Analytic Determination of the Accelerations of Vibrations for a Linear Viscoelastic Cinematic Element of a Crank and Connecting Rod Assembly Validated by Experiment

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Abstract: This work presents the analytic determination of vibrations effective accelerations for a cinematic element of a crank and connecting rod assembly and the experiment that validated it. The movement equations of the linear viscoelastic straight cinematic elements in plan-parallel motion are presented by using Hamilton's variation principle. In order to obtain the dynamic response, the Laplace integral transforms and the finite Fourier transforms are applied. Finally, for a crank and connecting rod assembly R (RRT) in concrete work conditions, a comparison between the experimental results and the analytic determination of the vibrations transversal accelerations is given.

Key-words: viscoelastic, vibrations, plan-parallel motion, vibration transversal acceleration.

1 Introduction

The tendency of replacing of metallic materials – steel – with rheological materials with reduced inertial forces and couples because of their lower specific mass goes to developing of the research in rheological domain. Generalizing the Hooke elasticity theory and the viscous fluid dynamic Newton theory it was developed the mechanical model that represents two models: one is perfect elastic model and the other one is represented by viscous fluid model. The variational principle of Hamilton can be used for the displacements equations in linear viscoelastic by replacing the Young's elasticity modulus, E, with its Laplace transform $\widetilde{E}(s)$.

For raising the accuracy of the work conditions for the cinematic elements of the mechanisms, the determination of the vibration of the elements of the assembly is very significant. In improving of the work conditions and increasing the productivity, using the linear viscoelastic materials have become ordinary.

2 The theoretical view and analytic determination of the displacements field for a viscoelastic cinematic element in plan parallel motion

By applying in the movement mathematical model of a linear elastic cinematic element bar type [1], the Laplace transform one-sided proportional to time and replacing the E modulus with $\widetilde{E}(s)$, the equation in matrix of the first approximation in Laplace images is obtained for the vibrations of the viscoelastic connecting rod of the R(RRT) mechanism, assimilated with the Maxwell model, as follows.

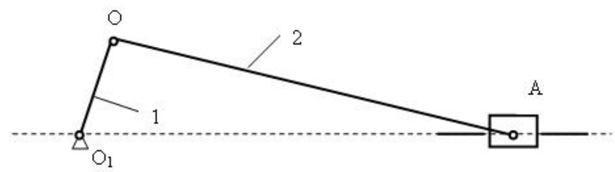

Fig. 1 - Crank and connecting rod mechanism

The solution in the „j"-th approximation will be [2]:

$$u_1^{(j)}(x,t) = \frac{1}{L}\cdot u_{1,c}^{(j)}(0,t) + \frac{2}{L}\sum_{n=1}^{n=\infty} u_{1,c}^{(j)}(n,t)\cdot \cos(\alpha_n \cdot x) \quad (1)$$

$$u_2^{(j)}(x,t) = \frac{2}{L}\sum_{n=1}^{n=\infty} u_{2,s}^{(j)}(n,t)\cdot \sin(\alpha_n \cdot x) \quad (2)$$

where: $u_{1,c}^{(j)}(n,t)$ and $u_{2,s}^{(j)}(n,t)$ are the finite Fourier transforms in cosine, respectively in sinus of the elastic longitudinal and transversal displacement.
The process of the successive approximation is considered complete when the difference between j and j-1 displacement becomes less than ε, where $\varepsilon > 0$ and low enough depending on the precision of the operation required.
And:

$$\{u^{(j)}\} = \{u_1^{(j)}(x,t), u_2^{(j)}(x,t)\}^T \quad (3)$$

$$\{u^{(j-1)}\} = \{u_1^{(j-1)}(x,t), u_2^{(j-1)}(x,t)\}^T \quad (4)$$

The connecting rod OA being double-articulated, the just on the line conditions which allowed the application of the two Fourier transforms, for initial functions and for their Laplace images, were the boundary conditions and initial conditions are given in [2].
The transversal displacements field, in the first approximation, in the case of free vibrations of the connecting rod OA belonging to the mechanism R(RRT) in figure 1, is represented by the equation (3),
where:

$$\{a_o\} = \left\{\omega_0^2 r\left[\frac{r}{L}\sin^2(\omega_0 t) - \cos(\omega_0 t)\right]; -\omega_0^2 r\left[\sin(\omega_0 t) + \frac{r}{2L}\sin(2\omega_0 t)\right]\right\}^T;$$

$$\omega = -\frac{r}{L}\omega_0\cos(\omega_0 t); \ \varepsilon = \frac{\omega_0^2 r}{L}\sin(\omega_0 t),$$

(5)

and will be supplied, as a result of the application of the preceding algorithm of integration, by the function (2) for j=1.

