Reduced Order Observer for Linear Time-Invariant Multivariable Systems with Unknown Inputs

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This paper presents the design of a new reduced order observer to estimate the state for a class of linear time-invariant multivariable systems with unknown inputs. The proposed design approach is a combination of the approaches proposed by Hou & Muller [11] and Boubaker [4]; matrix decompositions, state transformations, and substitutions based on coordinates change are used. It will be shown that the problem of reduced order observers for linear systems with unknown inputs can be reduced to a standard one (the unknown input vector will not interfere in the observer equations). The effectiveness of the suggested design algorithm is illustrated by a numerical example (aircraft lateral motion), and, for the same aircraft dynamics, we make a comparison between our new observer and other already existing observers from the existence conditions and dynamic characteristics' point of view; the superiority of the new designed observer is demonstrated.

Keywords: *Observer, Unknown input, Algorithm, Aircraft motion*

1. Introduction

A. Antecedents and motivations

A state observer is a dynamical system allowing the state reconstruction from the system model and the measurements of its inputs and outputs. The plant input and output signals are used to estimate the plant state, which is then employed to close the control loop [13]. The aim of the observers is to augment or replace sensors in a control system. Starting from the first observers, introduced by Luenberger, the observers for plants with both known and unknown inputs have been developed resulting in the so-called unknown input observer (UIO) architectures, such as, for example, those in [6, 7, 12, 17, 23].

A physical process is often subjected to disturbances which have as origin the noises due to its environment, uncertainty of measurements, fault of sensors or actuators. These disturbances affect the normal behavior of the process and the estimation of these disturbances is needed in order to conceive a control strategy able to minimize their effects. Disturbances are called unknown inputs if they affect the process input, their presence making difficult the state estimation [1].

The state estimation problem for linear time-invariant (LTI) multivariable system, subjected to unknown inputs, has received considerable attention in the last decades [3, 7, 15, 24, 26]. The dimension of the observer is considerably increased in [1] and, that is why, the approach of Wang et al. [26] is more interesting; they proposed a method to design reduced order observers without any knowledge of these inputs; existence conditions for this observer have been provided by Kudva et al. [15]. Silverman's inverse method [24], the generalized inverse matrix, and the singular value decomposition are useful in linear observers' design process [7].

Generally, observers can be designed for singular systems, unknown input systems, delay systems, and also uncertain system with time-delay perturbations [1, 8, 26]. There are 2 categories of papers describing observer design methods: the first one supposes a priori knowledge of information on these non-measurable inputs, while the second category proceeds either by estimation of the unknown inputs, or by their complete elimination from the equations of the system [1].

The design of the observers depends on the type and the complexity of the considered model. Two types of models are distinguished according to the linear or nonlinear character of the system [14]. Linear models have simple structures and they are the base of several applications and research works. In such cases, observers can be designed for uncertain systems with time delay perturbations [26] and unknown input systems [7]. However, in the majority of real cases the nonlinear nature of the process cannot be neglected. The assumption of linearity is checked only locally around an operating point. Real physical processes present complex behaviors with nonlinear laws. Because it is not easy to design an observer for a nonlinear system, the multiple model approaches constitute tools which are largely used in the modeling of nonlinear systems [14].

Easily verifiable system theoretic conditions, which are necessary and sufficient for the existence of UIOs, have been established (see [10] or [11]). One possible statement of these conditions is that the transfer function matrix between the unmeasurable input and the measured outputs must be minimum phase and relative degree one [9]. New conditions for the existence of reduced order linear functional state observers for linear systems with unknown inputs were presented in [25]. Systematic procedures for the synthesis of reduced order functional observers have been given, the attractive feature of the proposed observer being the simplicity with which the design process can be accomplished [25].

One of the most important algorithm for the design of reduced order observers for LTI systems with unknown inputs was suggested by Hou & Muller [11]; his reduced order observer for linear systems with unknown inputs decomposes the state equation of the system into two subsystems: the first one depends on the unknown inputs, while, in the second one, the unknown inputs may be dropped. One of this approach’s assumptions is that the state of the second subsystem may be obtained through the measurement equation. The designed observer has good results but there are many situations in which this hypothesis does not hold [11]. Maquin et al. made some modifications to the observer of Hou & Muller and a straightforward treatment allowing the unknown input estimation is proposed in [19]. Other important algorithm for the design of reduced order observers for LTI systems with unknown inputs was suggested by Boubaker [4]; any reduced order observer has the advantage of avoiding redundancy caused by reconstructing accessible states; transformations and substitutions based on coordinate system are used and existence conditions are provided.

The observer design problem is a very important problem that has various applications such as output feedback control, system monitoring, process identification, and fault detection [5]. The basic idea behind the use of observers for fault detection is to estimate the outputs of the system from the measurements by using some type of observer, and then construct the residual by a properly weighted output estimate error. By means of a fixed or adaptive threshold, the residual is examined for the likelihood of faults and certain decision rules can then be applied to determine if a fault has occurred [22].

For the majority of the existing approaches, the number of unknown inputs must be less than the number of outputs, and, moreover, additional structural requirements on the system to be observed are met [28]. Those conditions turn out to be rather restrictive because, for instance, they cannot cover the simplest class of mechanical systems with unknown inputs wherein only the position is measurable [2]. Another disadvantage of observers is that only asymptotic convergence to zero of the observation and error is guaranteed [20]. However, for instance, for hybrid systems, the finite time exact observation is quite important because the time of observation convergence must be less than the dwell time; for example, this happens in the case of walking robots [16].

We may conclude that there are a lot of observers for linear systems with unknown inputs. There are 3 important design methods: geometrical methods (introduced first time by Bhattacharyya [3]), algebraic methods (used in observer design by Kudva et al. [15], Hautus [10], Hou & Muller [11], Darouach et al. [7], Yang & Wild [27], O'Reilly [21] and so on), and methods that use the generalized inverse [4]. Each of them has advantages and disadvantages; achieving less restrictive existence conditions and more direct design procedures have always been a challenge in this area [5].

