Adaptive Control of the Helicopters' Pitch Angle and Velocity

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In this paper we discuss the flying objects adaptive control with direct application to the flight of helicopters. The authors suggest two new automatic adaptive control systems: the former is used for the pitch angle control, while the latter is used for the control of helicopter pitch angle and velocity; this second system is an extension of the first one. The adaptive control is based on the dynamic inversion principle and the using of neural networks. The two adaptive control systems have reference models, linear dynamic compensators, linear observers, and neural networks. The adaptive components of the automatic control laws compensate the approximation errors of the dynamic model nonlinear functions. The used actuators are linear or nonlinear. To eliminate the neural networks adapting difficulties, a Pseudo Control Hedging (PCH) block is inserted in the adaptive system; it limits the adaptive pseudo-control by means of a component which represents the estimation error of the actuator dynamics. Thus, the PCH block "moves back the reference model" i.e. it introduces a reference model response correction with respect to the actuator position estimation; the signal provided by PCH block represents a reference model additional input. For the two new automatic adaptive control systems, the Matlab/Simulink environment is used to obtain time characteristics of the adaptive systems with linear and nonlinear actuators. Phase trajectories of the two adaptive control systems with nonlinear actuators express the convergence of the nonlinear systems to stable limit cycles.

Subject headings: Adaptive control, Neural network, Dynamic inversion, Helicopter.

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Notation

$\overline{\boldsymbol{E}}$	=	signal used for the neural network training
$\boldsymbol{e}$	=	error vector
F_x, F_y, F_z	=	external forces
f, h	=	nonlinear functions
$H_d(\mathrm{s})$	=	flying object transfer function
$H_{dm}(\mathrm{s})$	=	reference model transfer function
h_r	=	nonlinear function
$\hat{h}_r$	=	h_r function best approximation
I	=	identity matrix
k_z, k_v	=	positive gain constants
k_z, k_v	=	coefficients of the proportional – derivative linear dynamic compensator
L	=	gain matrix
n	=	the number of the state variables
$P, \tilde{P}$	=	solutions of the Lyapunov equations
r	=	system relative degree
u	=	input vector of the system
$\hat{V}, \hat{W}$	=	neural network weights
$\hat{V}_0, \hat{W}_0$	=	initial values of the weights $\hat{V}, \hat{W}$
V_x, V_z	=	helicopter longitudinal and vertical velocities
v	=	pseudo-command (adaptive control law)
v_a	=	the adaptive component of the command (it compensates the approximation error ε)
v_{pd}	=	the output of the dynamic compensator
w	=	measuring sensor error
X, Y, Z	=	helicopter coordinates

48	x	=	state vector
49	$\bar{x}$	=	stable state
50	y	=	system output vector
51	$\bar{y}$	=	system imposed output
52	y_m	=	measured value of the vector y
53	$\lVert\hat{Z}\rVert_F$	=	Frobenius norm of the matrix $\hat{Z}$
54	$\bar{Z}$	=	neural network ideal matrix
55	α_1, γ_1	=	positive constants
56	β	=	longitudinal control of the principal rotor
57	δ	=	actuator output for the longitudinal cyclic command
58	δ_c	=	longitudinal cyclic
59	δ_m	=	engine command
60	ε	=	approximation error of the function h_r
61	ε_1	=	inversion error
62	φ	=	roll angle
63	η	=	neural network input vector
64	ψ	=	yaw angle
65	Γ_W, Γ_V	=	positive constants
66	σ	=	sigmoid function
67	σ'	=	Jacobian of the vector σ
68	ς	=	compensator state
69	ω_y	=	helicopter pitch angular rate
70	θ	=	helicopter pitch angle
71	θ_c	=	reference model input

Introduction

The complexity and incertitude which appear in the case of the nonlinear and instable aerodynamic phenomena are the main reasons which require the design of nonlinear adaptive structures for control and stabilization; in these cases, the linear models are far from a good description of the flying object dynamics. Another reason is the nonlinear character of the actuators because of the displacement/rate saturation of their elements.

The stability and maneuverability qualities have to be maintained even in the case of some sensors, actuators or force equipments failure. The flight control systems must be capable to identify the effects, to isolate the damaged elements, and to have a great flexibility in their real time architecture reconfiguration. The observers must be easily adaptable and their design algorithms must allow the flying object state estimation in the case of some sensors failure; thus, the signals provided by these sensors can not be used.

In recent years, lots of scientific researchers have applied the intelligent concepts for the aircrafts automatic control; they use the optimal synthesis $H_2, H_\infty, H_2 / H_\infty$ (Che and Chen 2001; Niewoenhner and Kaminer 1996; Kurdjnkov et al. 1996), the adaptive synthesis based on dynamic inversion theory, neural networks theory (Mori and Suzuki 2009; MIT Open Course 2007; Kargin 2007), and fuzzy techniques (Jantzen 1998; Mahfouf et al. 1999; Tudosie 2008). These intelligent techniques have the advantage of a very good adaptability, robustness, and software implementation capabilities.

In this paper we design two new adaptive control systems for the pitch angle control and for the pitch angle and velocity control, respectively; the second system is an extension of the first one. The two systems consist of: reference models, linear or nonlinear compensators, linear observers, and neural networks; to eliminate the neural networks' adapting difficulties, a Pseudo Control Hedging (PCH) block is introduced in order to limit the adaptive pseudo-control by means of a component which represents the estimation of the actuator dynamics. The two new adaptive control systems are validated in Matlab/Simulink environment for two concrete cases: adaptive system with linear actuator and adaptive system with nonlinear actuator.

In the second section of the paper, the flying object dynamics (in our case the dynamics of helicopters) is modeled by a nonlinear system consisting of a linear subsystem and a nonlinear subsystem described by a function h_r. The adaptive control law is expressed as a function $\hat{h}_r$ (the approximation of the function h_r with the error ε) (Calise et al. 2001; Lungu 1997). The pseudo-command v consists of r order derivative of the imposed output $(\bar{y})$, a proportional – derivative (PD) component, provided by a linear dynamic compensator, and a neural network, which must compensate the approximation error ε (Gregory 2000; Hovakimyan et al. 2002; Sharma and Calise 2001; Lungu 2000). The adapting (training) of each neural network, which models the pseudo-command adaptive component, is made by a command vector provided by a reduced order linear observer (Eltantawie 2010), whose input is the error of the adaptive system for the pitch angle and velocity control. The algorithm for the training of the neural network involves the calculation of the neural networks' weight matrices by the integration of some differential equations with respect to the state observer output vector (Chwa and Choi 2003; Balestrassi 2009). The adaptive component of the command (v_a) is calculated as a sigmoid function depending on the neural network input vector η (Chen et al. 2006; Ferrari 2005; Johnson and Calise 2001; Johnson et al. 2000); this vector has as components the input v and the output y of the linear subsystem with the relative degree $r,$ at different time moments. The sigmoid function depends on the neurons number from the neural network input layer.

