Comparison between Vibrations Transversal Displacements Analytic Determined for a Linear Elastic Connecting Rod and a Linear Viscoelastic one (After the Validation by Experiment of the Vibrations Effective Accelerations)

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Abstract: This work presents the comparison between vibrations transversal displacements analytic determined for a cinematic element of a crank and connecting rod assembly with linear elastic behaviour comportment and one with linear viscoelastic behaviour, considered in free vibrations and after the corresponding effective accelerations of vibrations were validated by experiment. The movement equations of the straight cinematic elements in plan-parallel motion are obtained by using the Hamilton's variational principle. In order to obtain the dynamic response, the Laplace integral transforms and the finite Fourier transforms are applied.

Key- words: viscoelastic, vibrations, plan-parallel motion, vibrations transversal displacements.

1 Introduction

In order to compare the vibrations transversal displacements for a connecting rod with linear elastic comportment and the ones of a connecting rod with linear viscoelastic comportment, there were determined the vibrations transversal displacements for three different frequencies of the leader element rotation, for both connecting rods [1].

The transversal displacements field is computed with the Mathematica program, using the linear properties of the material, starting from the mathematical model (1) and using the iterative method [4].

$$[L_0]\{u\}+[M_4]\{a_0\}+\{V_1\}+[M_7]\{f\}+\{V_2\}=\{0\}, \quad (1)$$

The displacements fields for free vibrations of a linear elastic cinematic element in plan parallel movement, in a first approximation are given by the following equations:

$$u_1^{(1)}(x,t)=\frac{1}{L}\cdot u_{1,c}^{(1)}(0,t)+\frac{2}{L}\sum_{n=1}^{n=\infty}u_{1,c}^{(1)}(n,t)\cdot\cos(\alpha_n\cdot x) \quad (2)$$

$$u_2^{(1)}(x,t)=\frac{2}{L}\sum_{n=1}^{n=\infty}u_{2,s}^{(1)}(n,t)\cdot\sin(\alpha_n\cdot x), \quad (3)$$

where $u_{1,c}^{(1)}(n,t)$ and $u_{2,s}^{(1)}(n,t)$ are the finite Fourier transforms in cosine (2), respectively in sine (3) of the elastic longitudinal and transversal displacement (1).

2 Numerical application for a connecting rod of a real R(RRT) mechanism with linear elastic comportment

It is considered the concrete case of the RRT mechanism as follows:

L = 1 [m]; r = 0,07 [m]; b = 0,011[m]; h = 0,011[m]; E = $2{,}1\cdot 10^{11}$[N/m^2]; ρ = 7800[Kg/m^3]; G = $8{,}1\cdot 10^{10}$ [N/m^2]; ν = 0.3. The three different established frequencies are: f_1 = 1.758 Hz; f_2 = 3.662 Hz; f_3 = 4.321 Hz. It is considered a point placed in the middle of the connecting rod and it is computed with the Mathematica program the variation of the vibrations transversal displacement in this point as the next figures show. The effective values of the vibrations transversal displacements in the considered point and for the specified frequencies are computed with the Mathematica program, too, and presented in Table 1.

