

Determination and Control of the Satellites' Attitude by using a Pyramidal Configuration of Four Control Moment Gyros

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Abstract: The paper presents a new architecture for mini-satellites' attitude control using a cluster consisting of four control moment gyros, in pyramidal configuration, and feedback from the quaternion and angular velocity vectors. The designed control law modifies the cluster's equivalent gyroscopic moment, the equivalent kinetic moment and the angular velocities' vector, this leading to the modification of the quaternion vector and to the change of the satellite's attitude. Matlab environment is used for the architecture's software implementation and validation, this being achieved for a mini-satellite involved in a typical motion around its own axis.

1 INTRODUCTION

To have multiple capabilities (missiles' tracking, land mobile targets' tracking etc.), the satellites must have good rotational handling and agility. Such satellites need an automatic system for their attitude's control (ASAC) by performing fast slewing maneuvers; the fast repositioning maneuvers are influenced by the physical limitations of the sensors and actuators, by the structural rigidity of the satellites and by the mission's type (Jovanovic, 2014; Bouwmeester and Guo, 2010; Lavet, 2010).

The performances of the ASACs are limited by the satellites' dimensions, especially in the case of small satellites' usage. In these cases, the gyroscopic wheels can not assure good agility because their limited capability to create control couples (maximum 1.5 Nm) (Wie et al., 2002; Berner, 2005; Lungu, 2008); therefore, in these cases, the control moment gyros (CMGs) are ideal because these can generate significant control couples (100-3000 Nm). As actuators, the CMGs have been used on large cosmic devices (Skylab, MIR, ISS) and, recently, on mini-satellites. The most important drawback of CMGs is related to the singularities' appearance (Ford and Hall, 2000); for particular values of the gyroscopic frames' angles, the CMGs do not produce big enough couples and the gyroscopic frames get stuck; therefore, special control laws must be designed to avoid such situations.

The CMGs may be single-gimbals type (SGCMG) of double-gimbals type (DGCMG); for the DGCMGs,

the control law can easily avoid the singularities, due to their extra degree of freedom, but the SGCMGs are simpler in terms of construction and hardware, having many other advantages related to their price, power and reliability (Berner, 2005).

By CMGs grouping, the clusters are obtained; the most used clusters are the pyramidal ones, having four CMGs; such configuration is presented in our paper.

In this paper, the authors propose a new architecture for the control of the mini-satellites' attitude; it is based on a cluster consisting of four control moment gyros and uses feedback from the system's quaternion vector (which gives information regarding the attitude of the satellite) and satellite's angular rates. The control law to be designed will modify the equivalent gyroscopic moment of the cluster, the equivalent kinetic moment, and the angular rates' vector, this leading to the modification of the quaternion and to the change of the satellite's attitude. The new architecture is implemented and validated through complex numerical simulations for the case of a mini-satellite involved in a typical maneuver around its own axis.

2 PYRAMIDAL CONFIGURATION WITH FOUR CMGS

We consider the motion of a mini-satellite which performs a typical maneuver (a complete cycle) around its own axis (with constrained angular speed);

the three phases of motion (the accelerated angular motion, the uniform angular motion and the braked motion) are described in detail in (Wie and Lu, 1995).

To control the satellite, a gyro system (a cluster having pyramidal configuration), consisting of four control moment gyros, is used. The pyramidal configuration is presented in Fig. 1. The gyros' rotation axes (the kinetic moments $\vec{K}_i, i = \overline{1,4}$) are initially oriented parallel to the sides of the pyramid base, the axes of the gyroscopic frames (the angular velocities associated to the frames' rotations $\vec{\gamma}_i, i = \overline{1,4}$) are

