Movements of an Aircraft

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Abstract: - This paper presents an algorithm for identification of the longitudinal and lateral movements of an aircraft. For identification a prediction state observer (Luenberger observer) has been projected. To perform the identification, the authors use the Matlab program from Appendix and an algorithm for the feedback gain matrix obtaining (ALGLX algorithm). With the obtained state observer a stabilization compensator has been made. The characteristics that express time variations of the state variables and indicial responses prove that the identification of the longitudinal and lateral movement of an aircraft has been made with small errors in good conditions. The presented Matlab program may be used with good results for identification and control of any system.

Key-Words: observer, algorithm, prediction, stabilization, compensator.

1 Introduction

To use a control law one must know the state variables. From the practical point of view, because of high prices of the transducers and interfaces, the measurement of all state variables isn't advisable. The state observers are the solution to this problem; they estimate the process' states using only the input and the output vectors of the process [1], [2], [3].

The use of a state observer leads to important savings, especially in the case of a state vector with n components. Instead of n transducers and n

measurement interfaces only one transducer and one interface will be used.

For estimate the states of an observer the system must be observable. That means that the rank of the matrix Q

$$Q^T = [C \ C \cdot A \ \dots \ C \cdot A^{n-1}] \quad (1)$$

must be equal with the lines' number of matrix A from the state general equations that describes a system

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx + Du. \end{aligned} \quad (2)$$

For doing this in Matlab one uses the instruction *obsv* with the following syntax

$$\mathbf{Q}=\text{obsv}(\mathbf{A},\mathbf{C}); \mathbf{r}=\text{rank}(\mathbf{Q});$$

The controllability and observability of a system are dual properties. That means that the pair (A, B) is controllable if pair (B^T, A^T) is observable. The pair (C, A) is observable if pair (A^T, C^T) is controllable.

2 The project of a prediction state observer

The equation that describes the Luenberger (prediction) state observer is [4]

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}), \quad (3)$$

where \hat{x} is the estimated state vector and L the gain matrix of the observer. This matrix has the same lines' number as matrix A and the same columns' number as matrix B .

Taking into account

$$\begin{aligned} y &= C \cdot x + D \cdot u, \\ \hat{y} &= C \cdot \hat{x} + D \cdot u, \end{aligned} \quad (4)$$

the first equation (2) leads to

$$\dot{e} = A \cdot e - L \cdot C \cdot e = (A - L \cdot C) \cdot e = \tilde{A} \cdot e, \quad (5)$$

where e is the estimation error of the prediction observer. Gain matrix of the observer (L) is calculated by assessing the eigenvalues for matrix $\tilde{A} = A - LC$ (the eigenvalues are contained by the column vector P) by the mean of command *acker* (Ackermann algorithm) or *place*

$$\begin{aligned} \mathbf{L} &= \text{acker}(\mathbf{A}', \mathbf{C}', \mathbf{P}); \mathbf{L} = \mathbf{L}'; \\ \mathbf{L} &= \text{place}(\mathbf{A}', \mathbf{C}', \mathbf{P}); \mathbf{L} = \mathbf{L}'; \end{aligned}$$

The condition that must be respected is the controllability of the pair (A^T, C^T) or the observability of the pair (C, A) .

For stabilization of the system described by equation (2) one uses the control law

$$u = -K\hat{x}; \quad (6)$$

matrix K is the gain matrix of the system. The calculus of this matrix is made using instructions *acker* and *place*; instead of matrix C matrix B is now used. This is another case when some eigenvalues must be asset for the matrix $A - BK$.

The obtaining of matrix K is made, both for longitudinal and lateral movements of the aircraft, using the authors' algorithm (ALGLX algorithm) [5]. Between the eigenvalues of the observer and the eigenvalues of matrix $A - BK$ there is the following relationship

$$|\text{Re}\{\lambda_{\min}\}_{\text{observer}}| > 5 \div 10 |\text{Re}\{\lambda_{\max}\}_{\text{system}}|. \quad (7)$$

That means that the eigenvalues of the observer are chosen such that their minimum real part is $5 \div 10$

times bigger than maximum real part of the system's eigenvalues.

The ensemble formed by the command law (matrix K) and the observer is called stabilization compensator (fig.1).