In the third section of this paper, we design a new system for the helicopters pitch angle adaptive command by using the model of an R-50 experimental helicopter (Calise et al. 2000). According to this model, the system relative degree with respect to the pitch angle, as an output variable, is $r = 3;$ we choose a three order reference model and we calculate the parameters of a PD linear dynamic compensator, the pseudo-command $v,$ the approximation error $\varepsilon,$ the observer gain matrix, and the neural network structure. The adaptive controllers are sensitive to actuator nonlinearity; that is why, in the architecture of these controllers, one may introduce a block which limits the pseudo-control by means of a component representing an estimation of the execution element dynamics (PCH - Pseudo Control Hedging) (Johnson and Calise 2001; Johnson et al. 2000; Calise et al. 2000; Calise et al. 2006; Hovakimyan et al. 2005; Johnson and Calise 2000; Johnson and Calise 2001).

In the forth section of this paper, we design a new adaptive system for the control of helicopters velocity and pitch angle by using the same algorithms as in the third section. The coupling of the two control systems (the system for the control of the helicopters velocity and the pitch angle control system) is made by means of the equations in (Prasad et al. 1999; Calise 2003). The dynamic compensators may also be designed by using fuzzy techniques (Zdenko and Stjepan 2006; Hampel et al. 2000; Tomescu 2001; Kumar et al. 2008; Corcau et al. 2009). The numerical simulations main results are presented in fifth section. Using the Matlab/Simulink environment, we plot time characteristics of the two adaptive control systems, with linear and nonlinear actuators.

Our paper brings contributions to other papers in the field of adaptive systems with neural networks. For example, one considers an adaptive feedback control of uncertain nonlinear systems, in which both the dynamics and the dimension of the regulated system may be unknown (Hovakimyan et al. 2002). Given a smooth reference trajectory, the contribution (Hovakimyan et al. 2002) is the design of a controller that forces the system measurement to track it with bounded errors. An observer for an uncertain nonlinear system is designed and the Lyapunov’s direct method is used to demonstrate the boundedness of the error signals. The theoretical results are illustrated in the design of a controller for a fourth-order nonlinear system of relative degree two and a high-bandwidth attitude command system for a model R-50 helicopter (Hovakimyan et al. 2002). Another interesting paper about an effective method to achieve altitude and attitude (pitch, roll and yaw) control of a helicopter in hovering and low-speed forward flight conditions is the paper of Zeng and Zhu (Zeng and Zhu 2006). Here a simplified multi-input- multi-output (MIMO) affine nonlinear model describing the angular rate responses of a helicopter is derived. A PID controller is used in the altitude loop and a dynamic inversion controller is used in the attitude loop. To compensate the dynamic inversion error caused by modeling uncertainties and disturbances, an adaptive compensating algorithm is employed. The system validation is made for the same system as the one in this paper (R-50 unmanned helicopter). The simulations in the paper of Zeng and Zhu (Zeng and Zhu 2006) prove that the algorithm is stable, robust, and it has good tracking performance and decoupling capability. In their paper (Hovakimyan et al. 2002), the authors propose a new approach for adaptive output feedback control of uncertain nonlinear systems using neural networks (a single hidden layer NN is introduced to cancel the inversion error). The network learns on line, and no off-line training is required. A simple linear observer is introduced to estimate the tracking error derivatives. These estimates are used in the adaptation laws for the NN parameters (Hovakimyan et al. 2002). The method is applicable to systems of unknown but bounded dimension, so long as the relative degree is known.

The main contributions (originality elements) of this paper are: 1) the design of the adaptive system for the control of the helicopters pitch angle; 2) the design of the adaptive system for the helicopters velocity and pitch angle control; 3) the dynamic characteristics of the two adaptive systems, which express good stationary and dynamic performances.

Adaptive Control Based on Dynamic Inversion

A flying object nonlinear dynamics, with one input u and one output y, is described by the equations:

$$\dot{x} = f(x,u), y = h(x), \tag{1}$$

with $x(n \times 1)$ – the state vector, n – the number of the state variables, f and h – nonlinear functions, generally unknown, u and y – the system input vector and the system output vector, respectively (measurable vectors). In this paper, we assume that $h(x)$ is a uniquely invertible function.

We use the hypothesis (Isidori 1995): the flying object dynamics, described by (1), satisfies the equations:

$$y^{(r)} = h_r(x,u), h_r = \frac{\mathrm{d}^r h}{\mathrm{d}t^r}, \tag{2}$$

$$\frac{\partial h_i}{\partial u} = 0, 0 \le i \le r, \frac{\partial h_r}{\partial u} \ne 0; \tag{3}$$

$r(\le n)$ is the relative degree of the system (1).

The adaptive control law (the pseudo-command) v is expressed as the following function (Calise et al 2001):

$$v = \hat{h}_r(y,u); \tag{4}$$

$\hat{h}_r(y,u)$ is the best approximation of the function:

$$h_r(y,u) = h_r(y(x),u) = h_r(x,u). \tag{5}$$

Under the assumption: $\hat{h}_r(y,u)$ is invertible, the command of the flying object, described by the equation:

$$u = \hat{h}_r^{-1}(y,v), \tag{6}$$

expresses the inverse dynamic model of the flying object nonlinear subsystem. If $\hat{h}_r = h_r$, then, according to equations (2) and (4), $y^{(r)} = v$, while, if $\hat{h}_r \ne h_r$, we have:

$$y^{(r)} = v + \varepsilon, \tag{7}$$

where

$$\varepsilon = \varepsilon(x,u) = h_r(x,u) - \hat{h}_r(x,u) \tag{8}$$

is the approximation error of the function h_r (the inversion error), which acts like a disturbing signal.