3 Numerical applications

For a real R(RRT) mechanism, a numerical application based on the experimental determination of the characteristics of the linear - viscoelastic material (polyvinyl chloride) was realised and studied.
The viscoelastic material characteristics were determined for the polyvinyl chloride in the Strength of Materials laboratory, Faculty of Mechanics from Craiova, an authorized laboratory for such determinations.
The Poisson's shear coefficient was considered invariable and equal to 0.3, even if it is a function of time, because of the difficulties in its experimental determination. This approximation doesn't affect considerably the obtained results.
The apparent modulus of elasticity E_{ap} was determined using the linear part of the load-amount of deflection curves. In order to determine the constant η afferent to the Newtonian component of the Maxwell model it was traced the creeping curve for the polyvinyl chloride. This constant is determined knowing the bending stress at maximum load σ_0 and the upgrade of the line which represents the stabilized creep zone [2].
The experimental determinations for the polyvinyl chloride were the following:

- the shear elasticity modulus: G = 1189.79[MPa];
- the bulk modulus: K = 2424.455[MPa];
- the constant η afferent to the Newtonian component of the Maxwell model:
  $\eta = 4.5*10^7$[MPah];
- the apparent modulus of elasticity $E_{ap} \cong E$ = 2909.347[MPa].

It is considered a numerical application of RRT mechanism, where the connecting rod is made of un-masticated polyvinyl chloride, as follows:
L = 1[m]; r = 0.07[m]; ω_0 = 31.41[s^{-1}]; b = 0.011[m]; h=0.011[m]; ρ=1213.3[kg/m^3].
The mathematical model (5) shows the influence of the cinematic parameters upon the transversal displacements and in the same time upon the accelerations of vibrations fields. These influences are very important in dimensioning and designing products.
Because the longitudinal displacement field has a negligible influence [1], it has continued the iterative/repeated process only for transversal displacements.
Using equation (6) and keeping the concrete data as above, the graphic representations are obtained using Mathematica program [3].
Figure 2 represents the longitudinal displacements in the first approximation $u_1 = u_1^{(1)}(x,t)$ and fig. 3 represents the transversal displacements in the first approximation $u_2 = u_2^{(1)}(x,t)$ for ω_0 = 16.568 rad/s.
The transversal displacement variation at x=L/4, $u_2 = u_2^{(1)}\left(\frac{L}{4}, t\right)$ is represented in Figure 4.