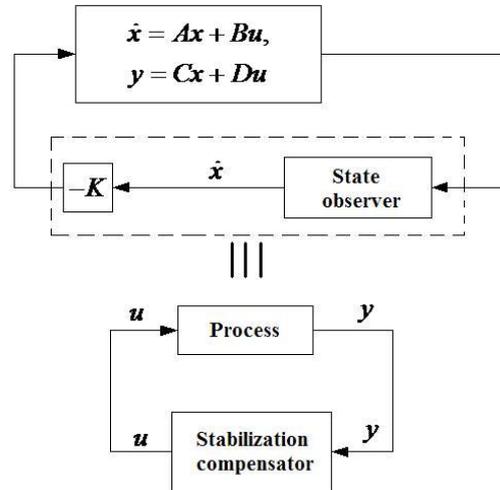


Fig.1. Block diagram of the stabilization compensator with prediction observer

For obtaining the equation of the stabilization compensator one uses equations (3) and (6); it results

$$\dot{\hat{x}} = (A - BK - LC)\hat{x} + LCx = \bar{A}\hat{x} + Ly \quad (8)$$

where

$$\bar{A} = A - BK - LC. \quad (9)$$

The state equations that describe the stabilization compensator are

$$\begin{aligned} \dot{\hat{x}} &= \bar{A}\hat{x} + Ly, \\ u &= -K\hat{x}. \end{aligned} \quad (10)$$

The input of the compensator is the output of the process (y) and the output of the process is the command vector (u).

The transfer function of the compensator is

$$H(s) = -K^T (sI - \bar{A})^{-1} L; \quad (11)$$

I is the identity matrix of order n .

From the stabilization compensator's equation (8) and the first state equation (2) one obtains

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} &= \begin{bmatrix} A & -BK \\ LC & \bar{A} \end{bmatrix} \cdot \begin{bmatrix} x \\ \hat{x} \end{bmatrix} \Leftrightarrow \\ \begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} &= \begin{bmatrix} A & -BK \\ LC & A - BK - LC \end{bmatrix} \cdot \begin{bmatrix} x \\ \hat{x} \end{bmatrix} = A_R \begin{bmatrix} x \\ \hat{x} \end{bmatrix}, \end{aligned} \quad (12)$$

where

$$A_R = \begin{bmatrix} A & -BK \\ LC & A - BK - LC \end{bmatrix}. \quad (13)$$

The eigenvalues of the compensator are the eigenvalues of the matrix A_R - the solutions of the characteristic equation

$$|\lambda I - A_R| = 0 \tag{14}$$

equivalent with

$$\begin{vmatrix} \lambda I - A & BK \\ -LC & \lambda I - A + BK + LC \end{vmatrix} = 0. \tag{15}$$

In the above determinant, one adds column 2 to column 1 and obtains

$$\begin{vmatrix} \lambda I - A + BK & BK \\ \lambda I - A + BK & \lambda I - A + BK + LC \end{vmatrix} = 0 \tag{16}$$

$$\stackrel{|2-1|}{\Rightarrow} \begin{vmatrix} \lambda I - A + BK & BK \\ 0 & \lambda I - A + LC \end{vmatrix} = 0$$

or

$$[\lambda I - (A - BK)] \cdot [\lambda I - (A - LC)] = 0. \tag{17}$$

So, the eigenvalues of the compensator represents the reunion between the eigenvalues associated to the command law and the eigenvalues of the observer.

3 Stabilization compensator with prediction observer for the longitudinal movement of an aircraft

Lets' consider the case of the non-dimensional longitudinal movement of an aircraft [5]

$$\begin{bmatrix} \dot{\hat{V}} \\ \dot{\hat{\alpha}} \\ \dot{\hat{\theta}} \\ \dot{\hat{\omega}}_y \end{bmatrix} = \begin{bmatrix} -0.026 & 0.025 & -0.1 & 0 \\ -0.36 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.4212 & -38.49 & 0 & -3.67 \end{bmatrix} \begin{bmatrix} \hat{V} \\ \hat{\alpha} \\ \hat{\theta} \\ \hat{\omega}_y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \delta_p, \tag{18}$$

where

$$\hat{V} = \frac{\Delta V}{V^*}, \hat{t} = \frac{t}{\tau_a}, \hat{\omega}_y = \frac{\bar{b}}{V^*} \omega_y, \hat{\theta} = \Delta \theta, \hat{\alpha} = \Delta \alpha; \tag{19}$$

$\tau_a = 2,1s$ – the aerodynamic time constant, ΔV – the variation of the flight velocity, θ – the pitch angle, α – the attack angle, V^* – rectilinear uniform flight's velocity, δ_p – the rudder deflection.