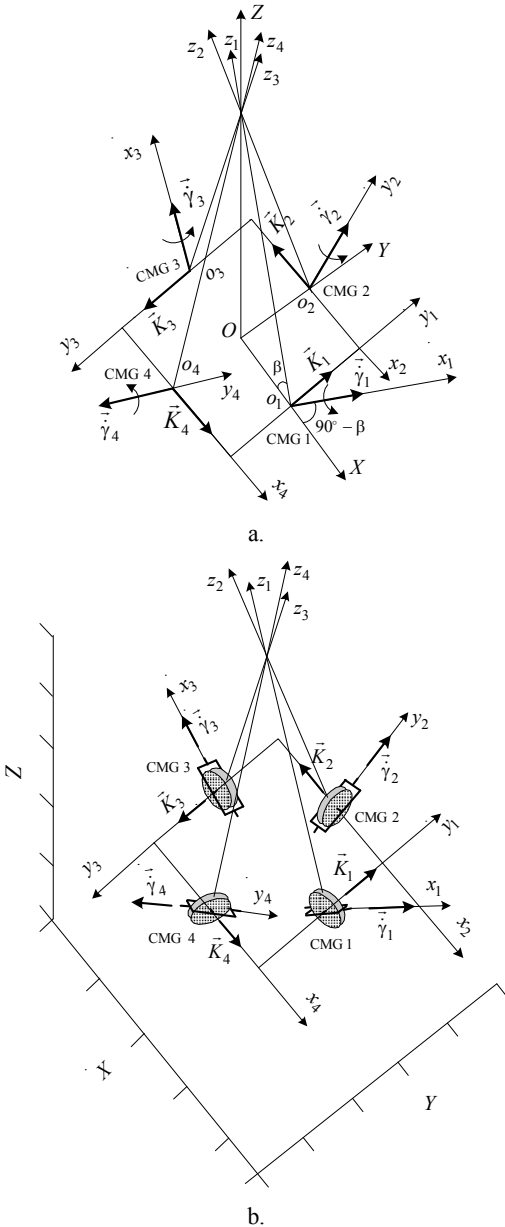


Figure 1: Pyramidal configuration with four CMGs.

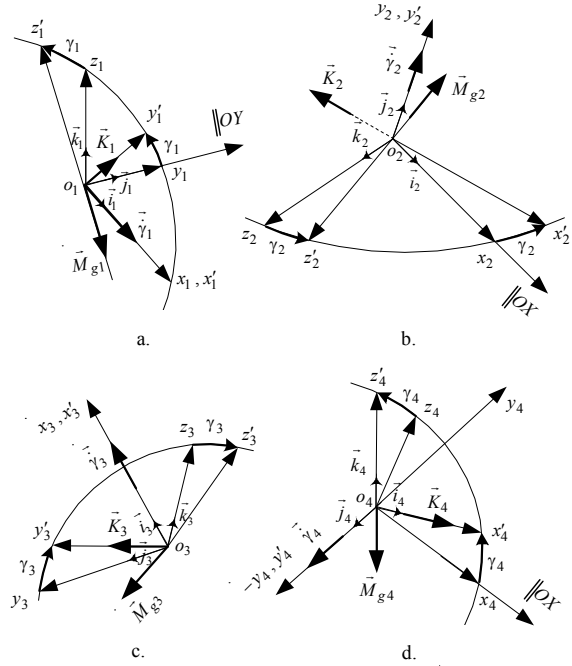


Figure 2: The rotations of the gyroscopic frames, angular variables, kinetic moments and gyroscopic couples.

perpendicular to the side faces of the pyramid, while the transversal axes of the CMGs are perpendicular to the gyro axes and gyroscopic frames. In Fig. 2, one presents the rotations of the gyroscopic frames, the angular variables, the kinetic moments and the gyroscopic couples. The frame S (satellite linked) is denoted with $OXYZ$. The gyroscopic tied frames are $o_i x_i y_i z_i, i = \overline{1,4}$; initially (in the absence of the gyroscopic frames' rotations, we have $\gamma_i = 0$); these frames are oriented towards the axes of the frames $o_i x_i y_i z_i$.

For the beginning, let us consider that the pyramid base is fixed ($\vec{\omega} = 0$) and, by rotation of the gyroscopic frames with the angular velocities $\vec{\gamma}_i, i = \overline{1,4}$, the gyros react by gyroscopic couples (Wie et al., 2002):

$$\vec{M}_{gi} = -\vec{K}_i = \vec{K}_i \times \vec{\gamma}_i, \vec{K}_i = d\vec{K}_i / dt. \quad (1)$$

In figs. 1 and 2, there are highlighted the rotations of the gyroscopic trihedrals with respect to the initial positions $o_i x_i y_i z_i$, the angular variables (γ_i and $\dot{\gamma}_i$), the directions of the kinetic moments \vec{K}_i after the rotations of the gyroscopic frames characterized by the rotation angles γ_i and the gyroscopic couples $-\vec{M}_{gi}$; $\vec{i}_i, \vec{j}_i, \vec{k}_i, i = \overline{1,4}$, represents the axes' versors of the trihedrals $o_i x_i y_i z_i$. By means of Fig. 1 and equation (1), one can obtain the four gyroscopic

moments:

$$\begin{aligned} \vec{M}_{g1} &= -\vec{K}_1 = \begin{vmatrix} \vec{i}_1 & \vec{j}_1 & \vec{k}_1 \\ 0 & K_1 \cos \gamma_1 & K_1 \sin \gamma_1 \\ \dot{\gamma}_1 & 0 & 0 \end{vmatrix} = \\ &= K_1 \sin \gamma_1 \cdot \dot{\gamma}_1 \vec{j}_1 - K_1 \cos \gamma_1 \cdot \dot{\gamma}_1 \vec{k}_1 = \\ &= \vec{M}_{g1y1} + \vec{M}_{g1z1} = \vec{M}_{g1z'1}; \end{aligned} \quad (2)$$