B. Main contribution

The classical reduced order observers are easier to be implemented from the software point of view; their disadvantages are related to the important number of constraints (existence conditions). This paper presents a new reduced order observer design for the estimation of the system state vector and unknown inputs. It will be shown that the problem of reduced order observers for linear systems with unknown inputs can be reduced to a standard one (the unknown input vector does not interfere in the observer equations). The existence conditions for the obtained observer are also given. The new observer will be obtained by combining other two reduced order observers: the first one has been suggested by Hou and Muller [11], while the second one belongs to Boubaker [4]. Moreover, a comparison between our new observer, the Boubaker observer, and the Hou & Muller observer is achieved; the superiority of the new designed observer is demonstrated from the dynamic characteristics' point of view and from the constraints' point of view. Our new observer main advantage is proved to be the lack of apriori restrictions on the class of systems that can be considered.

The paper is organized as follows: the design approach of the new reduced order observer (*ALGLIN* algorithm) is given in section 2; in the same section, the observer is validated by means of a numerical simulation for the case of an aircraft lateral motion. A comparison between our new algorithm and other two design approaches is achieved in section 3. Finally, some conclusions are shared in section 4.

2. Design of the new reduced order observer

A. Problem statement

In this section of the paper we design a new observer for the state estimation problem in the case of LTI systems with unknown inputs. The approach is original and it represents the main contribution of the paper. The observer has been obtained by combining other two reduced order observers: the observer designed by Hou & Muller [11] and the Boubaker observer [4]; two additional "while" loops have been added in the design procedure in order to obtain a reduced order observer without existence conditions. The new algorithm must make the design problem equivalent with the standard problem of the observers' design when all inputs are known. In order to obtain a better observer, we will also focus on the design procedure which also must be direct and simple.

Let us consider an LTI system described by [4]:

$$\dot{x} = Ax + Bu + Dd\,,\; y = Cx\,, \tag{1}$$

where $x \in \mathcal{R}^{n\times 1}$ is system state vector, $u \in \mathcal{R}^{p\times 1}$ – system known input vector, $d \in \mathcal{R}^{s\times 1}$ – system unknown input vector, and $y \in \mathcal{R}^{m\times 1}$ – output vector; known matrices A, B, C, D have appropriate dimensions $\left(A \in \mathcal{R}^{n\times n}, B \in \mathcal{R}^{n\times p}, C \in \mathcal{R}^{m\times n}, D \in \mathcal{R}^{n\times s}\right)$.

The state vector is divided into two state vectors: the first one is associated to the unknown inputs, while the second one depends on the system known inputs. We make a coordinate change by choosing a matrix $N \in \mathcal{R}^{n\times(n-s)}$ such that $T = [N \;\; D]$ is nonsingular and we transform the old system state (x) into a new one $(\bar{x})$ by using the transformation [19]:

$$x = T\bar{x} = T\left[\bar{x}_1^{\,T} \;\; \bar{x}_2^{\,T}\right]^T, \tag{2}$$

with $\bar{x}_1 \in \mathcal{R}^{m\times 1}, \bar{x}_2 \in \mathcal{R}^{(n-m)\times 1}$. Equations (1) are equivalent with [4]:

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{D}d\,,\; y = \bar{C}\bar{x}\,, \tag{3}$$

where the matrices $\bar{A}, \bar{B}, \bar{C}$, and $\bar{D}$ have the forms:

$$\begin{aligned} &\bar{A} = T^{-1}AT = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}, \bar{B} = T^{-1}B = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}, \\ &\bar{D} = T^{-1}D = \begin{bmatrix} 0_{(n-s)\times s} \\ I_s \end{bmatrix}, \bar{C} = CT = [CN \;\; CD] = \left[\bar{C}_1 \;\; \bar{C}_2\right]; \end{aligned} \tag{4}$$

matrices in the above equation have appropriate dimensions: $\bar{A}_{11} \in \mathcal{R}^{m\times m}, \bar{A}_{12} \in \mathcal{R}^{m\times(n-m)}$, $\bar{A}_{21} \in \mathcal{R}^{(n-m)\times m}, \bar{A}_{22} \in \mathcal{R}^{(n-m)\times(n-m)}, \bar{B}_1 \in \mathcal{R}^{m\times p}, \bar{B}_2 \in \mathcal{R}^{(n-m)\times p}, \bar{C}_1 \in \mathcal{R}^{m\times m}, \bar{C}_2 \in \mathcal{R}^{m\times(n-m)}$.

The partitioning of $\bar{x}$ points out the decomposition of the system (1) into an unknown-input depending subsystem and an unknown-input-free subsystem. Therefore, combining (2) and (4), we obtain [4]:

$$\begin{cases} \dot{\bar{x}}_1 = \bar{A}_{11}\bar{x}_1 + \bar{A}_{12}\bar{x}_2 + \bar{B}_1 u\,, \\ \dot{\bar{x}}_2 = \bar{A}_{21}\bar{x}_1 + \bar{A}_{22}\bar{x}_2 + \bar{B}_2 u + I_s d\,, \\ y = \bar{C}_1\bar{x}_1 + \bar{C}_2\bar{x}_2\,. \end{cases} \tag{5}$$

$\bar{x}_2$ depends on the unknown input d, while $\bar{x}_1$ does not; therefore, it is more judicious to estimate $\bar{x}_1$ rather than $\bar{x}_2$. As a consequence, we design a reduced order observer for the estimation of the state $\bar{x}_1$ by using the equations:

$$\begin{cases} \dot{\bar{x}}_1 = \bar{A}_{11}\bar{x}_1 + \bar{A}_{12}\bar{x}_2 + \bar{B}_1 u\,, \\ y = \bar{C}_1\bar{x}_1 + \bar{C}_2\bar{x}_2\,; \end{cases} \tag{6}$$

$\hat{\bar{x}}_2$ and $\hat{d}$ (the estimations of $\bar{x}_2$ and d, respectively) will be calculated with respect to $\hat{\bar{x}}_1$ (the estimation of $\bar{x}_1$).