The equation (7) expresses the presence of r ideal integrators between the pseudo-command (pseudo-control) v and the output y (Fig.1) (Lungu 1997). Imposing that the output y follows the bounded output variable $\bar{y}$, the signal v has the following form (Lungu et al. 2011; Lungu 2008):

$$v = \bar{y}^{(r)} + v_{pd} - v_a + \bar{v}; \tag{9}$$

the components of v have the significance in Fig. 2, where v_{pd} is the linear dynamic compensator output (used for stabilization), $v_a -$ the adaptive command which must compensate the error ε, while $\bar{v}$ has, for example, the form (Sharma and Calise 2001):

$$\bar{v} = k_z\left(\left\|\hat{Z}\right\|_F + \bar{Z}\right)\frac{\overline{\boldsymbol{E}}}{\left\|\overline{\boldsymbol{E}}\right\|}\left\|\hat{\boldsymbol{E}}\right\| + k_v\overline{\boldsymbol{E}}, \tag{10}$$

with the gain constants $k_z, k_v > 0$, $\left\|\hat{Z}\right\|_F -$ the ideal matrix Frobenius norm of the neural network $\hat{Z} = \begin{bmatrix} \hat{W} & 0 \\ 0 & \hat{V} \end{bmatrix}$, $\hat{W}, \hat{V} -$ the weights of a neural network, $\bar{Z} -$ the superior bound of $\left\|\hat{Z}\right\|_F$ (namely $\left\|\hat{Z}\right\|_F \le \bar{Z}$), and $\overline{\boldsymbol{E}} = \hat{\boldsymbol{E}}P\bar{B}$, with $\hat{\boldsymbol{E}}, P,$ and $\bar{B} -$ matrices.

Let us consider $H_d(\mathrm{s}) -$ the transfer function of the flying object linear system (Lungu 2000) with the input u_n, the output y, and the system relative order r (see Fig. 1):

$$H_d(\mathrm{s}) = \frac{b_p \mathrm{s}^p + b_{p-1}\mathrm{s}^{p-1} + \ldots + b_1\mathrm{s} + b_0}{s^r + \lambda_{r-1}\mathrm{s}^{r-1} + \ldots + \lambda_1\mathrm{s} + \lambda_0}, \; p \le r-1. \tag{11}$$

If $p = 0$, the linear subsystem with the input v and the output y is described by the equation:

$$y^{(r)} = -\lambda^T Y + b_0 v + \varepsilon, \tag{12}$$

where

$$\lambda^T = [\lambda_0 \ \lambda_1 \ \cdots \ \lambda_{r-1}], Y^T = [y \ \dot{y} \ \ldots \ y^{(r-1)}]. \tag{13}$$

The dynamic compensator is described by the state equations:

$$\dot{\varsigma} = A_c\varsigma + b_c e, v_{pd} = c_c\varsigma + d_c e, \tag{14}$$

where the vector ς has at least the dimension $(r-1)$; the error e is defined as follows:

$$e = \tilde{y} = \tilde{c}\boldsymbol{e}, \boldsymbol{e}^{\mathrm{T}} = [e \ \dot{e} \ \cdots \ e^{(r-1)}], \tilde{c} = [1 \ 0 \ 0 \ \cdots \ 0]_{1\times r}; \tag{15}$$

the vector $\boldsymbol{e}$ is defined later in the paper (equation (18)). The state equation of the subsystem in Fig. 1 with the input $(v+\varepsilon)$ and the output y is:

$$\dot{x} = Ax + b(v+\varepsilon), v = v_{pd} - v_a + \bar{v}, \tag{16}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{r\times r}, \ b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{r\times 1}. \tag{17}$$

The stable state $\bar{x}\left(\dot{\bar{x}} = v = \varepsilon = 0\right)$ verifies the equation $A\bar{x} = 0$ and, taking into account equation (16), we obtain the equation of the error vector $\boldsymbol{e} \equiv \tilde{x} = \bar{x} - x$,

$$\dot{\boldsymbol{e}} = A\boldsymbol{e} - bv_{pd} + b(v_a - \bar{v} - \varepsilon). \tag{18}$$

Using the notations:

$$\boldsymbol{E} = \begin{bmatrix} \boldsymbol{e} \\ \varsigma \end{bmatrix}, \bar{A} = \begin{bmatrix} A - d_c b\tilde{c} & -bc_c \\ b_c\tilde{c} & A_c \end{bmatrix}, \bar{b} = \begin{bmatrix} b \\ 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} \tilde{c} & 0 \\ 0 & I \end{bmatrix}, \tag{19}$$

where I is the identity matrix, we get:

$$\dot{\boldsymbol{E}} = \bar{A}\boldsymbol{E} + \bar{b}(v_a - \bar{v} - \varepsilon), z = \bar{C}\boldsymbol{E}; \tag{20}$$

A_c, b_c, c_c, d_c from equation (14) are calculated such that $\bar{A}$ is a Hurwitz matrix.

For the estimation of the vector $\boldsymbol{E}$, the paper authors introduce a linear state observer of order $(2r-1)$ in the linear dynamic compensator structure; the observer equations are:

$$\dot{\hat{\boldsymbol{E}}} = \bar{A}\hat{\boldsymbol{E}} + L(z - \hat{z}), \hat{z} = \bar{C}\hat{\boldsymbol{E}}, \tag{21}$$

with the gain matrix L calculated such that the matrix $\tilde{A} = \left(\bar{A} - L\bar{C}\right)$ is stable. Considering $w-$ the measuring sensor error, $y_m -$ the measured value of the output vector y, then $\tilde{y}_m = \bar{y} - y_m = \tilde{y} + w$, and the compensator equations become:

$$\dot{\boldsymbol{E}} = \bar{A}\boldsymbol{E} + \bar{b}\left(v_a - \bar{v} - \varepsilon\right) + Gw, z = \bar{C}\boldsymbol{E} + Hw, \tag{22}$$

with $H^T = [1 \;\; 0], G^T = [-bd_c \;\; b_c]$. If the compensator state ς is known, we use a reduced order observer for the estimation of the vector $\boldsymbol{e}$ (Eltantawie 2010):

$$\dot{\hat{\boldsymbol{e}}} = \bar{A}\hat{\boldsymbol{e}} + L_r\left(z_1 - \hat{z}_1\right), z_1 = c\boldsymbol{e}, \hat{z}_1 = c\hat{\boldsymbol{e}}. \tag{23}$$