$$\begin{aligned} \vec{M}_{g2} &= -\vec{K}_2 = \begin{vmatrix} \vec{i}_2 & \vec{j}_2 & \vec{k}_2 \\ -K_2 \cos \gamma_2 & 0 & K_2 \sin \gamma_2 \\ 0 & \dot{\gamma}_2 & 0 \end{vmatrix} = \\ &= -K_2 \sin \gamma_2 \cdot \dot{\gamma}_2 \vec{i}_2 - K_2 \cos \gamma_2 \cdot \dot{\gamma}_2 \vec{k}_2 = \\ &= \vec{M}_{g2x2} + \vec{M}_{g2z2} = \vec{M}_{g2z'2}; \end{aligned} \quad (3)$$

$$\begin{aligned} \vec{M}_{g3} &= -\vec{K}_3 = \begin{vmatrix} \vec{i}_3 & \vec{j}_3 & \vec{k}_3 \\ 0 & K_3 \cos \gamma_3 & K_3 \sin \gamma_3 \\ \dot{\gamma}_3 & 0 & 0 \end{vmatrix} = \\ &= K_3 \sin \gamma_3 \cdot \dot{\gamma}_3 \vec{j}_3 - K_3 \cos \gamma_3 \cdot \dot{\gamma}_3 \vec{k}_3 = \\ &= \vec{M}_{g3y3} + \vec{M}_{g3z3} = \vec{M}_{g3z'3}; \end{aligned} \quad (4)$$

$$\begin{aligned} \vec{M}_{g4} &= -\vec{K}_4 = \begin{vmatrix} \vec{i}_4 & \vec{j}_4 & \vec{k}_4 \\ K_4 \cos \gamma_4 & 0 & K_4 \sin \gamma_4 \\ 0 & -\dot{\gamma}_4 & 0 \end{vmatrix} = \\ &= K_4 \sin \gamma_4 \cdot \dot{\gamma}_4 \vec{i}_4 - K_4 \cos \gamma_4 \cdot \dot{\gamma}_4 \vec{k}_4 = \\ &= \vec{M}_{g4y4} + \vec{M}_{g4z4} = \vec{M}_{g4z'4}. \end{aligned} \quad (5)$$

The component \vec{M}_{g1y1} acts upon the gyroscopic frame without rotate it and, thus, it acts on the base (S), while the component \vec{M}_{g1z1} acts upon the gyro-frame ensemble and, thus, on the base. The component \vec{M}_{g2x2} acts on the gyroscopic frame, while \vec{M}_{g2z2} acts on the gyro-frame ensemble and, thus, both components act on the base (S). Same conclusion can be drawn regarding the components \vec{M}_{g3y3} and \vec{M}_{g3z3} – third gyro and for the components \vec{M}_{g4y4} and \vec{M}_{g4z4} – fourth gyro.

According to Fig. 1a, the projections of the gyroscopic couple \vec{M}_{g1} (first CMG) on the axes of the frame S (*OXYZ* – satellite linked) are:

$$\begin{aligned} M_{g1x} &= |M_{g1z1}| \cos \beta = K_1 \cos \gamma_1 \cos \beta \cdot \dot{\gamma}_1, \\ M_{g1y} &= M_{g1y1} = K_1 \sin \gamma_1 \cdot \dot{\gamma}_1, \\ M_{g1z} &= -|M_{g1z1}| \sin \beta = -K_1 \cos \gamma_1 \sin \beta \cdot \dot{\gamma}_1. \end{aligned} \quad (6)$$

Similarly, the projections of the gyroscopic couples \vec{M}_{g2} , \vec{M}_{g3} and \vec{M}_{g4} on the same axes are deduced as:

$$\begin{aligned} M_{g2x} &= M_{g2x2} = -K_2 \sin \gamma_2 \cdot \dot{\gamma}_2, \\ M_{g2y} &= |M_{g2z2}| \cos \beta = K_2 \cos \gamma_2 \cos \beta \cdot \dot{\gamma}_2, \end{aligned} \quad (7)$$

$$\begin{aligned} M_{g2z} &= -|M_{g2z2}| \sin \beta = -K_2 \cos \gamma_2 \sin \beta \cdot \dot{\gamma}_2; \\ M_{g3x} &= -|M_{g3z3}| \cos \beta = -K_3 \cos \gamma_3 \cos \beta \cdot \dot{\gamma}_3, \\ M_{g3y} &= -M_{g3y3} = -K_3 \sin \gamma_3 \cdot \dot{\gamma}_3, \end{aligned} \quad (8)$$

$$\begin{aligned} M_{g3z} &= -|M_{g3z3}| \sin \beta = -K_3 \cos \gamma_3 \sin \beta \cdot \dot{\gamma}_3; \\ M_{g4x} &= M_{g4x4} = K_4 \sin \gamma_4 \cdot \dot{\gamma}_4, \\ M_{g4y} &= -|M_{g4z4}| \cos \beta = -K_4 \cos \gamma_4 \cos \beta \cdot \dot{\gamma}_4, \end{aligned} \quad (9)$$

$$M_{g4z} = -|M_{g4z4}| \sin \beta = -K_4 \cos \gamma_4 \sin \beta \cdot \dot{\gamma}_4.$$