B. Design of the reduced order observer

The design of the new observer is concentrated into the following theorem:

Theorem 1:

Consider the LTI multivariable system (1); using the assumptions *n*>*m*, *n*>*s*, and (*C*, *A*) is an observable pair of matrices, we design the convergent reduced order observer for LTI systems with unknown inputs described by equations:

$$\begin{aligned} &\dot{\hat{\bar{x}}}_1 = \left(\tilde{A}_1 - LC_1\right)\hat{\bar{x}}_1 + \left(LH_{22}^T + \tilde{E}_1\right)y + \tilde{B}_1 u\,, \hat{\bar{x}}_2 = K_2 R_2^{-1} G^T H_2^T\left(y - \bar{C}_1\hat{\bar{x}}_1\right), \\ &\hat{d} = U_1\dot{y} - U_2 y - U_3\hat{\bar{x}}_1 - U_4\, u\,, \hat{x} = T\hat{\bar{x}} = T\left[\hat{\bar{x}}_1^T \ \ \hat{\bar{x}}_2^T\right]^T, \end{aligned} \tag{7}$$

where $H_{22} \in \mathcal{R}^{m\times m}$ is a sub-matrix of an orthogonal matrix $H_2 = \left[H_{21}\ H_{22}\right] \in \mathcal{R}^{m\times n}$, $C_1 = H_{22}^T\bar{C}_1$, $\tilde{A}_1 = \bar{A}_{11} - \bar{A}_{12}K_2R_2^{-1}G^T H_2^T\bar{C}_1$, $\tilde{B}_1 = \bar{B}_1$, $\tilde{E}_1 = \bar{A}_{12}K_2R_2^{-1}G^T H_2^T$, $G^T = \left[I_{n-m}\ \ 0\right]$, while R_2 (a non-singular matrix) and K_2 (an orthogonal matrix) are obtained by expanding the full column rank matrix $\bar{C}_2$ as following:

$$\bar{C}_2 = H_2\left[R_2^T\ \ 0\right]^T K_2^T; \tag{8}$$

L is calculated by choosing desired eigenvalues for the matrix $\left(\tilde{A}_1 - LC_1\right)$, such that $\left(\tilde{A}_1 - LC_1\right)$ is stable, while the matrices U_1, U_2, U_3, and U_4 have the expressions:

$$\begin{aligned} U_1 &= K_2R_2^{-1}G^T H_2^T\,, \\ U_2 &= K_2R_2G^T H_2^T\bar{C}_1\left(LH_{22}^T + \tilde{E}_1\right) + \bar{A}_{22}K_2R_2^{-1}G^T H_2^T\,, \\ U_3 &= K_2R_2^{-1}G^T H_2^T\bar{C}_1\left(\tilde{A}_1 - LC_1\right) + \bar{A}_{21} - \bar{A}_{22}K_2R_2^{-1}G^T H_2^T\bar{C}_1\,, \\ U_4 &= K_2R_2^{-1}G^T H_2^T\bar{C}_1\tilde{B}_1 + \bar{B}_2\,. \end{aligned} \tag{9}$$

Proof:

The expand of the matrix $\bar{C}_2$ (equation (8)) [11, 19] leads to the obtaining of the following matrices: $H_2 \in \mathcal{R}^{m\times n}, R_2 \in \mathcal{R}^{(n-m)\times(n-m)}, K_2 \in \mathcal{R}^{(n-m)\times(n-m)}$; analyzing the dimensions of the matrices R_2, K_2, and $\overline{D}$, new necessary conditions for the observer design results:

$$n - m > 0\,, n - s > 0\,. \tag{10}$$

In (8) the matrix H_2 must be orthogonal $\left(H_2 \cdot H_2^T = I_m\right)$, while R_2 must be nonsingular.

Next, by partitioning the matrix H_2 as follows [11, 19]: $H_2 = \left[H_{21}\;\; H_{22}\right]$, we obtain the matrices $H_{21} \in \mathcal{R}^{m\times(n-m)}, H_{22} \in \mathcal{R}^{m\times m}$, and we denote [11]:

$$\overline{y} = H_2^T y = \left[H_{21}\;\; H_{22}\right]^T y = \left[\overline{y}_1^{\,T}\;\; \overline{y}_2^{\,T}\right]^T; \tag{11}$$

the third equation (5) becomes: $y = \overline{C}_1\overline{x}_1 + H_2\left[R_2^T\;\; 0\right]^T K_2^T\overline{x}_2$. Left pre-multiplying both sides of the previous equation by H_2^T we obtain:

$$H_2^T y = H_2^T\overline{C}_1\overline{x}_1 + \underbrace{H_2^T H_2}_{I_m}\begin{bmatrix} R_2 \\ 0 \end{bmatrix} K_2^T\overline{x}_2 \Leftrightarrow \begin{bmatrix} H_{21}^T \\ H_{22}^T \end{bmatrix} y = \begin{bmatrix} H_{21}^T \\ H_{22}^T \end{bmatrix}\overline{C}_1\overline{x}_1 + \begin{bmatrix} R_2 \\ 0 \end{bmatrix} K_2^T\overline{x}_2 \tag{12}$$

and the measurement equation may be decomposed as follows:

$$\begin{cases} \overline{y}_1 = H_{21}^T\overline{C}_1\overline{x}_1 + R_2K_2^T\overline{x}_2, \\ \overline{y}_2 = C_1\overline{x}_1\,, \end{cases} \tag{13}$$

where $C_1 = H_{22}^T\overline{C}_1$. Now, with the notation $G^T = \left[I_{n-m}\;\; 0\right]$, one yields:

$$\overline{y}_1 = \left[I_{n-m}\;\; 0\right]\left[\overline{y}_1^{\,T}\;\; \overline{y}_2^{\,T}\right]^T \Leftrightarrow \overline{y}_1 = G^T\overline{y}\,. \tag{14}$$

Using the first equation (13) we write [11]:

$$\overline{x}_2 = \left(R_2K_2^T\right)^{-1}\left(\overline{y}_1 - H_{21}^T\overline{C}_1\overline{x}_1\right) = \left(R_2K_2^T\right)^{-1}\left(G^T\underbrace{H_2^T y}_{\overline{y}} - H_{21}^T\overline{C}_1\overline{x}_1\right) \tag{15}$$

or

$$\overline{x}_2 = K_2R_2^{-1}G^T H_2^T\left(y - \overline{C}_1\overline{x}_1\right); \tag{16}$$

Equation (16) has been obtained by using the equality $\left(R_2K_2^T\right)^{-1} = K_2R_2^{-1}$ (K_2 – orthogonal matrix) and the equation: $G^T H_2^T = \left[I_{n-m}\;\; 0\right]\left[H_{21}\;\; H_{22}\right]^T = H_{21}^T$.