The system is considered to be a system with one input (δ_p) and one output ($\hat{\omega}_y$).

$$C = [0 \ 0 \ 0 \ 1]; D = [0]. \tag{20}$$

Because the eigenvalues of the matrix $A - BK$ aren't known, the authors use the ALGLX algorithm [5] in order to obtain the gain matrix K . After this purpose is achieved, using the eigenvalues of matrix $A - BK$ and equation (7), the eigenvalues of matrix $A - LC$ are obtained (the eigenvalues of the observer).

One obtains

$$K = [0.547 \ -5.020 \ 6.897 \ 0.539]. \tag{21}$$

The Matlab program is presented in Appendix. For the system with stabilization compensator and prediction observer one obtained the gain matrix of

the observer (L_{pred}) the direct transfer function ($H_d(s)$), the transfer function of the feedback (the transfer function of the stabilization compensator - $H_{cp}(s)$), the transfer function of the closed loop system (positive feedback) - $H_{pred}(s)$, the poles of the closed loop system - pol_{pred} and the zeros of this system - $zero_{pred}$.

$$L_{pred} = [47.9578 \ -6.3245 \ -189.6605 \ 29.48],$$

$$H_d(s) = \frac{s^3 + 3.026s^2 + 0.087s + 1.332 \cdot 10^{-15}}{s^4 + 6.696s^3 + 49.68s^2 + 1.352s + 1.512},$$

$$H_{cp}(s) = \frac{785.4s^3 + 5204s^2 + 3.86 \cdot 10^{-4}s - 1764}{s^3 + 820.5s^2 + 2686s + 205.2},$$

$$H_{pred}(s) = \frac{s^6 + 823.5s^5 + 5169s^4 + 8404s^3 + 845.6s^2 + 17.85s + 2.73 \cdot 10^{-13}}{s^7 + 41.77s^6 + 649.9s^5 + 4537s^4 + 2.04 \cdot 10^4s^3 + 1.7 \cdot 10^4s^2 + 4492s + 310.3},$$

$$pol_{pred} = -16.9992 + 6.2155i; -16.9992 - 6.2155i; -3.3998 + 6.2155i;$$

$$-3.3998 - 6.2155i; -1.6414; -0.5361; -0.3283; -0.1072,$$

$$zero_{pred} = -16.9005 + 35.6617i; -16.9005 - 35.6617i; -2.997;$$

$$-2.837; -0.0762; -0.029; 0.0.$$

The block diagram of the stabilization compensator with prediction observer is the one from fig.2 (the subsystems are in fig.3).