The gain matrix L_r is obtained such that the matrix $\tilde{A} = \left(\bar{A} - L_r\tilde{c}\right)$ is stable. The vector $\hat{\boldsymbol{E}}^T = [\hat{\boldsymbol{e}} \;\; \varsigma]$ is obtained by means of the vectors $\hat{\boldsymbol{e}}$ and ς. The signal $\bar{\boldsymbol{E}} = \hat{\boldsymbol{E}}^T P\bar{b}$ is used for the neural network training; the weights $\hat{W}$ and $\hat{V}$ are obtained by using the equations (Balestrassi et al. 2009; Lungu 2008):

$$\dot{\hat{W}} = -\Gamma_W\left[2\left(\sigma - \sigma'\hat{V}^T\eta\right)\hat{\boldsymbol{E}}^T P\bar{B} + k\left(\hat{W} - \hat{W}_0\right)\right], \dot{\hat{V}} = -\Gamma_V\left[2\eta\hat{\boldsymbol{E}}^T P\bar{B}\sigma' + k\left(\hat{V} - \hat{V}_0\right)\right], \tag{24}$$

where the role of $\bar{B}$ is played by $\bar{b}$. The above equations have been obtained in other papers (Balestrassi et al. 2009; Lungu 2008) by using the Lyapunov stability conditions. In equations (24) σ is the sigmoid function (Chen et al. 2006; Ferrari 2005):

$$\sigma(z) = \left(1 + e^{-az}\right)^{-1}; \tag{25}$$

$\sigma' = \left.\frac{\mathrm{d}\sigma(z)}{\mathrm{dz}}\right|_{z=z_0}$ is the Jacobian of vector σ, $\hat{W}_0$ and $\hat{V}_0 -$ the initial values of the weights $\hat{W}$ and $\hat{V}$, $\Gamma_W, \Gamma_V -$ positive constants, $k > 2\left(k_1^2 + \gamma_1^2\left\|P\bar{B}\right\|^2\right)$, $k_1 = k_2\alpha_1 + \left\|P\bar{B}\right\|\gamma_1$, $k_2 = \left\|P\bar{B}\right\| + \left\|\tilde{P}\bar{B}\right\|$, $\gamma_1, \alpha_1 -$ positive constants, while P and $\tilde{P}$ are the solutions of the Lyapunov equations:

$$\bar{A}^T P + P\bar{A} = -Q, \; \tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} = -\tilde{Q}. \tag{26}$$

P from the signal used for NN training is the solution of first equation (26) with $\bar{A} = \left(A - d_c b\tilde{c}\right)$ (Chwa et al. 2004). The adaptive component v_a is calculated by using the formula (Shao et al. 2008):

$$v_a = \hat{W}^T \sigma\left(\hat{V}^T \eta\right), \tag{27}$$

with $\eta -$ the input vector of neural network NN_c (Hoseini et al. 2006)

$$\eta = \left[1 \ \bar{v}_d^T(t) \ \bar{y}_d^T(t)\right]^T, \tag{28}$$

$$\begin{aligned} \bar{v}_d^T(t) &= \left[v(t) \quad v(t) - v(t-d) \quad \left(v(t) - v(t-d)\right) - \left(v(t-d) - v(t-2d)\right)\right]^T, \\ \bar{y}_d^T(t) &= \left[y(t) \quad y(t) - y(t-d) \quad \left(y(t) - y(t-d)\right) - \left(y(t-d) - y(t-2d)\right)\right]^T. \end{aligned} \tag{29}$$

Adaptive Control System for the Helicopters' Pitch Angle Control

Let us consider now the model of the R-50 helicopter longitudinal dynamics (Calise et al. 2000):

$$\begin{bmatrix} \dot{V}_x \\ \dot{\omega}_y \\ \dot{\theta} \\ \dot{\beta} \\ \dot{V}_z \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} X_u & X_q & X_\theta & X_\beta & X_w & X_\delta \\ M_u & M_q & 0 & M_\beta & M_w & M_\delta \\ 0 & 1 & 0 & 0 & 0 & 0 \\ B_u & -1 & 0 & B_\beta & 0 & B_\delta \\ Z_u & Z_q & Z_\theta & Z_\beta & Z_w & Z_\delta \\ 0 & 0 & 0 & 0 & 0 & -1/\tau \end{bmatrix} \begin{bmatrix} V_x \\ \omega_y \\ \theta \\ \beta \\ V_z \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1/\tau \end{bmatrix} \delta_c . \tag{30}$$

with V_x and V_z $-$ the helicopters' longitudinal and vertical velocities, θ and ω_y $-$ the pitch angle and the pitch angular rate, respectively, $\beta -$ the principal rotor longitudinal control angle, δ_c $-$ the longitudinal cyclic input, $\delta -$ the actuator output for the longitudinal cyclic command.

Considering that the pitch angle $\left(y = \theta\right)$ is an output of the system, by derivation with respect to time of the second equation which derives from the state equation (30), and by elimination of $\dot{\delta}$ (from the sixth equation which derives from (30)), we obtain the equation:

$$\dddot{y} = \dddot{\theta} = M_u \dot{V}_x + M_q \ddot{\theta} + M_\beta \dot{\beta} + M_w \dot{V}_z - \frac{M_\delta}{\tau}\delta + \frac{M_\delta}{\tau}\delta_c . \tag{31}$$

Thus, the system relative degree is $r = 3$ and $u = \delta_c$.