Substituting $\overline{x}_2$ (expression (16)) into the first equation (6) we find [11]:

$$\dot{\overline{x}}_1 = \widetilde{A}_1\,\overline{x}_1 + \widetilde{B}_1 u + \widetilde{E}_1 y\,, \tag{17}$$

with

$$\widetilde{A}_1 = \overline{A}_{11} - \overline{A}_{12}K_2R_2^{-1}G^T H_2^T\overline{C}_1\,, \widetilde{B}_1 = \overline{B}_1\,, \widetilde{E}_1 = \overline{A}_{12}K_2R_2^{-1}G^T H_2^T\,. \tag{18}$$

In order to design the observer, the pair $\left(\widetilde{A}_1\,, C_1\right)$ must be observable or at least detectable. After the fulfillment of this condition, following the conventional

Luenberger observer design procedure, we design a reduced order observer for the unknown input free equation (17) as following [11, 19]:

$$\dot{\hat{\bar{x}}}_1 = \left(\tilde{A}_1 - LC_1\right)\hat{\bar{x}}_1 + \left(LH_{22}^T + \tilde{E}_1\right)y + \tilde{B}_1 u \,. \tag{19}$$

Equation (19) has been obtained by using the expression: $LC_1\bar{x}_1 = L\bar{y}_2 = LH_{22}^T y$; the matrix L is chosen such that $\sigma\left(\tilde{A}_1 - LC_1\right) \subset \mathrm{C}_-$ (the matrix $\tilde{A}_1 - LC_1$ is stable). If the condition is fulfilled, L has been obtained properly and the observer error converges asymptotically to zero (the proof is presented in [11] and [4]).

By means of (19), we determine $\hat{\bar{x}}_1$ and, after that, by using (16) we obtain:

$$\hat{\bar{x}}_2 = K_2 R_2^{-1} G^T H_2^T \left(y - \bar{C}_1 \hat{\bar{x}}_1\right); \tag{20}$$

the unknown input vector d is calculated from (5) as follows:

$$\hat{d} = \dot{\hat{\bar{x}}}_2 - \bar{A}_{21}\hat{\bar{x}}_1 - \bar{A}_{22}\hat{\bar{x}}_2 - \bar{B}_2 u \,, \tag{21}$$

where $\dot{\hat{\bar{x}}}_2$ has the form:

$$\dot{\hat{\bar{x}}}_2 = K_2 R_2^{-1} G^T H_2^T \left[\dot{y} - \bar{C}_1\left(\tilde{A}_1 - LC_1\right)\hat{\bar{x}}_1\right] - K_2 R_2^{-1} G^T H_2^T \left[\bar{C}_1\left(LH_{22}^T + \tilde{E}_1\right)y + \bar{C}_1\tilde{B}_1 u\right]. \tag{22}$$

Next, substituting (22) into (21) we find:

$$\hat{d} = U_1\dot{y} - U_2 y - U_3\hat{\bar{x}}_1 - U_4\, u, \tag{23}$$

where the matrices U_1, U_2, U_3, and U_4 have the forms (9).

The vector $\hat{\bar{x}}$ is determined by the concatenation of the vectors $\hat{\bar{x}}_1$ and $\hat{\bar{x}}_2$, while the estimated state vector $\hat{x}$ is calculated by means of an equation similar with (2); the theorem 1 is now demonstrated.

Remark 1:

The second condition (10) has been considered an assumption in the theorem 1 because the number of system unknown inputs (s) is commonly less than the number of states (n); moreover, the first condition (10) is an assumption for the approach because it generally holds; otherwise, it may be easily fulfilled by a judicious choice of matrix C.

Remark 2:

In order to reduce the observer constraints' number and to obtain a reduced order observer without existence conditions, we add in the design procedure two "while" loops. Because one of the existence conditions of the observer is related to the matrix $\bar{C}_2$, the first "while" loop is used to obtain a full column rank matrix $\bar{C}_2$, an orthogonal matrix H_2, and a nonsingular matrix R_2. On the other hand, to eliminate another constraint of our new approach (the pair $(\tilde{A}_1, C_1)$ must be observable or at least detectable), a second "while" loop is introduced. Thus, if the pair $(\tilde{A}_1, C_1)$ is not observable or at least detectable, we return to the selection of the matrix *N*, coordinates change (2), calculation of the matrices $\bar{A}, \bar{B}, \bar{C}, \bar{D}$, and so on; we repeat all these operations, in a second "while" loop, until this existence condition is fulfilled. There in no risk for an infinite "while" loop and, therefore, the approach will always provide an observable or at least detectable pair of matrices $(\tilde{A}_1, C_1)$.

Remark 3:

In two similar approaches (Boubaker observer and Hou & Muller observer, respectively) no remedy for the non-observability of the pair $(\tilde{A}_1, C_1)$ is presented; that is why, these two approaches have a disadvantage with respect to our new approach and, as a consequence, we obtained a design algorithm with a greater generality due its second "while" loop.

The observer design procedure for the state estimation problem in the case of LTI multivariable system, subjected to unknown inputs, is based on the ALGLIN algorithm, which is summarized below:

Step 1: We check if $\text{rank}(CD) = \text{rank}(D), n > m$, and $n > s$; if first two conditions are not met, we choose other matrix C) to satisfy the two conditions; the number of system unknown inputs (s) is commonly less than the number of states (n).

Step 2: We choose the matrix N such that the matrix $T = [N \;\; D]$ is nonsingular;

Step 3: The change of coordinates (2) is performed and the matrices $\bar{A}, \bar{B}, \bar{C}, \bar{D}$ are calculated; these 4 matrices are partitionated and the matrices $\bar{A}_{11}, \bar{A}_{12}, \bar{A}_{21}$, $\bar{A}_{22}, \bar{B}_1, \bar{B}_2, \bar{C}_1, \bar{C}_2$ are determined;

Step 4: We check if $\bar{C}_2$ is a full column rank matrix. If $\bar{C}_2$ is not a full column rank matrix, we return to step 2 and we repeat the steps 2-4 until the condition is fulfilled; otherwise, $\bar{C}_2$ is written under the form (8) and the matrices H_2, R_2, K_2 are obtained. The 3 matrices are calculated until we obtain a nonsingular matrix R_2 and two orthogonal matrices H_2, K_2;

Step 5: H_2 is partitionated $(H_2 = [H_{21} \;\; H_{22}])$ and the matrices H_{21} and H_{22} result;

Step 6: C_1 is calculated by means of equation: $C_1 = H_{22}^T \bar{C}_1$;