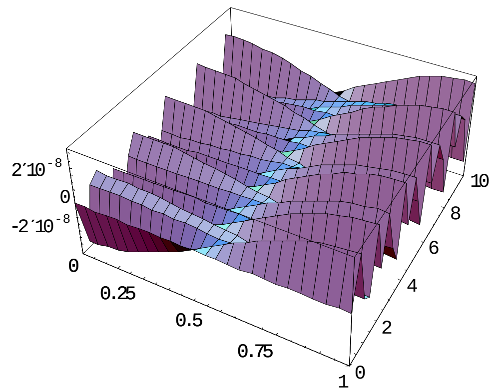

Fig. 2 The longitudinal displacement $u_1 = u_1^{(1)}(x,t)$

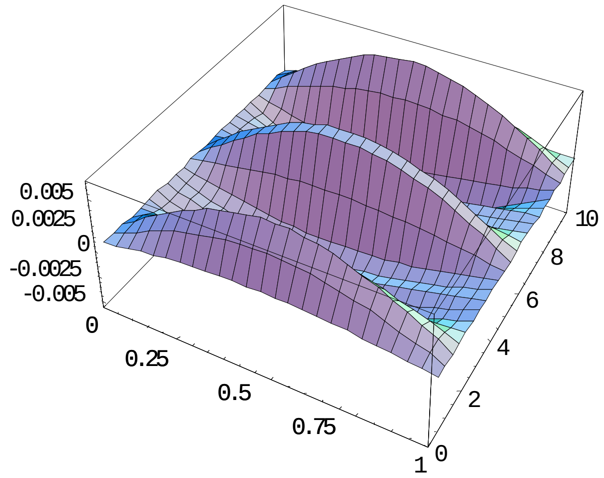

Fig. 3 The transversal displacement $u_2 = u_2^{(1)}(x,t)$

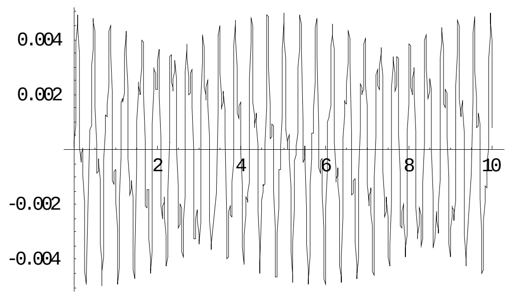

Fig.4 The transversal displacement variation at x=L/4: $u_2 = u_2^{(1)}\left(\frac{L}{4}, t\right)$

By derivation twice in relation to the time, vibrations transversal acceleration a_2 (L/4, t) is obtained. The diagram of its time variation in the point P considered above, placed at 1/4 length of the connecting rod from point A, the connecting rod end of action, at n = 158.2 rpm, is shown in Figure 5.

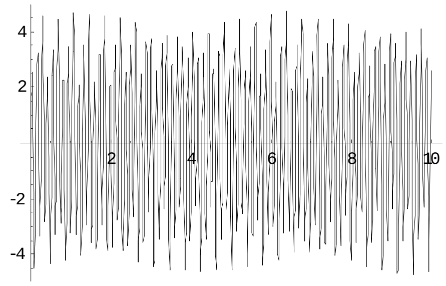

Fig. 5 Diagram of time variation of vibrations transversal acceleration at x=L/4: a_2 (L / 4, t)

Based on the calculation of vibrations acceleration performed using the program Mathematica and diagrams of time variation of the type presented above, there were determined the effective values of vibrations transversal accelerations for the entire range of speeds of experimental determinations, for the connecting rod made of un-masticated polyvinyl chloride[4].

These values are presented in Table 1 in order to be subsequently compared with those obtained experimentally.

Table1 Effective values of vibrations transversal accelerations for the connecting rod made of un-masticated polyvinyl chloride

No.	Material/Place of measurement/Frequency (Hz)	Effective values of vibrations transversal accelerations, $a_2(x,t)(m/s^2)$
1.	PVC/1_4/1.685	2.06087
2.	PVC/1_4/2.637	3.32693
3.	PVC/1_4/3.296	5.21912
4.	PVC/1_4/3.955	7.7047
5.	PVC/1_4/4.321	9.9738
6.	PVC/1_4/4.468	11.304
7.	PVC/1_2/1.685	1.6898
8.	PVC/1_2/2.344	1.96056
9.	PVC/1_2/2.930	2.746
10.	PVC/1_2/3.589	5.78867
11.	PVC/1_2/4.028	8.9791
12.	PVC/1_2/4.175	10.4491
13.	PVC/1_2/4.321	11.8914

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14.	PVC/1_2/4.688	15.1275
15.	PVC/3_4/1.831	1.6813
16.	PVC/3_4/2.710	2.1402
17.	PVC/3_4/3.662	2.5691
18.	PVC/3_4/4.028	2.8801
19.	PVC/3_4/4.395	3.297
20.	PVC/3_4/4.761	4.2338

In Figure 6 there are represented the vibrations transversal acceleration variations depending on the frequency of rotation of the leader element, for the three particular considered positions of point P (AP = L / 4, AP = L / 2, AP = 3L / 4) on the connecting rod made of un-masticated polyvinyl chloride [5].

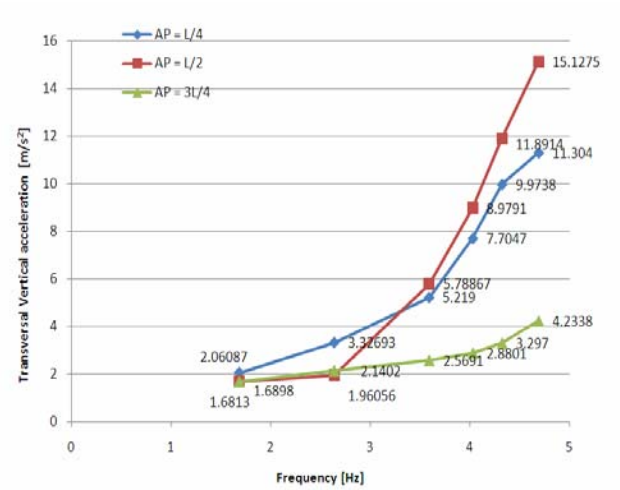

Fig. 6 Variations of the vibrations transversal acceleration depending on the frequency of rotation of the leader element

It can be noticed that the computed (theoretical) values of the vibrations acceleration on the vertical direction increases with an increasing frequency or speed of action, and the highest values are recorded in the point located at L/4 from the end A for low speed and in the middle of the connecting rod for greater speed.