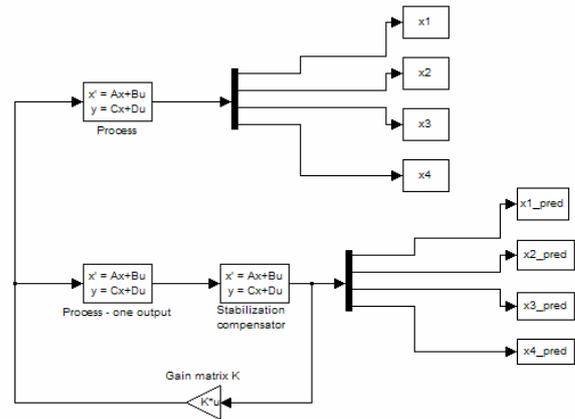


Fig.2. Block diagram of the stabilization compensator with prediction observer

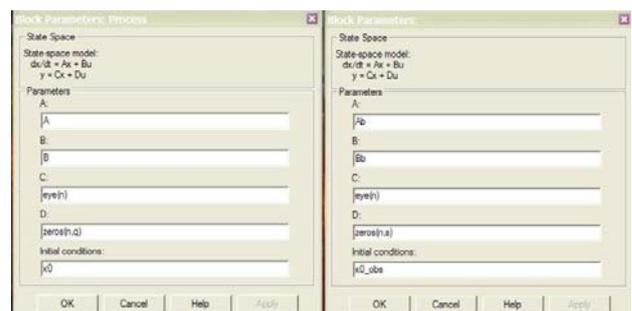


Fig.3. The subsystems of system from fig.2

Using the Matlab program from Appendix and the model Matlab/Simulink from fig.2 one obtains the indicial response of the closed loop system (fig.4), time variations of the components of the state vectors x and \hat{x} (x with blue color and \hat{x} with red color) (fig.5) and time variations of the four components of error vector $e = x - \hat{x}$ (fig.6).

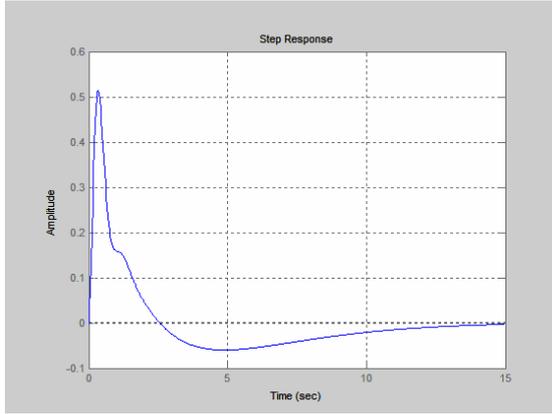


Fig.4. The indicial response of the closed loop system (longitudinal movement)

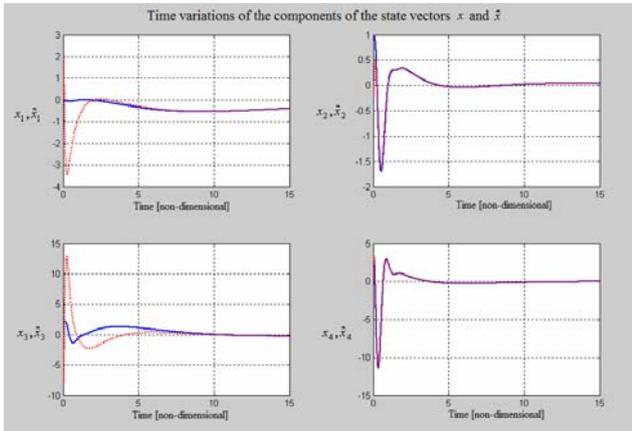


Fig.5. Time variations of the components of the state vectors x and \hat{x} (longitudinal movement)

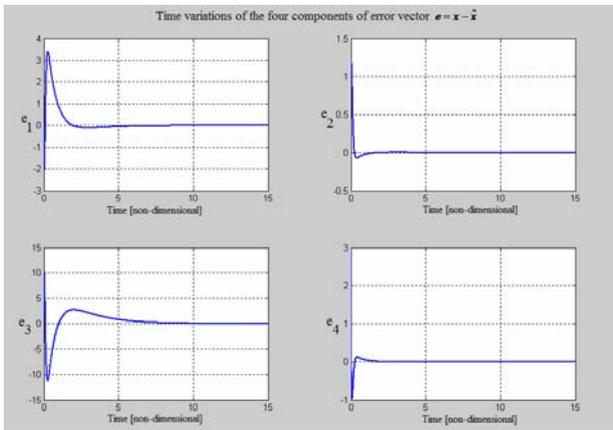


Fig.6. Time variations of the four components of error vector $e = x - \hat{x}$ (longitudinal movement)

4 Stabilization compensator with prediction observer for the lateral movement of an aircraft

Let's consider now the case of the lateral movement of a fight aircraft which flies with $M=1.5$, at $H=10000$ ft, with values from [6]; the state vector and the input vector are

$$x^T = [\Delta\beta \quad \Delta\omega_x \quad \Delta\omega_z \quad \Delta\varphi], u^T = [\delta_f \quad \delta_d]; \quad (22)$$