Step 7: Using (18), we determine $\tilde{A}_1, \tilde{B}_1, \tilde{E}_1$. We check if pair $(\tilde{A}_1, C_1)$ is observable and, if the condition holds, we go to the next step; otherwise we return to step 2 and we repeat steps 2-7 until the fulfillment of the observer existence conditions;

Step 8: The Luenberger observer described by equations (19) is designed, the matrix L being chosen such that $\sigma(\tilde{A}_1 - LC_1) \in C_-$. The reduced order observer estimates the vector $\bar{x}_1$; its estimation $(\hat{\bar{x}}_1)$ is obtained;

Step 9: $\hat{\bar{x}}_2$ is calculated by means of (20);

Step 10: We calculate the matrices U_1, U_2, U_3, U_4 (equation (9)) and, after that, the unknown input vector $\hat{d}$ is obtained by using (23);

Step 11: $\hat{\bar{x}}$ is determined by the concatenation of the vectors $\hat{\bar{x}}_1$ and $\hat{\bar{x}}_2$, while the estimated state vector $\hat{x}$ is obtained by using an equation similar with (2).

The new reduced order observer with unknown inputs reconstruction has 3 assumptions ($n > m, n > s$, (C, A) – observable pair of matrices) and 4 existence conditions (constraints): 1) $\text{rank}(CD) = \text{rank}(D)$; 2) $\bar{C}_2$ is a full column matrix; 3) the matrix R_2 is nonsingular, while the matrix H_2 is orthogonal; 4) the pair $(\tilde{A}_1, C_1)$ is observable or at least detectable. Condition 1) is sometimes called the *observer matching condition*, and it is the analogue of the well-known *matching condition* for a sliding mode controller to be insensitive to matched perturbations. This condition may be met easily by a judicious choice of matrix C (the choice of the output vector y); the existence conditions 2) and 3) are met in the first “while loop” by choosing a suitable matrix N, while the fulfillment of the constraint 4) is made in the second “while” loop. If condition 4) is fulfilled, the observer gain matrix L has been obtained properly, the dynamics of the observer error has a homogeneous form, and, therefore, the observer error converges asymptotically to zero.

In conclusion, choosing a suitable matrix C, a suitable matrix N (first “while” loop), and an observable pair of matrices $(\tilde{A}_1, C_1)$ – second “while” loop, all the existence conditions for the observer design are met. In these circumstances, our approach has no existence conditions and only 3 assumptions.

Remark 4:

The only minor disadvantage of our new observer with respect to other design algorithms in the specialty literature is related to the use of pole placement technique which is easy to implement but has some disadvantages: 1) it becomes difficult to be used for systems with big order or for poorly controlled systems; 2) if we choose fast poles for the observer, the advantage is that the observer estimation error decays rapidly, but the disadvantage is that the system needs perfect sensors and/or noise free environment; 3) if we choose slow poles for the observer, the advantage is that the system is less sensitive to process disturbances and measurement noise, but the disadvantage is that the observer estimation error decays slowly. The use of the pole placement technique is not a fundamental problem since most observers use this method for determination of the observer gain matrix; therefore the disadvantage of our new observer is a minor one.

C. Validation of the ALGLIN algorithm

The validation of our new algorithm for a reduced order observer design is performed, in this section, in Matlab/Simulink environment, for the case of lateral motion of a Boeing 747 flying with 0.8 Mach number at the altitude H=40000 ft [18]. Aircraft flight is often influenced by disturbances like longitudinal or vertical wind shears, atmospheric turbulences or errors of the sensors. From the aircraft dynamics' point of view, these represent unknown inputs; an observer for systems with unknown inputs must estimate these unknown inputs and, in the same time, estimate the system states with very small errors. Here, we validate our new reduced order observer for the case of an aircraft flight but the observer may be used, with good results, in any other examples due to its generality character. Because our new observer main advantage is the lack of apriori restrictions, we have to know only the system linear dynamics and to put the dynamics under the form of state equations (1); in our case, the state equations associated to the Boeing 747 lateral motion have the form (1) with [18]:

$$x = [\Delta\beta \;\; \Delta r \;\; \Delta p \;\; \Delta\varphi]^T, u = [\delta_r \;\; \delta_a]^T,$$

$$A = \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.598 & -0.115 & -0.0318 & 0 \\ 0.305 & 0.388 & -0.465 & 0 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix}, B = D = \begin{bmatrix} 0.0073 & 0 \\ -0.475 & 0.123 \\ 0.153 & 1.063 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}; \quad (24)$$

β is the aircraft sideslip angle, r and p are the yaw and roll angular rates, φ – the roll angle, δ_r – the rudder deflection, and δ_a – the ailerons deflection, while Δ is associated with the deviation of the variables from their nominal values; $d \in \mathcal{R}^{s\times 1}$ is considered to be a vertical wind shear. In this simulation, the input signal is calculated, by using the ALGLX algorithm [18]; thus, we consider the gain matrix $\overline{K}$ (feedback of the closed loop system) and the input vector of the system

$u = [\delta_r \;\; \delta_a]^T = -\overline{K}\hat{x}$. We obtained the graphic characteristics in Fig.1 (the 4 states $x_i, i = \overline{1,4}$ – solid line and the 4 estimated states $\hat{x}_i, i = \overline{1,4}$ – dashed line); the graphics of the system states are superposed over the estimated states' graphics.

3. Comparison between our new observer and other observers for LTI systems with unknown inputs

In this section, we compare our new observer with other already existing observers for LTI systems with unknown inputs. We briefly present 2 important observers, representative for the research area of LTI systems with unknown inputs, and we compare these observers with our new observer. These observers have been designed by Boubaker [4] and Hou & Muller [11], respectively.