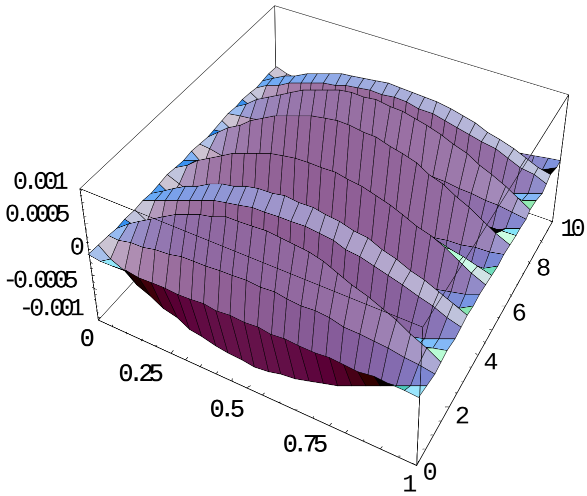

Fig. 1 The transversal displacement $u_2 = u_2^{(1)}(x,t)$ for $f_1 = 1.758$ Hz

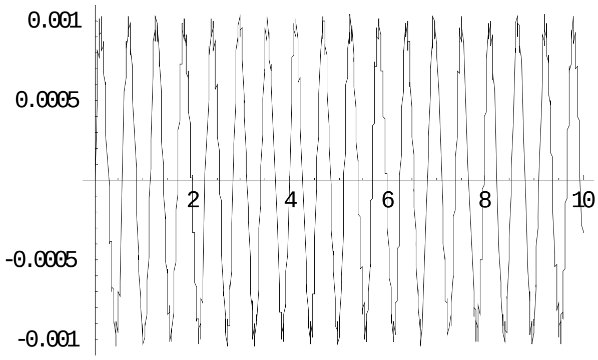

Fig. 2 The transversal displacement variation at for $f_1 = 1.758$ Hz and x=L/2: $u_2 = u_2^{(1)}\left(\frac{L}{2},t\right)$

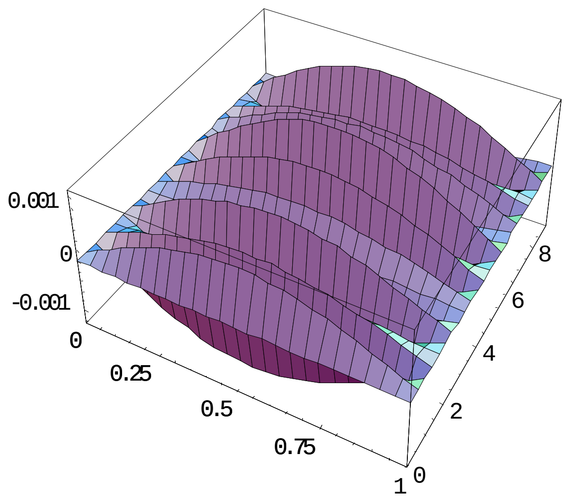

Fig. 3 The transversal displacement $u_2 = u_2^{(1)}(x,t)$ for $f_2 = 3.662$ Hz

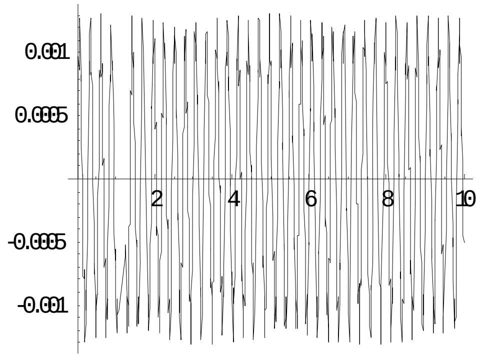

Fig. 4 The transversal displacement variation at $f_2 = 3.662$ Hz and x=L/2: $u_2 = u_2^{(1)}\left(\frac{L}{2},t\right)$