β is the sideslip angle, ω_x – the roll angular velocity, ω_z – the yaw angular velocity, φ – the roll angle, δ_f – the flaps deflection, δ_d – the direction deflection. The matrices from state equations (2) are

$$A = \begin{bmatrix} -0.493 & 0.015 & -1 & 0.02 \\ -61.176 & -7.835 & 4.991 & 0 \\ 31.804 & -0.235 & -0.994 & 0 \\ 0 & 1 & -0.015 & 0 \end{bmatrix}, B = \begin{bmatrix} -0.002 & 0.002 \\ 8.246 & 1.849 \\ 0.249 & -0.436 \\ 0 & 0 \end{bmatrix}, \quad (23)$$

$$C = [0 \ 0 \ 0 \ 1], D = [0 \ 0].$$

This time the system is a system with two inputs (the deflection of the flaps and the deflection of the direction) and one output ($\Delta\varphi$). As a consequence will be obtained two transfer functions (the first associated to the input δ_f and output $\Delta\varphi$ and the second associated to the input δ_d and the output $\Delta\varphi$).

The methodology is the same with the one presented above for the longitudinal movement of the aircraft. With the ALGLX algorithm one obtains the gain matrix K

$$K = \begin{bmatrix} 8.209 & 0.967 & 0.658 & 4.223 \\ -7.844 & -0.183 & -7.952 & -2.619 \end{bmatrix}.$$

A similar Matlab program to the one presented in Appendix leads to the obtaining of the gain matrix of the Luenberger observer (L_pred), of the poles of the two closed loop transfer functions (pol_pred1 pol_pred2), and of the two closed loop transfer functions ($H_pred1(s) = \Delta\varphi(s)/\delta_f(s), H_pred2(s) = \Delta\varphi(s)/\delta_d(s)$).

$$L_pred = 10^3 \cdot [-0.3418 \ 2.1219 \ 1.5711 \ 0.0935]^T;$$

$$pol_pred1 = -71.9509; -16.5483 + 24.2263i; -16.5483 - 24.2263i$$

$$-13.2191; -0.7656 + 5.8092i; -0.7656 - 5.8092i;$$

$$-1.7782 + 0.5769i; -1.7782 - 0.5769i;$$

$$pol_pred2 = -62.2109; -24.3838 + 10.9214i; -24.3838 - 10.9214i;$$

$$-3.5340 + 8.6595i; -3.5340 - 8.6595i;$$

$$-2.2605 + 4.5679i; -2.2605 - 4.5679i; -0.7868;$$

$$H_{pred1}(s) = \frac{-8.88 \cdot 10^{-15} s^7 + 8.24 s^6 + 953 s^5 + 3.61 \cdot 10^4 s^4 + 7.78 \cdot 10^3 s^3 + 5.19 \cdot 10^6 s^2 + 2.83 \cdot 10^7 s + 9.83 \cdot 10^7}{s^8 + 123.4 s^7 + 5276 s^6 + 1.33 \cdot 10^5 s^5 + 1.56 \cdot 10^6 s^4 + 9.3 \cdot 10^6 s^3 + 4.93 \cdot 10^7 s^2 + 1.16 \cdot 10^8 s + 9.82 \cdot 10^7}$$

$$H_{pred2}(s) = \frac{1.85 s^8 + 212.1 s^7 + 7822 s^6 + 1.61 \cdot 10^5 s^5 + 8.24 \cdot 10^5 s^4 + 2.7 \cdot 10^6 s^3 + 1.12 \cdot 10^7}{s^8 + 123.4 s^7 + 5276 s^6 + 1.08 \cdot 10^5 s^5 + 1.2 \cdot 10^6 s^4 + 9.76 \cdot 10^6 s^3 + 4.12 \cdot 10^7 s^2 + 1.27 \cdot 10^8 s + 7.93 \cdot 10^7}$$

To obtain indicial responses (fig.7 and fig.8), time variations of the components of the state vectors x and \hat{x} (x with blue color and \hat{x} with red color) (fig.9) and time variations of the four components of error vector $e = x - \hat{x}$ (fig.10) the diagram block from fig.2 is used.