We choose a three order reference model described by the equation:

$$\bar{y} = \frac{p\omega_{r0}^2}{(\mathrm{s} + p)(\mathrm{s}^2 + 2\xi_0\omega_{r0}\mathrm{s} + \omega_{r0}^2)} y_c \;, \; p = 25, \xi_0 = 0.7, \omega_{r0} = 10 \text{ rad/s}\,. \tag{32}$$

Analyzing the equation (31) and the transfer function (11), $H_d(\mathrm{s})$ gets the form:

$$H_d(\mathrm{s}) = \frac{1}{\mathrm{s}^2(\mathrm{s}+1)}; \tag{33}$$

we have chosen $b_0 = 1; \lambda_0 = \lambda_1 = 0, \lambda_2 = 1$. Equation (12) becomes:

$$\ddot{y} = \dddot{\theta} = -\ddot{\theta} + v + \varepsilon, \tag{34}$$

with $\ddot{\theta}$ described by the second equation which derives from (30):

$$\ddot{\theta} = \dot{\omega}_y = M_u V_x + M_q \omega_y + M_\beta \beta + M_w V_z + M_\delta \delta. \tag{35}$$

Using (34) and (35) we obtain the equation:

$$\dddot{y} = \dddot{\theta} = M_u \dot{V}_x + M_\beta \dot{\beta} + M_w \dot{V}_z - \frac{M_\delta}{\tau}\delta + \frac{M_\delta}{\tau}\delta_c + M_q\left(M_u V_x + M_\beta \beta + M_w V_z + M_\delta \delta\right) + M_q^2 \dot{\theta}, \tag{36}$$

with $\dot{V}_x, \dot{V}_z$, and $\dot{\beta}$ obtained from (30), i.e.:

$$\begin{bmatrix} \dot{V}_x \\ \dot{\beta} \\ \dot{V}_z \end{bmatrix} = \begin{bmatrix} X_u & X_\beta & X_w \\ B_u & B_\beta & 0 \\ Z_u & Z_\beta & Z_w \end{bmatrix} \begin{bmatrix} V_x \\ \beta \\ V_z \end{bmatrix} + \begin{bmatrix} X_\theta & X_q & X_\delta \\ 0 & -1 & B_\delta \\ Z_\theta & Z_q & Z_\delta \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ \delta \end{bmatrix}. \tag{37}$$

So, equation (36) has the form (7); now, we separate v and ε. v has the form (4) and it contains terms which depend on $u = \delta_c$ and y or which depend on $\dot{y} = \dot{\theta}$; the other terms are included in ε. Thus,

$$v = \hat{h}_r(y, \delta_c) = \frac{M_q}{\tau}\delta_c + M_q(M_q + 1)\dot{\theta}; \tag{38}$$

$$\delta_c = u = \hat{h}_r^{-1}(x, v) = \frac{\tau}{M_\delta}\left[v - M_q(M_q + 1)\,\dot{\theta}\right] \tag{39}$$

and ε has the following form:

$$\varepsilon = \left(M_q + 2b_0\right)\left[M_u \;\; M_\beta \;\; M_w \;\; M_\delta\right]\begin{bmatrix} V_x \\ \beta \\ V_z \\ \delta \end{bmatrix} + \left[M_u \;\; M_\beta \;\; M_w\right]\cdot\left\{\begin{bmatrix} X_u & X_\beta & X_w & X_q \\ B_u & B_\beta & 0 & B_\delta \\ Z_u & Z_\beta & Z_w & Z_\delta \end{bmatrix}\cdot\begin{bmatrix} V_x \\ \beta \\ V_z \\ \delta \end{bmatrix} + \begin{bmatrix} X_\theta & X_q \\ 0 & -1 \\ Z_\theta & Z_q \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}\right\} - \frac{M_\delta}{\tau}\delta \quad (40)$$

with $\delta -$ the actuator equation solution:

$$\tau\dot{\delta} + \delta = \delta_c . \quad (41)$$

The coefficients k_p and k_d of the proportional - derivative linear dynamic compensator are calculated by imposing desired roots for the characteristic equation of the linear closed loop subsystem with unitary and negative feedback $(\hat{v}_a = 0)$; we obtain $k_p = 100, k_d = 15$. The dynamic compensator is a first order compensator, having only one state variable $\tilde{y} = e$. As a consequence, according to (19),

$$\boldsymbol{E}^T = \boldsymbol{e}^T = \left[\tilde{y} \;\; \dot{\tilde{y}} \;\; \ddot{\tilde{y}}\right], \overline{A} = \left(A - d_c b\tilde{c}\right), z = \overline{C}\boldsymbol{E} = \tilde{c}e = \tilde{y} . \quad (42)$$

The state observer is described by equations (21); $\hat{\boldsymbol{E}} = \hat{\boldsymbol{e}}, \hat{z} = \hat{e}$. The matrix L is calculated such that the matrix $\tilde{A} = \overline{A} - L\overline{C}$ is asymptotically stable. The adaptive component v_a is obtained by using equation (27), with $\hat{W}$ and $\hat{V} -$ the solutions of the equations (24) and the input vector η having the form (28), with the components in (29); $d = 0.05, \Gamma_W = 23, \Gamma_V = 12.5, k = 0.16$; P and $\tilde{P}$ are the solutions of the Lyapunov equations (26);

$$\eta^T = \left[1 \;\; v(t) \;\; v(t) - v(t-d) \;\; \left(v(t) - v(t-d)\right) - \left(v(t-d) - v(t-2d)\right) \;\; y(t) \;\; y(t) - y(t-d) \;\; \left(y(t) - y(t-d)\right) - \left(y(t-d) - y(t-2d)\right)\right]. \quad (43)$$

The function σ is calculated by means of (25), with $z = \hat{V}^T\eta$ and $a = \left[1 \;\; 0.9 \;\; 0.8 \;\; 0.7 \;\; 0.6 \;\; 0.5 \;\; 0.4\right]$. The component $\bar{v}$ is obtained by using the equation (10), with $k_z = 0.8, k_v = 0.7$, and $\overline{Z} = 50$.

The block diagram of the automatic control system for the helicopters' pitch angle control, having the general form in Fig. 2, is presented in Fig. 3; its reference model is presented in Fig. 4, with $v_h = 0$ if the actuator is linear, or $v_h \neq 0$ if the actuator is nonlinear. In the case of a nonlinear actuator, the second input of the command filter (reference model) is the signal v_h provided by the subsystem in Fig. 6. If the system has a linear actuator, the only input of the reference model (described by equation (32) and Fig. 4) is $y_c = \theta_c$.

Input saturation and input rate saturation are significant problems in the case of adaptive control; saturation violates the assumption of affinity in control and the assumption that the sign of the effect of the control is known/non-zero, since the effect of additional control input is zero once saturation is entered. Saturation is also related to the controllability and the invertability issues during saturation; this temporary loss of control effectiveness violates necessary conditions for dynamic inversion (Johnson and Calise 2000). The main purpose of the Pseudo Control Hedging (PCH) control method is to prevent the adaptive element of an adaptive control system from trying to adapt to a class of system input characteristics (characteristics of plant or of the controller). To achieve this issue, the adaptive law is prevented from "seeing" the system characteristic. A plain-language conceptual description of the method belongs to Johnson and Calise: *The reference model is moved backwards (hedged) by an estimate of the amount the plant did not move due to system characteristics the control designer does not want the adaptive control element to 'know' about*. In other words, in the context of an adaptive control law involving dynamic inversion, "movement" should be replaced by some system signal. Preventing the adaptive element from 'knowing' about a system characteristic means to prevent that adaptive element from seeing the system characteristic as model tracking error (Johnson and Calise 2000).

