Automatic Landing Control using H-inf Control and Dynamic Inversion

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Abstract— The paper presents the automatic control of the aircraft in the longitudinal plane during landing, taking into account the sensor errors and the wind shears. The H-inf control provides robust stability with respect to the uncertainties caused by different disturbances and noise type signals, while the dynamic inversion provides good precision tracking. These techniques are combined and a robust automatic landing system (ALS) is obtained; by adding an optimal observer and two reference models providing the desired altitude and velocity, one obtained a new automatic landing system which is very suited for landing control in the longitudinal plane. The optimal control law is calculated in two ways, this improving the generality, applicability, and simplicity degree of the ALS. The theoretical results are validated by numerical simulations for a Boeing 747 landing; the simulation results are very good (Federal Aviation Administration accuracy requirements for Category III are met) and show the robustness of the algorithm even in the presence of wind shears and sensor errors. Moreover, the designed control law has the ability to reject the sensor measurement noises, wind gust, and wind shears with low intensity.

Keywords: Landing, H-inf control, Dynamic inversion, Observer, Reference model

Notation

$\hat{a}_{ij}$	=	elements of the matrix $\hat{A}_{11}$
A	=	linear model of the aircraft motion
$\hat{B}_1^+$	=	pseudo-inverse of the matrix $\hat{B}_1$
H, $\overline{H}$	=	aircraft altitude and its imposed value, respectively
I	=	identity matrix
K_∞ , L_∞	=	controller gain matrix and observer gain matrix, respectively
q	=	aircraft pitch angular rate

P_∞ , P_∞^*	=	stabilizing solution of the matrix Riccati equation
r_1 , r_2	=	relative degrees
$\boldsymbol{u}$	=	command vector
u	=	aircraft longitudinal velocity
$\boldsymbol{u_w}$	=	vector of disturbances
$\boldsymbol{u}_\infty$	=	aircraft optimal command
$\bar{\boldsymbol{u}}$	=	aircraft command calculated by using the dynamic inversion
T	=	transformation matrix
T_0	=	flight time period inside the wind shear
V	=	aircraft velocity in longitudinal plane
V_{vx}, V_{vz}	=	components of the wind velocity along the longitudinal and vertical axes of the aircraft
V_{vx_0} , V_{vz_0}	=	maximum absolute values of wind velocities with respect to aircraft longitudinal and vertical axes
w	=	aircraft vertical velocity
$\boldsymbol{x}$	=	system state vector
$\hat{\boldsymbol{x}}$	=	estimated state vector of the system
X	=	horizontal distance covered during landing
X_{p_0}	=	coordinate, horizontally measured, of the point where the glide slope intersects the horizontal axis
y	=	output vector of the system
z	=	system containing the system's controllable output variables
$\bar{z}$	=	system containing the reference variables
$z_i^{(r_i-1)}$	=	the (r_i-1) order derivative of z_i
δ_e	=	elevator deflection
δ_T	=	engine command
ξ	=	state vector consisting of the controlled variables and their derivates
γ_c	=	imposed slope angle of the aircraft trajectory during landing
τ	=	time constant that defines the exponential curvature (flare landing phase)
θ	=	aicraft pitch angle

I. Introduction

A. Antecedents and motivations

For aircraft landing, the Automatic Landing Systems (ALSs) are used; among the first such systems one mentions the one tested in September 1947, allowing the complete fully automatic takeoff and landing [1]. Most aircraft have ALSs based on the Instrumental Landing System (ILS), using different conventional control laws (proportional-derivative – PD, proportional-integral – PI, proportional-integral-derivative - PID) for the altitude and descent velocity control [2-6], PD or PID conventional laws for the pitch angle and pitch rate control as well as different laws based on the state vector, dynamic inversion concept, with command filters, dynamic compensators, and state observers [7-13].