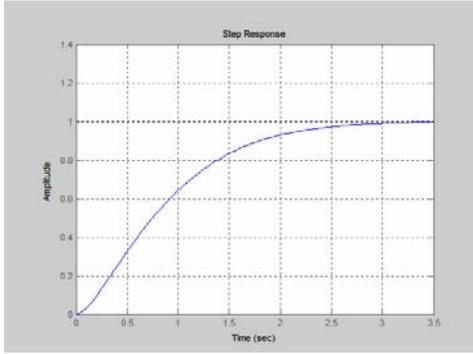


Fig.7. Indicial response of the system (input 1) for the lateral movement

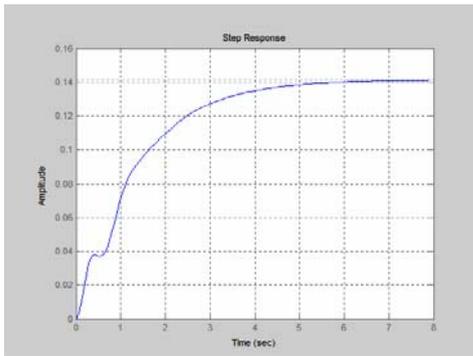


Fig.8. Indicial response of the system (input 2) for the lateral movement

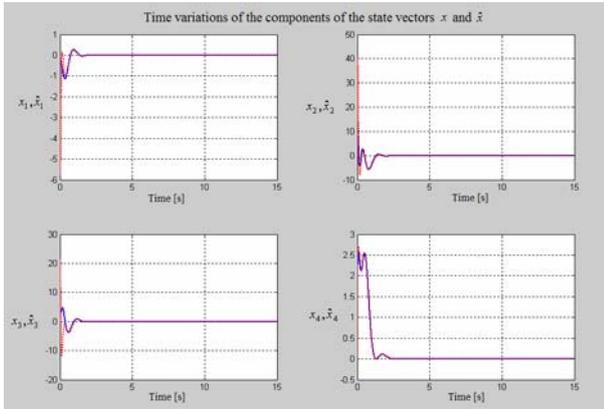


Fig.9. Time variations of the components of the state vectors x and \hat{x} (lateral movement)

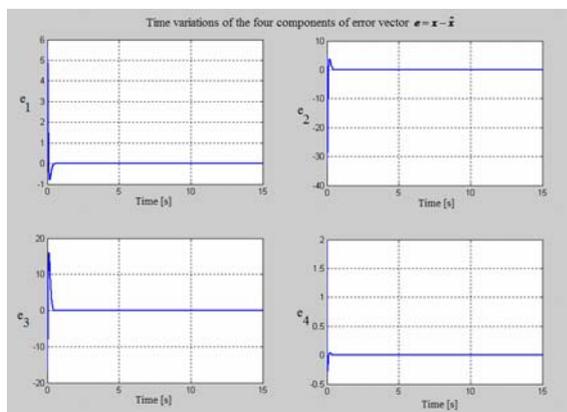


Fig.10. Time variations of the four components of error vector $e = x - \hat{x}$ (lateral movement)