On the other hand, the adaptive controllers are sensitive to actuator nonlinearity. That is why in these controllers architecture one may introduce a block which limits the pseudo-control by means of a component (PCH) representing an estimation of the execution element dynamics (Johnson and Calise 2001; Johnson et al. 2000; Calise et al. 2000; Calise et al. 2006; Hovakimyan et al. 2005; Johnson and Calise 2000; Johnson and Calise 2001); PCH modifies the neural network adapting signal. The neural networks have adapting difficulties because of the execution element characteristics: time delays, displacement/velocity saturation, and the execution element dynamics. The aim of the PCH is to prevent neural networks adapting to these characteristics. Although the execution elements are in the saturation zone, the neural network of the adaptive control systems (the two new systems presented in this paper) compensates the inversion error. That is why we introduce a PCH block, which limits the adaptive pseudo-control v_a (and of course the variable v) with a component representing the actuator dynamics estimation. Thus, PCH "moves back" the reference model, introducing a reference model response correction with respect to the estimation of the execution element position. The signal provided by PCH is a reference model additional input (Johnson and Calise 2001; Johnson et al. 2000; Calise et al. 2000; Calise et al. 2006; Hovakimyan et al. 2005; Johnson and Calise 2000; Johnson and Calise 2001).

According to Fig. 2, the adaptive command v_a modifies the error $\tilde{y} = \bar{y} - y$ by changing of the signal y. To avoid the adapting difficulties of NN_c when the execution element works in the saturation zones (position and velocity saturation zones), we must make the neural network “not to see” the execution element operation in the saturation zones and to adapt itself in order to compensate the control system dynamic inversion (the flying object dynamics). Therefore, we may operate with $\bar{y}$ to have the equivalent effect of the modification of y and, as a consequence, it results a modification of $\tilde{y}$. Thus, we introduce an additional input v_h in the reference model. In the following figure and in the following sections we denote with A – the flying object dynamics and with $\hat{\mathrm{A}}^{-1}$ – the flying object (helicopter) inverse estimated dynamics.

In Fig. 5 we present the system consisting of the control system model (the flying object dynamics - A), the actuator (execution element), and the estimated model of A^{-1} (i.e. $\hat{\mathrm{A}}^{-1}$). x is a part of the state vector and it contains the state variables associated (attached) to the r order subsystem; the rest of the states are included in the vector χ.

Because the dependence between δ and δ_c (see Figs. 5 and 6) is expressed by means of a nonlinear function $h_a\left(\delta_c, \dot{\delta}_c\right)$, it results:

$$\hat{h}_r(x, \delta_c) \neq \hat{h}_r(x, \hat{\delta}); \tag{44}$$

we obtain the difference between the two functions:

$$v_h = \hat{h}_r\left(x, \delta_c\right) - \hat{h}_r(x, \hat{\delta}); \tag{45}$$

$\delta = \hat{\delta}$ because we considered that δ is the measured variable ($\hat{\delta}$ has the significance of an estimated variable).

Taking into account that:

$$\hat{h}_r\left(x, \delta_c\right) = \hat{h}_r\left(x, \hat{h}_r^{-1}\left(x, v\right)\right) = v, \tag{46}$$

the function (45) becomes:

$$v_h = v - \hat{h}_r(x, \hat{\delta}). \tag{47}$$

If we use a linear actuator (execution element), the actuator is described by equation (41); if the actuator is nonlinear, instead of the model described by (41), we use the model which is a subsystem of the system in Fig. 6. In the nonlinear actuator case, the system in Fig. 3 is augmented with the system in Fig. 6, where $x = \dot{\theta}$; we choose $T = 0.03\,\mathrm{s}$, while the actuator control limits, in displacement and velocity, are $5\deg$ and $50\deg/\mathrm{s}$, respectively.

Adaptive Control System for the Control of the Helicopters' Velocity and Pitch Angle

In this section of the paper we design a new adaptive system for the control of helicopters velocity and pitch angle by using the same algorithms as in the third section; in fact, the adaptive system which is to be designed here is an extension of the system presented in the third section.

The dependence between the helicopter accelerations with respect to its three coordinate axes $(\ddot{X}, \ddot{Y}, \ddot{Z})$ and the external forces (F_x, F_y, F_z), which act on the flying object dynamics (A), is (Prasad et al. 1999):

$$\begin{bmatrix}\ddot{X} & \ddot{Y} & \ddot{Z}\end{bmatrix}^T = L(\varphi, \theta, \psi)\begin{bmatrix}F_x / m & F_y / m & F_z / m\end{bmatrix}^T + \begin{bmatrix}0 & 0 & g\end{bmatrix}^T, \quad (48)$$

where $L(\varphi, \theta, \psi)$ is the coordinates transformation matrix; the transformation is made between the local horizontal frame and the helicopter tied frame; g is the gravitational acceleration. To obtain the helicopter displacement between two points with the coordinates (X, Y, Z) and (X_c, Y_c, Z_c), the following linear dynamic compensator equations are used (Sharma and Calise 2001):

$$U_1 = k_x(\overline{X} - X) + k_{\dot{x}}(\dot{\overline{X}} - \dot{X}) + \ddot{\overline{X}}, U_2 = k_y(\overline{Y} - Y) + k_{\dot{y}}(\dot{\overline{Y}} - \dot{Y}) + \ddot{\overline{Y}}, U_3 = k_z(\overline{Z} - Z) + k_{\dot{z}}(\dot{\overline{Z}} - \dot{Z}) + \ddot{\overline{Z}}. \quad (49)$$