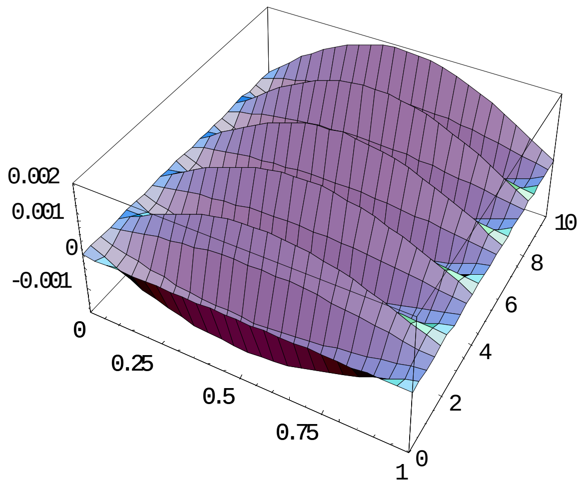

Fig. 5 The transversal displacement $u_2 = u_2^{(1)}(x,t)$ for $f_3 = 4.321$ Hz

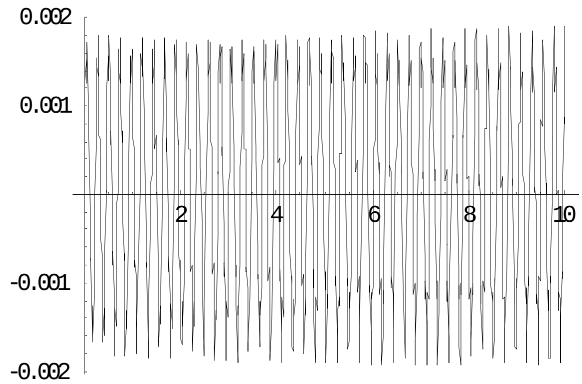

Fig. 6 The transversal displacement variation at $f_3 = 4.321$ Hz and x=L/2: $u_2 = u_2^{(1)}\left(\frac{L}{2},t\right)$

By derivation $u_2 = u_2^{(1)}(L/2,t)$ twice in relation to the time, vibrations transversal accelerations were obtained and their effective values were compared with the values experimentally obtained [2].

Because of the small registered differences (up to 6%), it was considered that the mathematical model and the method of solving it was correct. That is why the effective values of the vibrations transversal displacements from the Table 1 are validated by experiment.

3 Numerical application for a connecting rod of a real R (RRT) mechanism with linear viscoelastic comportment (un-masticated polyvinyl chloride – PVC-U)

By applying in the movement mathematical model of a linear elastic cinematic element bar type (1), the Laplace transform one-sided proportional to time and replacing the E modulus with $\widetilde{E}(s)$, the equation in matrix of the first approximation in Laplace images is obtained for the vibrations of the viscoelastic connecting rod of the R (RRT) mechanism as follows [4]:

$$\left[\widetilde{L}_0(s)\right]\left\{\widetilde{u}^{(1)}\right\}+\left[M_4\right]\left\{\widetilde{a}_0\right\}+\left\{\widetilde{V}_1\right\}+\left[M_7\right]\left\{\widetilde{f}\right\}+\left\{\widetilde{V}_2\right\}+ \\ \vdots \qquad +\left[M_2\right]\frac{d^2\left\{V_3\right\}}{dx^2}+\left[M_4\right]\left\{V_3\right\}=\left\{0\right\} \quad (4)$$

The connecting rod OA being double-articulated, the just on the line conditions which allowed the application of the two Fourier transforms, for initial functions and for their Laplace images, were the boundary conditions and initial conditions are given in [3].

The Laplace images of the vibrations displacements will be:

$$\widetilde{u}_i^{(1)}(x,s)=\frac{2}{L}\sum_{n=1}^{\infty}\widetilde{u}_{i,s}^{*(1)}(n,s)\sin(\alpha_n x)$$

or

$$\widetilde{u}_i^{(1)}(x,s)=\frac{1}{L}\widetilde{u}_{i,c}^{*(1)}(0,s)+\frac{2}{L}\sum_{n=1}^{\infty}\widetilde{u}_{i,c}^{*(1)}(n,s)\cos(\alpha_n x),$$

(5)

where i = 1, 2 and $\alpha_n=\frac{n\pi}{L}$.

The transversal displacements field, in the first approximation, in the case of free vibrations of the connecting rod belonging to the mechanism R (RRT) are obtained by inversing in (5) the Laplace transform.

It is considered the same concrete case of the RRT mechanism, with a connecting rod made of un-masticated polyvinyl chloride, with the following geometric characteristics:

L = 1[m]; r = 0.07[m]; b = 0.011[m]; h=0.011[m].