We denote with \mathbf{K} and \mathbf{M}_g – the equivalent kinetic and gyroscopic moments, respectively; these can be expressed as vectors having the components upon the three axes and verifying the equations:

$$\mathbf{K} = [K_x \ K_y \ K_z]^T, \mathbf{M}_g = [M_{gx} \ M_{gy} \ M_{gz}]^T; \quad (10)$$

putting together the equations (1)-(5), we get:

$$\dot{\mathbf{K}} = -\mathbf{M}_g \quad (11)$$

or $[\dot{K}_x \ \dot{K}_y \ \dot{K}_z]^T = -[M_{gx} \ M_{gy} \ M_{gz}]^T$, where

$$\begin{aligned} M_{gx} &= M_{g1x} + M_{g2x} + M_{g3x} + M_{g4x}, \\ M_{gy} &= M_{g1y} + M_{g2y} + M_{g3y} + M_{g4y}, \\ M_{gz} &= M_{g1z} + M_{g2z} + M_{g3z} + M_{g4z}. \end{aligned} \quad (12)$$

Replacing (6)-(9) in (12), the equation (11) becomes:

$$\dot{\mathbf{K}} = \mathbf{Q} \dot{\boldsymbol{\gamma}}, \quad (13)$$

with (Haruhisa, 1997) $\dot{\boldsymbol{\gamma}} = [\dot{\gamma}_1 \ \dot{\gamma}_2 \ \dot{\gamma}_3 \ \dot{\gamma}_4]^T$ and

$$\mathbf{Q} = J_g \Omega \begin{bmatrix} -c\beta c\gamma_1 & s\gamma_2 & c\beta c\gamma_3 & -s\gamma_4 \\ -s\gamma_1 & -c\beta c\gamma_2 & s\gamma_3 & c\beta c\gamma_4 \\ s\beta c\gamma_1 & s\beta c\gamma_2 & s\beta c\gamma_3 & s\beta c\gamma_4 \end{bmatrix}, \quad (14)$$

where $J_g \Omega$ (J_g – the inertia moment and Ω – the vector of the gyros' angular velocities) is the module of

the gyros' kinetic moments (of the gyroscopic wheels), $c\beta = \cos\beta$, $s\beta = \sin\beta$, $c\gamma_i = \cos\gamma_i$, $s\gamma_i = \sin\gamma_i$.

Furthermore, taking into account the base's rotation with the angular velocity $\vec{\omega}$, the equation of the pyramidal cluster (containing 4 CMGs – Fig. 1) becomes (Wie et al., 2002): $\dot{\vec{K}} + \vec{\omega} \times \vec{K} = -\vec{M}_g$ or, under an algebraic form,

$$\dot{\vec{K}} + \vec{\omega} \times \vec{K} = -\vec{M}_g, \quad (15)$$

with \vec{K} and \vec{M}_g of forms (10) and (Lungu, 2008):

$$\vec{\omega} \times = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}, \quad (16)$$

where $\omega_1, \omega_2, \omega_3$ are the components of the satellite's angular velocity upon the axes of the $OXYZ$ frame.

3 SATELLITE'S ATTITUDE CONTROL BY USING THE PYRAMIDAL STRUCTURE

In this section of the paper, the control of the satellite's attitude will be achieved by means of the complex system presented in Fig. 3a; the system is mainly based on a control law with 2 components: a proportional-integrator component with respect to the quaternion vector (q) and a proportional component with respect to the satellite's angular velocity vector (ω). As the actuators' system, we will use the cluster (pyramidal configuration with four CMGs) presented in the previous section; the actuators' saturation will be considered both from the generated gyroscopic couples' point of view and from the gyroscopic frame angular velocities' point of view. The structure of the system for the calculation of the vector $\dot{\gamma}$ (the vector containing the angular velocities of the CMGs' frames) is presented in Fig. 3b.

The command M_c (the command couple of the system) may be associated to the gyroscopic couple M_g from equation (15); this equation becomes:

$$\dot{\vec{K}} = -M_c - \vec{\omega} \times \vec{K}, \quad (17)$$

modelled within the pyramidal cluster (Fig. 3b); for the calculation of the angular velocity vector $\dot{\gamma}_c$, we use the equations (17) and (13); we get: $\dot{\gamma}_c = Q^+ \dot{\vec{K}}$, with Q^+ – the pseudo-inverse of matrix $Q = Q(\gamma)$.