To briefly present the Boubaker observer, we consider the same LTI system described by (1). All the equations (1)-(6) and the notations remain valid. The observer design is concentrated into the following theorem:

Theorem 2 [4]:

Consider the LTI multivariable system (1); using the assumptions $n > m$ and (C, A) is an observable pair of matrices, we design the convergent observer for LTI systems with unknown inputs described by the equation [4]:

$$\dot{\hat{\overline{x}}}_1 = \left(\tilde{A}_1 - L\tilde{C}_1\right)\hat{\overline{x}}_1 + \overline{B}_1 u + \tilde{L}y \,,\; \hat{\overline{x}}_2 = U_1 y - U_1 CN\hat{\overline{x}}_1 \,, \\ \hat{d} = U_1\dot{y} + G_1\hat{\overline{x}}_1 + G_2 y + G_3 u \,,\; \hat{x} = T\hat{\overline{x}} = T\left[\hat{\overline{x}}_1^T \;\; \hat{\overline{x}}_2^T\right]^T, \tag{25}$$

where $\tilde{A}_1 = \overline{A}_{11} - \overline{A}_{12}U_1 CN$, $\tilde{C}_1 = U_2 CN$, $E_1 = \overline{A}_{12}U_1$, $\tilde{L} = LU_2 + E_1$, L has been calculated by choosing desired eigenvalues for the matrix $\left(\tilde{A}_1 - L\tilde{C}_1\right)$ such that $\sigma\left(\tilde{A}_1 - L\tilde{C}_1\right) \subset \mathrm{C}_-$, while $U_1 \in \mathcal{R}^{m\times p}$ and $U_2 \in \mathcal{R}^{(p-m)\times p}$ have been obtained by choosing a nonsingular matrix $U = [CD \;\; Q], Q \in \mathcal{R}^{p\times(p-m)}$; U^{-1} has been partitioned as $U^{-1} = [U_1^T \;\; U_2^T]^T$. The matrices U and U^{-1} must fulfill the condition [4]:

$$U^{-1}U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}[CD \;\; Q] = \begin{bmatrix} U_1 CD & U_1 Q \\ U_2 CD & U_2 Q \end{bmatrix} = \begin{bmatrix} I_m & 0 \\ 0 & I_{p-m} \end{bmatrix}; \tag{26}$$

the matrices G_1, G_2, G_3 are calculated as below [4]:

$$G_1 = U_1 CN\left(LU_2 CN + \overline{A}_{12}U_1 CN\right) - U_1 CN\overline{A}_{11} - \overline{A}_{21} + \overline{A}_{22}U_1 CN \,, \\ G_2 = -U_1 CNLU_2 - U_1 CN\overline{A}_{12}U_1 - \overline{A}_{22}U_1 \,,\; G_3 = -U_1 CN\overline{B}_1 - \overline{B}_2 \,.$$

The theorem proof is presented in [4]. Our new observer design approach has borrowed from the Boubaker design procedure 5 steps: steps 1, 2, 3, 10, and 11).

The Boubaker observer has 2 assumptions ($n > m$, (C, A) – observable pair of matrices) and 3 existence conditions (constraints). The first existence condition of the Boubaker observer is related to the dimensions of the matrix Q; thus, the observer first constraint is *p*>*m*. This constraint is sometimes difficult to be met and no remedy is presented in the Boubaker approach; for example, if the LTI multivariable system has only one known input, the observer can not be designed (matrix C does not exist). Therefore, the observer may be designed and used only for LTI multivariable systems with multiple known inputs. The other 2 Boubaker observer existence conditions are given by the following two theorems [11]:

Theorem 3 [11]:

For the given system (1), observer (25) exists if and only if:

1) $\operatorname{rank}(CD) = \operatorname{rank}(D)$; 2) $\operatorname{rank}\begin{bmatrix} \mathbf{s}I_{n-s} - \bar{A}_{11} & -\bar{A}_{12} \\ CN & CD \end{bmatrix} = n, (\forall)\mathbf{s} \in \mathrm{C}, \operatorname{Re}\{\mathbf{s}\} \geq 0$.

Theorem 4 [11]:

If $\operatorname{rank}(CD) = \operatorname{rank}(D) = s$, then the following statements are equivalent:

1) pair $(\tilde{A}_1, \tilde{C}_1)$ is observable or at least detectable; 2) $\operatorname{rank}\begin{bmatrix} \mathbf{s}I_{n-s} - \bar{A}_{11} & -\bar{A}_{12} \\ CN & CD \end{bmatrix} = n$, $(\forall)\mathbf{s} \in \mathrm{C}, \operatorname{Re}\{\mathbf{s}\} \geq 0$; 3) $\operatorname{rank}\begin{bmatrix} \mathbf{s}I_n - A & D \\ C & 0 \end{bmatrix} = n + s, (\forall)\mathbf{s} \in \mathrm{C}, \operatorname{Re}\{\mathbf{s}\} \geq 0$.

The disadvantages of this observer are related to the number of existence conditions: 1) the number of system outputs is less than the number of system known inputs; 2) $\operatorname{rank}(CD) = \operatorname{rank}(D)$; 3) the pair $(\tilde{A}_1, \tilde{C}_1)$ is observable or at least detectable. Condition 2) is not a serious problem because it may be easily fulfilled by a judicious choice of matrix *C*; condition 1) is very restrictive and it represents a serious problem in the case of LTI systems with only one known input. If condition 3) is not met, the algorithm stops; no solution for solving this problem is included in the Boubaker approach. In contrast with the Boubaker observer, our observer solves this problem by means of a “while” loop which includes the steps 2-7. By a judicious choice of matrix C_1, our algorithm has no existence condition due to the “while” loops, while the Boubaker observer has 2 constraints without any remedy. Our new observer has one additional assumption which generally holds (the number of states is greater than the number of unknown inputs). Both design procedures use the pole placement technique, this being a minor disadvantage.

The second observer presented in this section belongs to Hou & Muller [11]. We consider the observable LTI system described by (1); Hou & Muller observer can be designed only if C is a full raw rank matrix and D is a full column rank matrix [11]. The matrix C can be conveniently chosen by selecting the system outputs, while the matrix D, associated to the unknown inputs of the system, can not be chosen by the designer. Considering the set consisting of these two conditions as the first assumption of the observer for the state estimation of an observable system, we perform the first matrix decomposition [11]:

$$D = H\begin{bmatrix} R^T & 0 \end{bmatrix}^T K^T, \tag{27}$$

where $H \in \mathcal{R}^{n\times n}$ and $K \in \mathcal{R}^{s\times s}$ are orthogonal matrices, while $R \in \mathcal{R}^{s\times s}$ – non-singular matrix. Using the coordinates change $\bar{x} = H^T x, \bar{d} = K^T d$, and the decomposition of the new state vector $\bar{x} = \begin{bmatrix} \bar{x}_1 & \bar{x}_2 \end{bmatrix}^T, \bar{x}_1 \in \mathcal{R}^{s\times 1}, \bar{x}_2 \in \mathcal{R}^{(n-s)\times 1}$, the system (1) gets the form [11]:

$$\begin{cases} \dot{\bar{x}}_1 = \bar{A}_{11}\bar{x}_1 + \bar{A}_{12}\bar{x}_2 + \bar{B}_1 u + R\bar{d}\ , \\ \dot{\bar{x}}_2 = \bar{A}_{21}\bar{x}_1 + \bar{A}_{22}\bar{x}_2 + \bar{B}_2 u\ , \\ y = \bar{C}_1\bar{x}_1 + \bar{C}_2\bar{x}_2\ , \end{cases} \tag{28}$$

with $\bar{A} = H^T AH, \bar{B} = H^T B, \bar{C} = CH, \bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}, \bar{B} = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}, \bar{C} = \begin{bmatrix} \bar{C}_1 & \bar{C}_2 \end{bmatrix}$; matrices in the previous equation have appropriate dimensions: $\bar{A}_{11} \in \mathcal{R}^{s\times s}, \bar{A}_{12} \in \mathcal{R}^{s\times(n-s)}$, $\bar{A}_{21} \in \mathcal{R}^{(n-s)\times s}, \bar{A}_{22} \in \mathcal{R}^{(n-s)\times(n-s)}, \bar{B}_1 \in \mathcal{R}^{s\times p}, \bar{B}_2 \in \mathcal{R}^{(n-s)\times p}, \bar{C}_1 \in \mathcal{R}^{m\times s}, \bar{C}_2 \in \mathcal{R}^{m\times(n-s)}$. Analyzing the dimensions of the matrices $\bar{A}_{12}, \bar{A}_{21}$ or $\bar{A}_{22}$, it results a new necessary condition for the design of the observer: $n>s$ (the number of system unknown inputs must be less than the number of system states). Because $\bar{x}_1$ depends on the unknown input d, while $\bar{x}_2$ does not, it is more judicious to estimate $\bar{x}_2$; as a consequence, Hou & Muller observer estimates the state $\bar{x}_2$, while the estimations of $\bar{x}_1$ and d are calculated with respect to $\hat{\bar{x}}_2$ (the estimation of $\bar{x}_2$).

Hou & Muller observer design is concentrated into the following theorem:

Theorem 5 [11]:

Consider the LTI multivariable system (1) written under the form (28); using the assumptions $n>s$, (C, A) is an observable pair of matrices, C is a full raw rank matrix, and D is a full column rank matrix, we design the convergent reduced order observer for LTI systems with unknown inputs described by the equations:

$$\dot{\hat{x}}_2 = \left(A_2 - LC_2\right)\hat{x}_2 + B_2 u + \left(D_2 + LH_{12}^T\right)y\ ,\ \hat{x}_1 = K_1 R_1^{-1} G^T H_1^T \left[y - \overline{C}_2 \hat{x}_2\right],$$
$$\hat{d} = KR^{-1}\dot{\hat{x}}_1 - KR^{-1}\overline{A}_{11} R_1^{-1} G^T H_1^T y + \left(KR^{-1} R_1^{-1} G^T H_1^T \overline{C}_2 - KR^{-1}\overline{A}_{12}\right)\hat{x}_2 - KR^{-1}\overline{B}_1 u\ , \qquad (29)$$

where $\overline{C}_1 = H_1 \left[R_1^T \;\; 0\right]^T K_1^T$ ($\overline{C}_1$ must be a full column rank matrix, $H_1 \in \mathcal{R}^{m\times m}$, $K_1 \in \mathcal{R}^{s\times s}$ – orthogonal matrices, $R_1 \in \mathcal{R}^{s\times s}$ –nonsingular matrix), $H_1 = \left[H_{11} \;\; H_{12}\right]$, $H_{11} \in \mathcal{R}^{m\times s}, H_{12} \in \mathcal{R}^{m\times(m-s)}, C_2 = H_{12}^T \overline{C}_2\ , G^T = \left[I_s \;\; 0\right], A_2 = \overline{A}_{22} - \overline{A}_{21} K_1 R^{-1} G^T H_1^T$, $B_2 = \overline{B}_2\ , D_2 = \overline{A}_{21} K_1 R^{-1} G^T H_1^T ; L$ is calculated by choosing desired eigenvalues for the matrix $\left(A_2 - LC_2\right)$ such that $\sigma\left(A_2 - LC_2\right) \subset C_-$. Vector $\hat{\bar{x}}$ is determined by the concatenation of the vectors $\hat{\bar{x}}_1$ and $\hat{\bar{x}}_2$, while the estimated state vector $\hat{x}$ is calculated by means of an equation similar with (2). ALGLIN observer design procedure has borrowed 6 steps (steps 4-9) from the Hou & Muller approach.

Hou & Muller observer has 4 assumptions (*n*>*s*, (*C*,*A*) - observable pair of matrices, *C* - full raw rank matrix, and *D* - full column rank matrix) and 5 constraints: 1) $\mathrm{rank}(CD) = \mathrm{rank}(D)$; 2) $\overline{C}_1$ is a full column matrix; 3) matrix *R* is nonsingular, while matrix *H* is orthogonal; 4) matrix R_1 is nonsingular, while the matrix H_1 is orthogonal; 5) the pair $\left(A_2\ , C_2\right)$ is observable or at least detectable.

2 of the 4 assumptions (*n*>*s*, (*C*, *A*) - observable pair of matrices) is an observable pair of matrices) are common to ALGLIN observer and Hou & Muller observer; the observability of the pair (*C*,*A*) is met easily by a judicious choice of matrix *C*, while the assumption *n*>*s* is generally valid. The other 2 assumptions make the Hou & Muller observer less general than the ALGLIN observer, this being the first disadvantage of the Hou & Muller observer with respect to our new observer. The existence condition 1) is not a serious problem because it may be fulfilled easily by a judicious choice of the matrix *C*. Constraint 2) is solved in the ALGLIN algorithm by using a "while" loop; in the Hou & Muller observer no solution is presented for the case when this existence condition is not fulfilled, this being the second disadvantage of the Hou & Muller observer. The constraint 3) of the ALGLIN algorithm is similar to the existence conditions 3) and 4) in the Hou & Muller approach. If in the ALGLIN design procedure, the constraint is eliminated by introducing the first "while" loop, in the Hou & Muller design procedure no remedy is presented for the case when these two conditions are not met (the third disadvantage of the observer presented in [11]). Finally, the constraint 4) of the ALGLIN algorithm is similar to the existence condition 5) of the Hou & Muller approach. If the pair $\left(A_2\ , C_2\right)$ is not observable, the Hou &

Muller design procedure stops (the fourth disadvantage of the observer presented in [11]) unlike the ALGLIN algorithm, where the second "while" loop (the loop which includes the steps 2-7) assures the fulfillment of this constraint.