The experimental determinations for the un-masticated polyvinyl chloride PVC-U characteristics of material were the following:

- the shear elasticity modulus: G = 1189.79[MPa];
- the bulk modulus: K = 2424.455[MPa];
- the apparent modulus of elasticity $E_{ap} \cong E$ = 2909.347[MPa];
- the density: ρ=1213.3[kg/m^3];
- the constant η afferent to the Newtonian component of the Maxwell model: $\eta = 4.5*10^7$[MPah].

Using Mathematica program there were determined the vibrations transversal displacements variations for the same three different frequencies of the leader element rotation, as in the case of the steel connecting rod: f_1 = 1.758 Hz; f_2 = 3.662 Hz; f_3 = 4.321 Hz.

It is computed with the Mathematica program the variation of the vibrations transversal displacement in the point placed in the middle of the connecting rod, too, in order to determine the effective values of the vibrations transversal displacements and compare with the ones obtained for the steel connecting rod (Table 1).

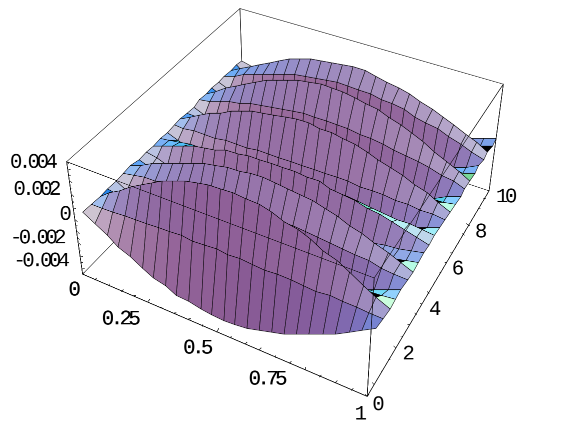

Fig. 7 The transversal displacement $u_2=u_2^{(1)}(x,t)$ for f_1 = 1.758 Hz

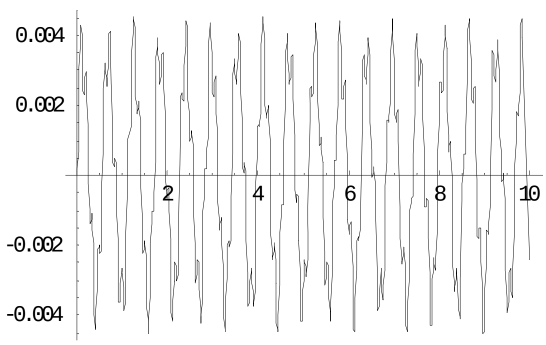

Fig. 8 The transversal displacement variation at for

f_1 = 1.758 Hz and x=L/2: $u_2 = u_2^{(1)}\left(\frac{L}{2}, t\right)$

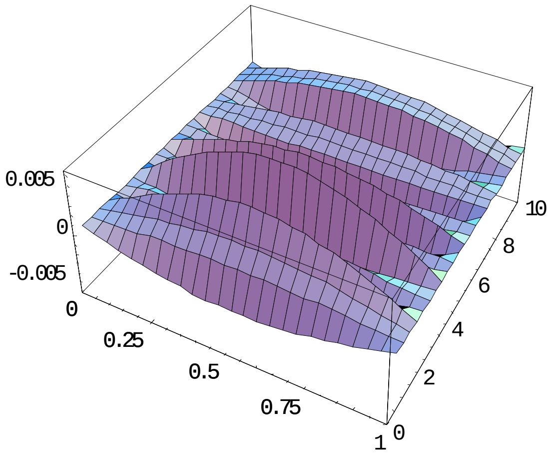