According to (48), the dynamic inversion based control laws are:

$$v_1 = U_1, v_2 = U_2, v_3 = U_3 - g, \quad (50)$$

while the equations expressing the dependence between these command laws and the imposed altitude (Prasad et al. 1999) are:

$$\overline{\theta} \cong \mathrm{arctg}\frac{U_1 \cos\overline{\psi} + U_2 \sin\overline{\psi}}{U_3 - g}, \overline{\varphi} \cong \arcsin\frac{-U_1 \sin\overline{\psi} + U_2 \cos\overline{\psi}}{\sqrt{U_1^2 + U_2^2 + (U_3 - g)^2}}. \quad (51)$$

If we desire a constant altitude flight, without deviation from the flight direction $(\Delta\overline{\psi} = 0)$, then $U_3 \cong 0$ and the equations (51) get the form (Calise 2003):

$$\bar{\theta} \cong -\frac{U_1}{g}, \bar{\varphi} \cong \frac{U_2}{\sqrt{U_1^2 + U_2^2 + g^2}}. \quad (52)$$

In the case when we impose a displacement velocity $V_x = \dot{X} = \text{const} = \dot{X}_c$, the first equation (49) becomes:

$$U_1 = k_{\dot{x}}\left(\dot{\bar{X}} - \dot{X}\right) + \ddot{\bar{X}}; \quad (53)$$

in the complex domain, according to (53), $\bar{\theta}$ becomes:

$$\bar{\theta} = \frac{k_{\dot{x}}\mathrm{s}\left(\bar{X} - X\right) + \mathrm{s}^2\bar{X}}{-g} = \frac{k_{\dot{x}}\left(\bar{V}_x - V_x\right) + \dot{\bar{V}}_x}{-g}. \quad (54)$$

Taking into account equation (54) and the expression $\theta = -\ddot{X}/g \ \left(\theta = -\mathrm{s}^2 X(\mathrm{s})/g\right)$, it results:

$$\frac{V_x(\mathrm{s})}{\bar{V}_x(\mathrm{s})} = \frac{\dot{X}(\mathrm{s})}{\dot{\bar{X}}(\mathrm{s})} = \frac{b_0 k_p \mathrm{s} + b_0 k_p k_{\dot{x}}}{\mathrm{s}^4 + b_0 \mathrm{s}^3 + b_0 k_d \mathrm{s}^2 + b_0 k_p \mathrm{s} + b_0 k_p k_{\dot{x}}}. \quad (55)$$

We want to transform this transfer function into a three order transfer function (our aim is to compensate a zero with a pole); for doing this, we choose a Visnegradski three order transfer function and we obtain the values of the coefficients $b_0, k_p, k_d, k_{\dot{x}}$.

The velocity equation from (30) is:

$$\dot{V}_x = X_u V_x + X_q \dot{\theta} + X_\theta \theta + X_\beta \beta + X_W V_z + X_\delta \delta + X_m \delta_m, \quad (56)$$

where we added a term containing the engine command (δ_m). Equation (56) relative degree is $r = 1$ and, therefore, $H_{dx}(\mathrm{s})$ – the linear system transfer function, with the general form in Fig. 1, having the output $y = V_x$ and the relative degree $r = 1$, and the transfer function of the reference model $H_{dm}(\mathrm{s})$ are:

$$H_{dx}(\mathrm{s}) = \frac{1}{\mathrm{s} + c}, H_{dm}(\mathrm{s}) = \frac{1}{\mathrm{s} + p}. \quad (57)$$

Equation (12), with $b_0 = 1$, gets the form:

$$\dot{V}_x = -\lambda_0 V_x + v_1 + \varepsilon_1; \quad (58)$$

from (55) and (58), by identification, we obtain:

$$\delta_m = \hat{h}_m^{-1}(V_x, v_1) = \frac{1}{X_m}\left[(X_u + \lambda_0)V_x + v_1\right], \tag{59}$$

$$\varepsilon_1 = X_q\dot{\theta} + X_\theta\theta + X_\beta\beta + X_W V_z + X_\delta\delta. \tag{60}$$

We choose the coefficients: $p = 0.5$, $c = 0.3$, $k_{\dot{x}} = 0.5$, $b_1 = 1$, $P_1 = 1/k_{p_1} = 1$, $n_1 = 5$, $d = 0.05$, $\Gamma_V = 30$, $\Gamma_W = 40$, $k = 0.1$, while η_1 has the form:

$$\begin{aligned}
&\eta_1 = \left[1\ \bar{v}_{d1}^T(t)\ \bar{y}_{d1}^T(t)\right]^T, \\
&\bar{v}_{d1}^T(t) = \left[v_1(t)\ \ v_1(t) - v_1(t-d)\ \ (v_1(t) - v_1(t-d)) - (v_1(t-d) - v_1(t-2d))\right]^T, \\
&\bar{y}_{d1}^T(t) = \left[y_1(t)\ \ y_1(t) - y_1(t-d)\ \ (y_1(t) - y_1(t-d)) - (y_1(t-d) - y_1(t-2d))\right]^T.
\end{aligned} \tag{61}$$

The component $\bar{v}_1$ is calculated by means of equation (10), where $k_z = 0.6$, $k_v = 0.8$, $\bar{Z} = 20$; $V_{x_c} = 20$ m/s.

For the calculation of the inversion error ε_1, the differential equations of β and V_z from (30) are used, i.e.:

$$\begin{aligned}
&\dot{\beta} = B_u V_x - \dot{\theta} + B_\beta\beta + B_\delta\delta, \\
&\dot{V}_z = Z_u V_x + Z_q\dot{\theta} + Z_\theta\theta + Z_\beta\beta + Z_W V_z + Z_\delta\delta + X_z\delta_m,
\end{aligned} \tag{62}$$

or

$$\begin{bmatrix}\dot{\beta} \\ \dot{V}_z\end{bmatrix} = \begin{bmatrix}B_\beta & 0 \\ Z_\beta & Z_W\end{bmatrix}\begin{bmatrix}\beta \\ V_z\end{bmatrix} + \begin{bmatrix}0 & -1 & B_u & 0 & 0 \\ Z_\theta & Z_q & Z_u & Z_\delta & X_z\end{bmatrix}\left[\theta\ \dot{\theta}\ V_x\ \delta\ \delta_m\right]^T. \tag{63}$$

Thus, ε_1 is calculated by using equation (60), with θ, $\dot{\theta}$, and δ determined by the subsystem in Fig. 3; β and V_z are calculated by means of equation (63), while δ_m is obtained with (59). The block diagram of the adaptive control system for the control of the helicopters velocity and pitch angle is presented in Fig. 7.