5 Appendix

```

close all; clear all;
A=[-0.026 0.025 -0.1 0;-0.36 -3 0 1;0 0 0 1;
    0.4212 -38.49 0 -3.67]; B=[0;0;0;1];
Q=[10 0 0 0;0 10 0 0;0 0 100 0;0 0 0 1];R=[2];
[K,P,E] = LQR(A,B,Q,R);I2=[1 0;0 1];
N3=randn(4,3); T(:,1)=B(:,1);
for i=1:4
    for j=1:3
        T(i,j+1)=N3(i,j);
    end
end
Ab=(inv(T))*A*T; Bb=(inv(T)*B);
Kb=place(Ab,Bb,E);
e=eig(Ab-Bb*Kb);
k1=Kb(1);k21=Kb(2);k22=Kb(3);k23=Kb(4);
r1=1;Rb=[r1];R=Rb;
Pb=r1*[k1 k21 k22 k23;k21 1 0 0;
        k22 0 1 0; k23 0 0 1];
ee=eig(Rb);
Qb=-((Pb*Ab+(transpose(Ab))*Pb-Pb*Bb*Kb));
PPP=transpose(inv(T))*Pb*inv(T);
KKK=inv(R)*transpose(B)*PPP;
EEE=eig(A-B*KKK);m=rank(T);
while real(EEE(1))>0 | real(EEE(2))>0 |
    real(EEE(3))>0 | real(EEE(4))>0 | m<4
    N3=randn(4,3);T(:,1)=B(:,1);
    for i=1:4
        for j=1:3
            T(i,j+1)=N3(i,j);
        end
    end
    Ab=(inv(T))*A*T;Bb=(inv(T)*B);
    Kb=place(Ab,Bb,E);
    e=eig(Ab-Bb*Kb);
    k1=Kb(1);k21=Kb(2);k22=Kb(3);k23=Kb(4);
    r1=5;Rb=[r1];R=Rb;
    Pb=r1*[k1 k21 k22 k23;k21 1 0 0;
            k22 0 1 0; k23 0 0 1];
    ee=eig(Rb);
    Qb=-((Pb*Ab+(transpose(Ab))*Pb-Pb*Bb*Kb));
    PPP=transpose(inv(T))*Pb*inv(T);
    KKK=inv(R)*transpose(B)*PPP;
    EEE=eig(A-B*KKK);m=rank(T);
end
Q=transpose(inv(T))*Qb*inv(T);R=Rb;
[KK,PP,EE] = LQR(A,B,Q,R);
K=KK;
% Longitudinal movement n=4
clear Ab; clear Bb; clear i;
% System's matrices
n=size(A,1); q=size(B,2);
C=[0 0 0 1]; s=size(C,1);
D=zeros(s,q);
x0=[0;1;2;3]; % Initial system's state
    
```

```

% Process transfer function
[num_d,den_d]=ss2tf(A,B,C,D);
% Eigenvalues of matrix(A-B*K)
pp=eig(A-B*K);
% Eigenvalues of the prediction observer
for j=1:length(pp)
R(j)=real(pp(j));
I(j)=imag(pp(j));
RR=R*5; II=I;
ee(j)=RR(j)+II(j)*i;
qq=ee';
end
OB=obsv(A,C);CO=ctrb(A,B);
r1=rank(OB);r2=rank(CO);
% Matrix L_pred determination
L_pred=place(A',C',qq);
L_pred=L_pred';
% State equations of the stabilization compensator
Ab=A-B*K-L_pred*C;
Bb=L_pred;
Cb=-K;
Db=D;
% Initial state of the observer
x0_obs=zeros(size(x0));
% Transfer function of the compensator
[num_cp,den_cp]=ss2tf(Ab,Bb,Cb,Db);
% Closed loop transfer function
[num_pred,den_pred]=feedback(num_d,den_d,
num_cp,den_cp,+1);
pol_pred=roots(den_pred)
zero_pred=roots(num_pred);
sys_d=tf(num_d,den_d)
sys_cp=tf(num_cp,den_cp)
sys_pred=tf(num_pred,den_pred)
% State variables time variations
sim('Comp_obs_pred_long');
e1=x1-x1_pred;e2=x2-x2_pred;
e3=x3-x3_pred;e4=x4-x4_pred;
t=linspace(0,15,length(x1));
step(sys_pred);grid;h=figure;
subplot(221);plot(t,x1,t,x1_pred,':r');
grid; xlabel('Time [adim]');
subplot(222);plot(t,x2,t,x2_pred,':r');
grid; xlabel('Time [adim]');
subplot(223);plot(t,x3,t,x3_pred,':r');
grid; xlabel('Time [adim]');
subplot(224);plot(t,x4,t,x4_pred,':r');
grid; xlabel('Time [adim]');
h=figure;
subplot(221);plot(t,e1);grid;xlabel('Time [adim]');
subplot(222);plot(t,e2);grid;xlabel('Time [adim]');
subplot(223);plot(t,e3);grid;xlabel('Time [adim]');
subplot(224);plot(t,e4);grid;xlabel('Time [adim]');

```

6 Conclusion

For identification of the longitudinal and lateral movement of an aircraft, a prediction state observer (Luenberger observer) has been projected. In Appendix is presented a Matlab program for the longitudinal movement of the aircraft. The obtained results show that the algorithm may be used with good results to any system's identification. All the variables tend to zero; this is very good because these variables are preceded by the symbol Δ (the variables are, in fact, variations of some angles, angular velocities and so on). To perform the identification, the authors has used the algorithm for the feedback gain matrix obtaining (ALGLX) [5].

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