Fig. 9 The transversal displacement $u_2 = u_2^{(1)}(x,t)$ for f_2 = 3.662 Hz

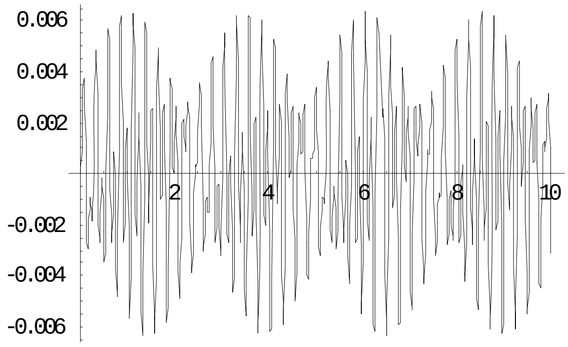

Fig. 10 The transversal displacement variation at f_2 = 3.662 Hz and x=L/2: $u_2 = u_2^{(1)}\left(\frac{L}{2}, t\right)$

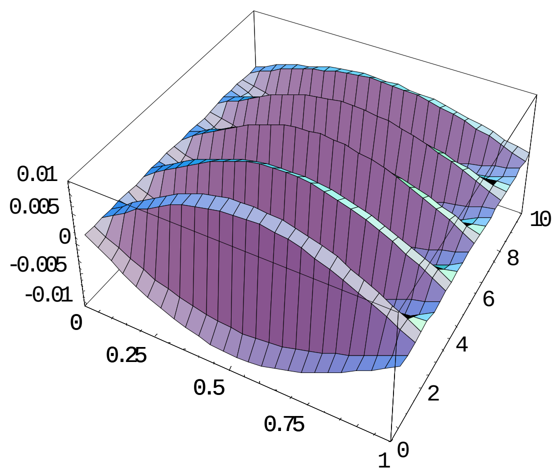

Fig. 11 The transversal displacement $u_2 = u_2^{(1)}(x,t)$ for f_3 = 4.321 Hz

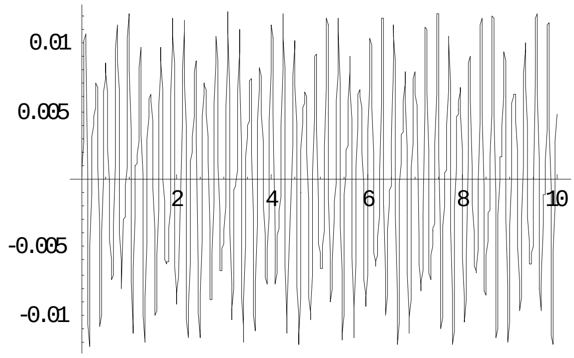

Fig. 12 The transversal displacement variation at f_3 = 4.321 Hz and x=L/2: $u_2 = u_2^{(1)}\left(\frac{L}{2}, t\right)$

The vibrations transversal accelerations were computed also in this case by derivation $u_2 = u_2^{(1)}(L/2, t)$ twice in relation to the time and their effective values were compared with the values experimentally obtained. The mathematical model and the method of solving were validated as correct because of the small registered differences (up to 9%) [8].

4 Comparison between the comportment of the two connecting rods

The effective values of the vibrations transversal displacements $u_2^{(1)}(L/2, t)$ for the free vibrations of the connecting rods made of steel and, respectively, un-masticated polyvinyl chloride PVC-U are presented in Table 1.

Table 1. Effective values for the transversal displacements $u_2^{(1)}(L/2, t)$

Frequency [Hz]	1.758	3.662	4.321
$u_2^{(1)}(L/2,t)$[m]	0.00073	0.00092	0.00136
$u_2^{(1)}(L/2,t)$[m]	0.00319	0.00448	0.00869