The signal $\dot{\gamma}_c$ is applied to the gyroscopic frames'

drive motors by means of the dynamic correction filter having the transfer matrix:

$$H_f(s) = \frac{\omega_0^2}{s^2 + 2\xi_0\omega_0s + \omega_0^2} I_{4 \times 4}. \quad (18)$$

The vector of the satellite's command moments is $M_g = -Q\dot{\gamma}$. For satellite's attitude control system in Fig. 3, the following design equations are used:

$$K_p = k_p J, K_p P = k_p J P = kJ, K_d = k_d J, \quad (19)$$

with (Lungu, 2008; Wie and Lu, 1995):

$$k = 2 \left(\omega_n^2 + \frac{2\xi\omega_n}{T_i} \right), k_d = 2\xi\omega_n + \frac{1}{T_i}, \quad (20)$$

$$k_p = \text{diag}[k_{p1} \ k_{p2} \ k_{p3}]^T, k_{pi} = k_d \frac{|q_i(0)|}{|q(0)|} |\omega_i|_{\max}. \quad (21)$$

From (19), one yields:

$$P = kK_p^{-1}J. \quad (22)$$

For sat $(Pq)^*$, with $L = [L_1 \ L_2 \ L_3]^T$, the limits $L_i, i = \overline{1,3}$, are calculated from the stabilized regime condition ($u_c=0$). i.e.: $K_p L = K_d \|\omega\|_{\max}$ or

$$k_p L = k_d \|\omega\|_{\max}, \quad (23)$$

equation equivalent with the following one:

$$L_i = \frac{k_d}{k_{pi}} |\omega_i|_{\max} \stackrel{(21)}{=} \frac{\|q(0)\|}{|q_i(0)|}, i = \overline{1,3}. \quad (24)$$

The satellite's attitude (Euler angles – θ, φ and ψ) may be defined by means of two quaternion vectors ($q = [q_1 \ q_2 \ q_3]^T$ and $\hat{q} = [q_1 \ q_2 \ q_3 \ q_4]^T$); the significances of these angles are similar to the ones expressing the attitude of an aircraft with respect to the Earth tied frame: φ is associated to the roll angle, θ – associated to the pitch angle and ψ – associated to the direction angle. The differential equations of the quaternions are (Heiberg et al., 2000):

$$\dot{q} = -\frac{1}{2} \vec{\omega} \times q + \frac{1}{2} q_4 \vec{\omega}, \dot{q}_4 = -\frac{1}{2} \vec{\omega}^T q; \quad (25)$$

the correlation formulas between the components of the quaternion vector \hat{q} and the satellite's attitude angles are (Wen and Delgado, 1991):

$$\begin{aligned} \theta &= \operatorname{atan} \frac{2(q_1q_3 + q_2q_4)}{-q_1^2 - q_2^2 + q_3^2 + q_4^2}, \\ \varphi &= \operatorname{asin} [2(q_1q_4 - q_2q_3)], \\ \psi &= \operatorname{atan} \frac{2(q_1q_2 + q_3q_4)}{-q_1^2 + q_2^2 - q_3^2 + q_4^2}. \end{aligned} \quad (26)$$

For the calculation of the angular velocities' vector ($\dot{\gamma}_c$), the pseudo-inverse of matrix Q must be obtained first. From the methods for the calculation of the pseudo-inverse, we can choose the robust type inverse and the equation: $Q^+ = Q^T(QQ^T + \lambda I)^{-1}$, with I - the identity matrix. This formula does not always guarantee the avoidance of single frame CMGs' singularities. Such a CMG can get into the singularity zone in the presence of sensor noise. Furthermore, if the system is controlled such that it goes into the singularity zone, it can not escape from this state. Therefore, to remove this drawback, we will use a simple but effective equation for the calculation of the pseudo-inverse of matrix Q (Wie et al., 2002; Haruhisa, 1997); it is designed especially for reorientation maneuvers, stage in which it is not necessary to achieve certain accuracy.

The pseudo-inverse is obtained by using the formula (Wie et al., 2002; Haruhisa, 1997):