From the constraints and design point of view, the ALGLIN observer is better, its main advantages being the lack of apriori restrictions on the class of systems that can be considered. Now, we want to make a brief comparison between our new observer and the 2 observers summarized in this section from the dynamic characteristics' point of view; therefore, we implemented in Matlab/ Simulink all the 3 observers by using the same aircraft dynamics, same flight data, and same unknown input vector. Thus, in Fig.2 we represent the time histories of the 4 state estimation errors for the ALGLIN observer (solid line), for the Boubaker observer, and for the Hou & Muller observer, respectively.

From the dynamic characteristics' point of view, the comparison between the 3 reduced order observers for systems with unknown inputs leads to the following conclusions: 1) All the 3 observers are convergent - the 4 components of the state estimation error tend to zero); 2) ALGLIN observer is characterized by a convergence speed of 3 seconds, while the convergence speed associated to other 2 observers is about 4-5 seconds; this means an advantage of our new observer from the convergence speed point of view (a decrease of 33.3-66.6 % of the convergence speed); 3) The oscillations' amplitudes of the state estimation errors are smallest if the ALGLIN observer is used. For a correct comparison, same desired eigenvalues have been used for all the three observers. Moreover, in Remark 4 we specified the following well-known issue: "*if we choose slow poles for observer, the advantage is that the system is less sensitive to process disturbances and measurement noise, but the disadvantage is that the observer estimation error decays slowly*". Thus, for a correct comparison, the solution was to choose the same slow poles for all the 3 observers; this way, the sensors' errors will not affect the measurements. On the other hand, although slow poles have been chosen, our observer estimation error does not decay slowly, 3 seconds representing a very good convergence speed in the research area of the LTI systems with unknown inputs.

4. Conclusions

In this paper we design a new approach for the state estimation problem in the case of LTI multivariable systems with unknown inputs. The approach is original and it represents the main contribution of the paper; the observer has been obtained by combining other two reduced order observers: the observer designed by Hou & Muller [11] and the Boubaker observer [4]. The effectiveness of the suggested design algorithm is illustrated by a numerical example (aircraft motion), and, for the same aircraft dynamics, we made a comparison between our new observer, Boubaker observer, and Hou & Muller observer, respectively; the superiority of the new designed observer has been demonstrated especially from the constraints' point of view (our new algorithm has 4 existence conditions, but all of them can be eliminated by means of a judicious choice of the system outputs and two "while" loops). The design procedure presented in this paper can be extended in the future and a new observer with unknown inputs' reconstruction can be designed; it can be a subsystem (fault detection/diagnosis scheme) of a typical fault-tolerant control system.

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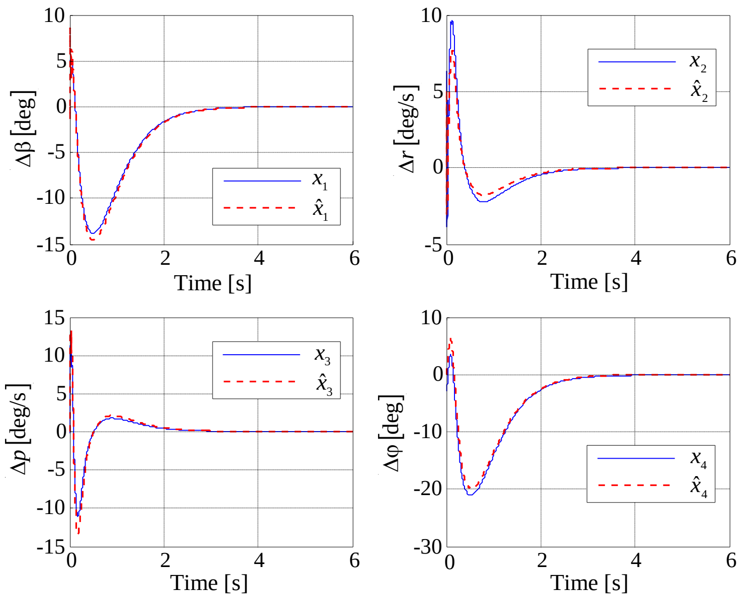

Fig. 1 State estimation errors by using the ALGLIN algorithm

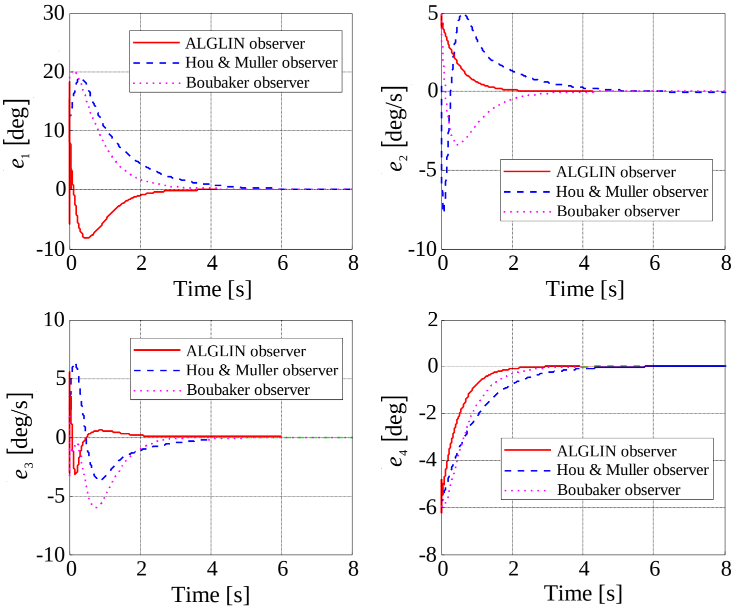

Fig. 2 Comparison between the ALGLIN observer, Boubaker observer, and Hou & Muller observer, respectively