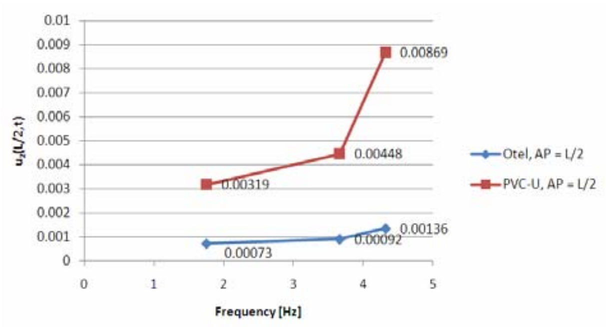

Fig. 13 The variations of the vibrations transversal displacements for the point placed in the middle of the connecting rods depending on the action frequencies of the leader element

In figure 13 the variations of the vibrations transversal displacements of the point placed in the middle of the connecting rods depending on the action frequencies of the leader element are presented for the connecting rod made of steel and, respectively, un-masticated polyvinyl chloride PVC-U. It is noticed that the computed values of the vibrations transversal displacements are greater in the case of the connecting rod with viscoelastic comportment [5], [6].

If there are assesed transversal displacements of vibration when there are used cinematic elements with viscoelastic comportment, similar to those registered using cinematic elements with elastic comportment, this can be achieved by growing the dimensions of the transversal section of the cinematic element with viscoelastic comportment. The materials with viscoelastic comportment are more advantageous because they have considerably less density than metallic materials, so less moments of inertia, at bargain price and in comparable stiffness conditions [7].

So, for a connecting rod made of PVC-U having 0.011 x 0.016 [m] the dimensions of the transversal section, that is 50% thicker than the studied one, the variations of the transversal displacements depending on time for the action frequency at the leader element f = 3.662 Hz is presented in fig. 14.

It can be noticed that the diagram of the variation in this case is similar to the one presented in fig. 3 corresponding to the connecting rod made of steel for the same action frequency of the leader element, but having 0.011 x 0.016 [m] the dimensions of the transversal section.

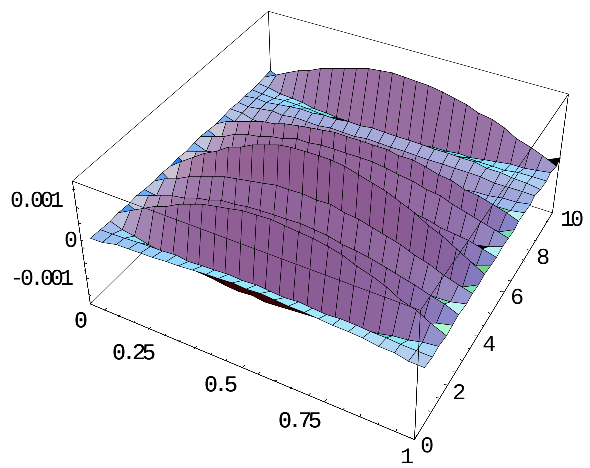

Fig. 14 The variations of the transversal displacements $u_2 = u_2^{(1)}(x,t)$ for the connecting rod made of PVC-U and the action frequency at the leader element f = 3.662 Hz, having 0.011 x 0.016 [m] the dimensions of the transversal section.

5 Conclusions

A first conclusion is that the computed (theoretical) values of vibrations transversal displacements are greater in the case of the connecting rod made of un-masticated polyvinyl chloride PVC-U than in the case of the one made of steel.

The dimensions of the transversal section of the cinematic element with viscoelastic comportment can be increased in order to obtain vibrations transversal displacements similar to those of cinematic element with linear elastic comportment.

In terms of comparable stiffness, the materials with vascoelastic comportment are more advantageous both in terms of cost and because they have considerably less density in comparison with metallic materials, so both forces and moments of inertia are reduced.

The graphical representations show the significant influence of the cinematic parameters of the mechanism movement in the maximum values of the vibrations transversal displacements.

This paper is useful in the machine designing. The calculus of the transversal displacements field is very important in determination of the influence of the vibrations and of the stress and strain state of the cinematic element, which represents important data in machine designing and dimensioning.

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