$$Q^+ = Q^T(QQ^T + \lambda E)^{-1}, \quad (27)$$

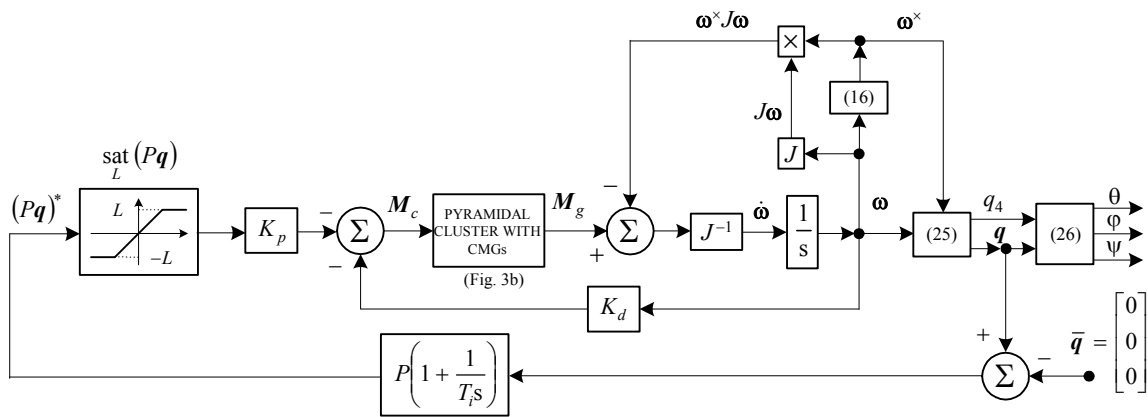
With

$$E = \begin{bmatrix} 1 & \varepsilon_3 & \varepsilon_2 \\ \varepsilon_3 & 1 & \varepsilon_1 \\ \varepsilon_2 & \varepsilon_1 & 1 \end{bmatrix} > 0, \quad (28)$$

where ε_i is continuously modulated:

$$\varepsilon_i = \varepsilon_0 \sin(\omega_i t + \Phi_i), i=1,3; \quad (29)$$

this guarantees that $Q^+ u_k = Q^+ \dot{K} \neq 0$. This solution does not ensure the avoidance of singularity, but rather its proximity and transit.



a.

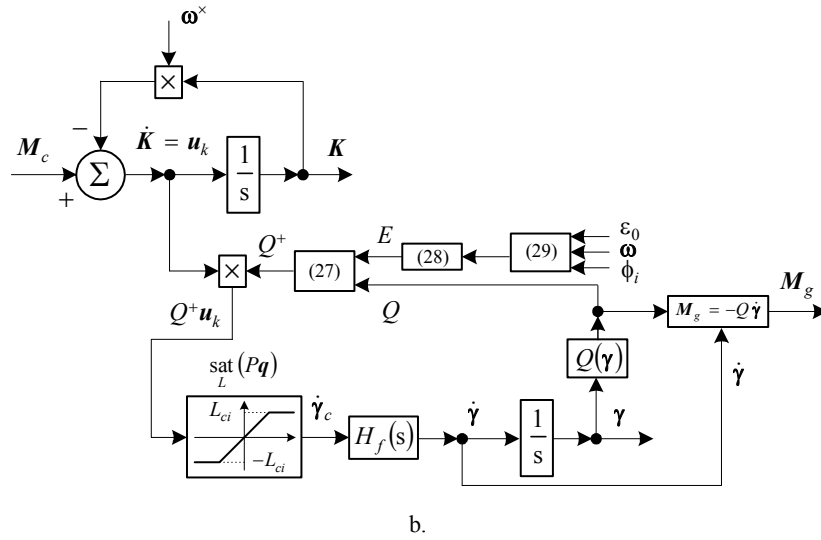


Figure 3: Satellite's attitude control system using a pyramidal cluster with four CMGs (a) and a subsystem for the calculation of the vector $\dot{\gamma}$ (b).

4 NUMERICAL SIMULATION RESULTS

In this section, the satellite's attitude control system (Fig. 3), using a cluster consisting of four CMGs (Fig. 1), is software implemented and validated in Matlab/Simulink environment, for the case of a mini-satellite. The attitude of the satellite (the angles θ , φ and ψ) will be controlled by means of the quaternion vectors ($q = [q_1 \ q_2 \ q_3]^T$ and $\hat{q} = [q_1 \ q_2 \ q_3 \ q_4]^T$) and of the gyroscopic moment M_g (the output signal of the pyramidal cluster – Fig. 3b); M_g modifies the vector of angular velocities (ω) and the equivalent kinetic moment vector K ; according to (11) and (13), the modification of K leads to the actualization of the vectors γ , $\dot{\gamma}$ and of the matrix $Q(\gamma)$; according to (25), the modification of the vector ω leads to other expressions of the quaternion vectors (q and \hat{q}); the modification of the quaternions is equivalent with the change of the satellite's attitude (see eq. (26)).

Let us consider the matrix of the satellite's inertia moments (Wie et al., 2002):

$$J = \begin{bmatrix} 21.4 & 2100 & 1800 \\ 2100 & 20.1 & 500 \\ 1800 & 500 & 5000 \end{bmatrix} \text{kg m}^2 \quad (30)$$

and the values: $\omega_n = 3 \text{ rad/s}$, $\xi = 0.8$, $T_i = 12 \text{ s}$. With these, we obtain $k = 18.8$, $k_d = 4.88$; k_{pi} is calculated with (21) for

$|\omega_i|_{\max} = 10 \text{ deg/s} = 0.174 \text{ rad/s}$ and the quaternion vector $\hat{q}(0) = [q_1(0) \ q_2(0) \ q_3(0) \ q_4(0)]^T = [0.45 \ 0.5 \ -0.5 \ 0.5454]^T$.