In the experiment, measurements were made at different frequencies of action in the range 1.5 ... 5 Hz, the acceleration response is determined at the distances of L/4, L/2 and 3L/4 from the end of action, where L is the length of the connecting rod.

Thus, theoretical values of the vibrations transversal acceleration obtained after solving the mathematical model were compared with those obtained experimentally; errors are recorded as 9.3%.

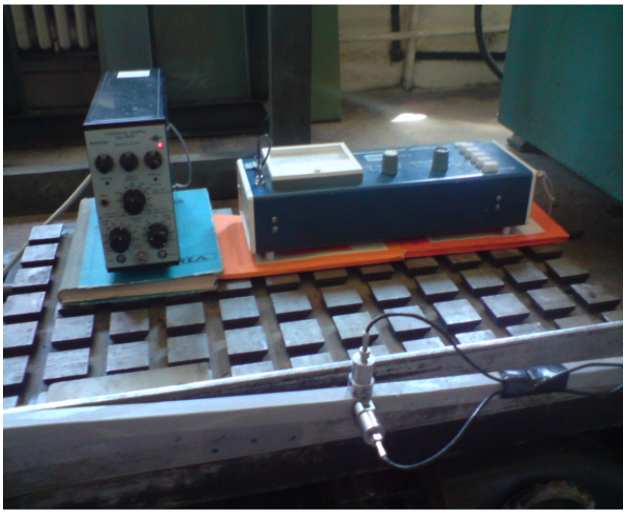

Fig. 7. Experimental assembly

The experimental determination of the acceleration of vibration for the connecting rod element, made of un-masticated polyvinyl chloride, is represented in Fig. 9 and the spectral analysis is given by Fig. 8.

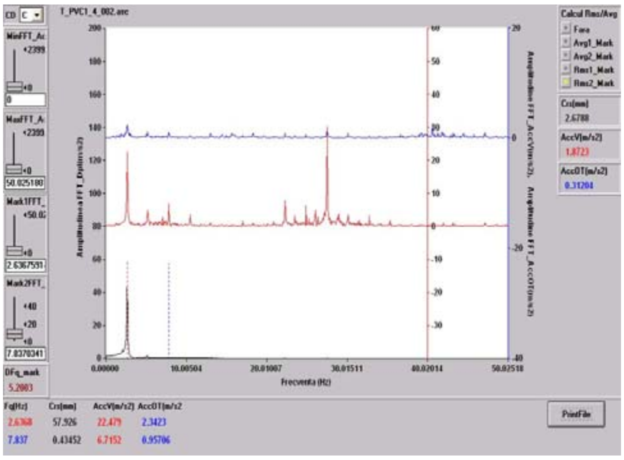

Fig.8 The spectral analysis at x=L/4. Frequency = 2.637 Hz

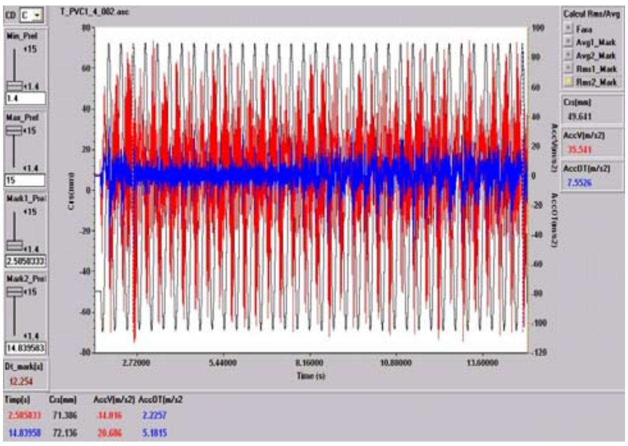

Fig.9 The acceleration of vibration at x=L/4, Frequency = 2.637 Hz

Spectral analysis showed the following:
1. The spectral components of superior frequency are reduced in the point A in the case of the connecting rod made of un-masticated polyvinyl chloride; there we notice the fundamental component and the harmonica order 1.
2. When the measuring point is placed in the middle of the connecting rod, spectral components of high frequency are reduced and, besides the fundamental component, it can be noticed the harmonica order 1 with priority. Oscillations on the transverse-horizontal direction are of much smaller amplitude.