Similarly with the above presented system we can design a new system for the control of the vertical velocity V_z; in this case $U_3 \neq 0$ and

$$\theta_c = 1/\tan\left(\frac{U_1}{U_3 - g}\right); \tag{64}$$

U_3 has the following form:

$$U_3 = k_z\left(\bar{V}_z - V_z\right) + \dot{\bar{V}}_z. \tag{65}$$

The structure of this system is similar with the one in Fig. 7, where: V_x is replaced with $V_z = y_2$, U_1 is replaced with U_3, v_1 becomes v_3, and ε_1 becomes ε_3.

Simulation Results

The coefficient values of the model (30) have been borrowed from Calise's work (Calise et al. 2000):

$$\begin{aligned}&X_u = -0.0553,\ X_q = 1.413,\ X_\theta = -32.1731,\ X_\beta = -19.9033,\ X_w = 0.0039,\ M_u = 0.2373,\ M_q = -6.9424,\\ &M_\beta = 68.2896,\ M_w = 0.002,\ B_u = 0.0101,\ B_\beta = -2.1633,\ Z_u = -0.0027,\ Z_q = -0.0236,\ Z_\theta = -0.2358,\\ &Z_\beta = -0.1233,\ Z_w = -0.5727,\ X_\delta = 11.2579,\ M_\delta = -38.6267,\ B_\delta = -4.2184,\ Z_\delta = 0.0698,\\ &\hat{M}_\delta = 0.5M_\delta,\ \hat{M}_q = 2M_q,\ \tau = 0.05.\end{aligned}$$

The solution P of the first equation (26), with $b^T = [0\ 0\ 1]$ and $\bar{C} = [1\ 0\ 0]$, for $Q = I_3$, and the linear observer gain matrix are, respectively:

$$P = \begin{bmatrix} 3.3593 & 3.4555 & 2.5981 \\ 3.4555 & 5.8130 & 3.9555 \\ 2.5981 & 3.9555 & 4.4555 \end{bmatrix},\ L = \begin{bmatrix} -0.2494 \\ 0.3966 \\ -0.2640 \end{bmatrix}.$$

In Fig. 8 the characteristics $\bar{\theta}(t), \theta(t), \varepsilon(t), v_a(t), \hat{\delta}(t), \delta(t)$, and $v(t)$ ($\bar{\theta}, \varepsilon, \hat{\delta} -$ solid line, $\theta, v_a, \delta -$ dashed line) are presented. If we use a linear actuator it is found that $\theta \to \bar{\theta}$, $v_a \to \varepsilon$ (the adaptive component of the command compensates the approximation error of the function h_r); also, $\delta \to \hat{\delta}$ and $v \to 0$. If a nonlinear actuator is used, the characteristics in Fig. 9 result; we remark additional graphics with respect to Fig. 8: the characteristics $v_h(t)$ and $\dot{\theta}(\theta)$. $v_h = 0$ means that the actuator is in saturation state, while $v_h \neq 0$ means that the actuator works in the linear zone. The system phase portrait (the characteristic $\dot{\theta}(\theta)$) shows that the nonlinear system tends to stable limit cycle.

Remark 1:

The new designed system works both with linear and nonlinear actuator; in fact, we talk about models of the linear or nonlinear actuators. In real cases, the actuator of the system may be close to a linear or a nonlinear actuator model. Usually, the real actuator is closer to the nonlinear model because the deflection angles and the angular rates are limited and, furthermore, these actuators are characterized by a delay time. By using PCH, the system works in the linear zones of the nonlinearities which can be found in the model of the nonlinear actuator (Fig. 6). Therefore, for the linear zones, the subsystem with unitary negative feedback is described by equation (41), i.e. the equation of a linear model. Analyzing the characteristics in Figs. 8 and 9, we conclude that the characteristics in Fig. 9 are better because the rate of change for the state and command variables is smaller.

In Fig. 10 we represent the time characteristics of the system for the automatic control of helicopter velocity and pitch angle (linear actuator), while, in Fig. 11, the same characteristics are represented for the nonlinear actuator case. From Figs. 10 and 11 we may conclude that: $V_x \to \overline{V}_x \to \overline{V}_{x_c}$, $\theta \to \overline{\theta} \to \theta_c$; $v_a \to \varepsilon$, $v_{a_1} \to \varepsilon_1$ (the adaptive components compensate the approximation errors ε and ε_1); $\delta, \delta_c, \delta_m \to 0$; $v, v_1 \to 0$ and $u, u_1 \to 0$; the transient regime periods are small and the overshoot is small too. The nonlinear dynamic regimes are stable (the phase trajectories tend to stable limit cycles).

Remark 2:

The obtained results have been briefly compared with the results in (Prasad et al. 1999) and we may remark the superiority of our new control system. Our conclusion is based on the comparison between the dynamic and stationary performances presented in our work and in Prasad's work; the quality indicators (overshoot, stationary error, transient regime period, rate of change for the state and command variables) are generally better for our system. A rigorous comparison between our system and the system presented in the work of Prasad, from the results point of view, can not be made because we do not know if a tuning process was taken into account in Prasad's work and how deep he tuned the parameters of the controllers, if he did. On the other hand, we have not the trim values used by Prasad et al; in this paper simulation we have judiciously chosen the trim values, but these are different from the ones considered in Prasad's work. However, we may remark the proper functioning of our new designed system and its good results.

Conclusions

In this paper the authors design an adaptive control system for the command and control of the helicopters' pitch angle and an adaptive control system for the command and control of an experimental R-50 helicopter velocity and pitch angle by using the dynamic inversion concept, linear dynamic compensators, and nonlinear adaptive controllers with neural networks. The adaptive components of the automatic control laws have to compensate the approximation errors of the dynamic models nonlinear functions; we use either linear or nonlinear actuators. In the case of nonlinear actuators use, we introduce in the automatic control system a PCH (Pseudo Control Hedging) block to limit the adaptive pseudo-control in order to cancel the neural networks' training difficulties; the PCH block introduces an additional input in the reference model.

For the two adaptive control systems presented in this paper, Matlab environment is used to obtain the time characteristics (with linear and nonlinear actuators) describing the two systems. These characteristics prove the good dynamic and stationary performances of the two automatic control systems. The phase trajectories of the systems with nonlinear actuators tend to stable limit cycles.

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