Using (26) and (28), it results:

$$P = \begin{bmatrix} 36.9765 & 0.0703 & 0.0603 \\ 0.0460 & 41.4923 & 3.8713 \\ -0.0049 & -0.4770 & 36.5669 \end{bmatrix}, \quad (31)$$

$$L = [L_1 \ L_2 \ L_3]^T = [1.862 \ 1.677 \ 1.677]^T.$$

The nonlinearity $\text{sat}_L(Pq)^*$ is described by the

$$\text{equation } \text{sat}_L(Pq)^* = \text{sat}_L(\tilde{P}_i) = \begin{cases} -L_i, & \tilde{P}_i < -L_i, \\ \tilde{P}_i, & \tilde{P}_i \in [-L_i, L_i], \\ L_i, & \tilde{P}_i > L_i, \end{cases}$$

with $L_i, i = \overline{1, 3}$, having the values in (31).

The limits of the saturation zone are chosen as: $L_c = [L_{c1} \ L_{c2} \ L_{c3} \ L_{c4}]^T = [1 \ 1 \ 1 \ 1]^T \text{ rad/s}$. For the transfer matrix $H_f(s)$ of form (18), we choose the values: $\omega_0 = 50 \text{ rad/s}$, $\xi_0 = 0.7$, while, for the cluster in Fig. 1, we consider $\beta = 55 \text{ deg}$ and the initial vector $\gamma(0) = [\gamma_1(0) \ \gamma_2(0) \ \gamma_3(0) \ \gamma_4(0)]^T = [\pi/2 \ -\pi/2 \ -\pi/2 \ \pi/2]^T$. The matrix Q has the form (23), with $K_i = J_g \Omega = 1000 \text{ Nms}$, $i = \overline{1, 4}$ and $K(0) = [K_x(0) \ K_y(0) \ K_z(0)]^T = 1000[1 \ 1 \ 1]^T \text{ Nms}$; $\omega(0) = [\omega_1(0) \ \omega_2(0) \ \omega_3(0)]^T = [0 \ 0 \ 0]^T \text{ deg/s}$.

For the system presented in Fig. 3, we choose (Wie et al., 2002):

$$\begin{aligned} \varepsilon_i &= 0.01 \sin\left(\frac{\pi}{2}t + \Phi_i\right), \Phi_1 = 0, \Phi_2 = \frac{\pi}{2}, \Phi_3 = \pi; \\ \lambda &= 0.01 \exp[-10 \det(QQ^T)]. \end{aligned} \quad (32)$$

In Fig. 4a we present the time histories of the satellite's attitude angles (θ, φ, ψ) , angular velocities $(\omega_1, \omega_2, \omega_3)$, angular accelerations $(\dot{\omega}_1, \dot{\omega}_2, \dot{\omega}_3)$, components of the command couple of the system $- \mathbf{M}_c (M_{c1}, M_{c2}, M_{c3})$, components of the gyroscopic moment $- \mathbf{M}_g (M_{g1}, M_{g2}, M_{g3})$ and components of the kinetic moment vector $- \mathbf{K} (K_x, K_y, K_z)$; in Fig. 4b we present the time histories associated to the components of the quaternions $\hat{\mathbf{q}} (q_1, q_2, q_3, q_4)$, to the components of the vector $\boldsymbol{\gamma} (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ and the components of its derivative $\dot{\boldsymbol{\gamma}} (\dot{\gamma}_1, \dot{\gamma}_2, \dot{\gamma}_3, \dot{\gamma}_4)$.

As one can see in Fig. 3a, the control of the satellite's attitude is achieved by controlling the quaternion vector \mathbf{q} and the satellite's vector $(\boldsymbol{\omega})$. Actually, the first component of the designed control law (\mathbf{M}_c) is proportional-integrator type and assures the convergence of the quaternion vector \mathbf{q} to the desired quaternion $\bar{\mathbf{q}} = [0 \ 0 \ 0]^T$ (see Fig. 4b); the

second component of the control law cancels the deviation of the angular velocity vector from the one associated to the reference frame (see Fig. 4a). The closed loop control system has good convergence, global asymptotically stability and $\hat{\mathbf{q}}(t) \rightarrow 0, \boldsymbol{\omega}(t) \rightarrow 0$; on the other hand, as one can notice from Fig. 4, the cancel of the vectors $\hat{\mathbf{q}}$ and $\boldsymbol{\omega}$ leads to the cancel of other variables: the components of the command couple $- \mathbf{M}_c (M_{c1}, M_{c2}, M_{c3})$, the components of the gyroscopic moment $- \mathbf{M}_g (M_{g1}, M_{g2}, M_{g3})$ and of the kinetic moment vector $- \mathbf{K} (K_x, K_y, K_z)$.