For the modal analysis of the mechanism, the elements composing the crank and connecting rod mechanism were modeled first in SolidWorks design environment.

Then, after they were assembled and it was simulated the movement of the mechanism, it was performed the modal analysis using the module "visualNASTRAN INSIDE SolidWorks, an application for numerical modeling with finite element, used in integrated designing.

The "INSIDE visualNASTRAN" module associated to SolidWorks Program allows the use of finite element analysis with the latest capabilities of MSC / NASTRAN in order to automatically simulate the behavior of parts and assemblies modeled in SolidWorks.

In Figure 10 it is represented the mechanism having meshed connecting rod in order to realize the modal analysis.

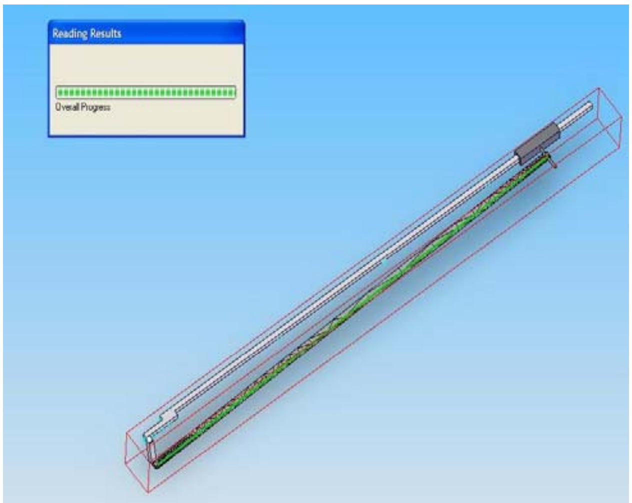

Fig. 10 The mechanism having meshed connecting rod

The results of the modal analysis are presented in Figure 11 for the frequency of rotation of the leader element f = 4.321 Hz.

It can be noticed that the characteristic frequencies are quite the same found by the spectral analysis. The values of rotations and displacements are given and the extreme values for the averaged stress computed by Von Mises theory.

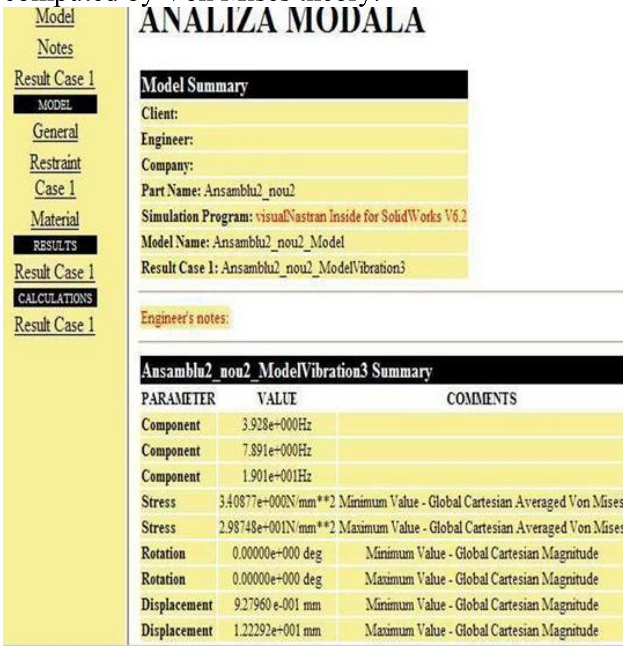

Fig. 11 The results of the modal analysis

4 Conclusions

The equations (4) and the graphical representations show the significant influence of the cinematic parameters of the mechanism movement in the maximum values of vibrations displacements and, as a consequence, in the maximum values of the stress and strain components, which represents important data in machine designing and dimensioning.

For a linear viscoelastic connecting rod, the computed (theoretical) values of the vibration acceleration on the vertical direction increases with an increasing speed or action frequency. The experiment, by measuring the values of the transversal vibrations acceleration, validates the computed (theoretical) values of vibrations displacements fields.

So, this paper is useful in the machine designing because the calculus of the transversal displacements field is very important in the determination of the influence of the vibrations and of the stress and strain state of the cinematic element.

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