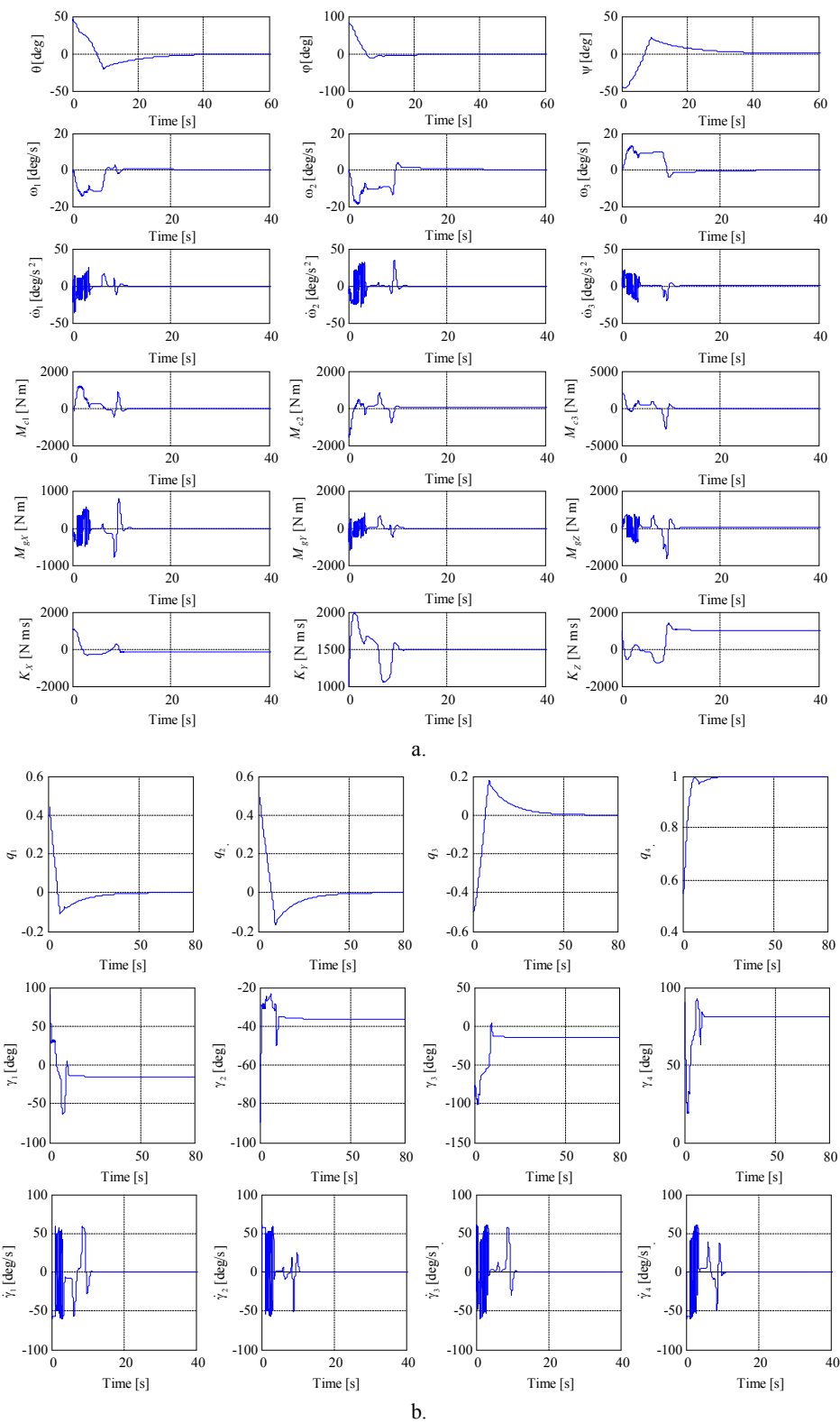


Figure 4: Time histories of the main variables associated to the satellite's attitude control system.

5 CONCLUSIONS

The purpose of this study was to design a new architecture for mini-satellites' attitude control using a cluster consisting of four CMGs and feedback from the quaternion and angular velocity vectors; the equivalent gyroscopic moment modifies the satellite's equivalent kinetic moment and the vector of angular velocities, this leading to the modification of the satellite's quaternion vector and of its attitude. The new architecture has been software implemented and validated through complex numerical simulations for the case of a mini-satellite involved in a typical motion around its own axis; the closed loop control system has been proved to be characterized by convergence and global asymptotically stability.

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