In recent years lots of scientific researchers have applied the intelligent concepts for the automatic landing of the aircraft; they use the optimal synthesis $H_2, H_\infty, H_2 / H_\infty$ [5], [14], [15], the adaptive synthesis based on dynamic inversion theory and neural networks theory [16-19] or fuzzy techniques [20-22]. In the research area of optimal synthesis, Shue and Agarwal [23] have developed a mixed technique for the H_2/H_∞ control of the landing, while Ochi and Kanai [24] have used the H_∞ control technique to design aircraft automatic approach and landing. In these papers, the authors did not analyze the robustness of the designed controllers in the presence of sensor errors and wind shears – issue which is considered in our paper. This is achieved in [25], where a PD-type fuzzy control system is developed for automatic landing control of both linear and nonlinear aircraft models; here, the robustness for a wide range of initial conditions was demonstrated successfully [26]. The drawback is that the authors only set up the wind disturbance as the initial condition; persistent wind disturbance is not considered. The learning scheme using fuzzy controllers with the BPTT (Back-propagation Through Time), presented in [26], guides the aircraft to a safe landing and makes the controller more robust and adaptive to the ever-changing environment.

Most of the improvements in the ALS have been based on the guidance instruments, such as the GNSS Integrity Beacons, Global Positioning System, Microwave Landing System, and Autoland Position Sensor [27-30]. By using improvement calculation methods and high-accuracy instruments, these systems provide more accurate flight data to the ALS to make the landing smoother; however, these studies did not include weather factors such as wind disturbances [26]. Same conclusions can be drawn from the work of Singh and Padhi [31] where a nonlinear control has been designed using the dynamic inversion approach for the automatic landing of unmanned aerial vehicles (UAVs) along with associated path planning. The obtained algorithm is not tested in the presence of wind shears and wind gusts, this being a disadvantage of the algorithm. Other methods to design the control for aircraft landing involve the use of neural networks [32], [33], but the weakness of these methods is that they do not control the aircraft to track the desired flight path accurate. For example, the paper [34] presents an intelligent automatic landing system that uses a time delay neural network controller and a linearized inverse aircraft model to improve the performance of conventional automatic landing systems, a learning-through-time process being used in the controller training; the disadvantage of the designed ALS is that it is enabled only under limited conditions. If severe wind disturbances are encountered, the pilot must handle the aircraft due to the limits of the automatic landing system.

B. Main contribution

This paper focuses on the automatic control of aircraft in the longitudinal plane, during landing, by using the linearized longitudinal dynamics of aircraft, taking into consideration the longitudinal and vertical wind shears and the errors of the sensors. It aims to the design of a new landing flight control system in the longitudinal plane by using the H-inf control and the dynamic inversion concept; H-inf control optimizes the system's performances and assures a robust stability.

The controlled output vector (z) consists of the aircraft flight altitude and velocity, the disturbances of the system being the wind shears and the sensor measurement errors (sensor noises). The reference altitude, velocity, and their derivatives up to relative degrees of the system are provided by reference models by means of the landing geometry dynamics. By using the states of the reference models, one calculated the aircraft desired state $(\bar{\boldsymbol{x}})$ and the reference vector $(\bar{y})$ for the measured outputs y. The optimal control is made on-line after the deviation (error) $\Delta\hat{\boldsymbol{x}}$ between the estimated state $(\hat{\boldsymbol{x}})$ and the aircraft desired state $(\Delta\boldsymbol{x} = \hat{\boldsymbol{x}} - \bar{\boldsymbol{x}})$. The estimation of the error $\Delta\boldsymbol{x} = \boldsymbol{x} - \bar{\boldsymbol{x}}$, i.e. $\Delta\hat{\boldsymbol{x}}$, is calculated by means of an optimal observer [35].

The ALS designed in this paper represents an improved version of the automatic landing system designed in [7] and it differs from other similar automatic landing systems from the specialty literature; our ALS is designed for the control of landing in the longitudinal (vertical) plane but it can also be applied to the lateral-directional motion of the aircraft during landing or other flight trajectories. Our new automatic landing system has some additional elements with respect to the one presented in [7]: 1) an optimal observer, which is used for the estimation of the error $\Delta\boldsymbol{x} = \hat{\boldsymbol{x}} - \bar{\boldsymbol{x}}$ in the presence wind shears and sensor errors; 2) two reference models which provide the desired altitude, velocity on the landing curve, and their derivatives up to relative degrees of the system; 3) the optimal control law is calculated in two ways, this improving the generality, applicability, and simplicity degree of the ALS.

The inputs of these reference models are the altitude and the velocity calculated for the two stages of the landing (the glide slope and the flare). By means of the two reference models' states one calculates the desired component $\bar{\boldsymbol{u}}$ (guidance component) of the control law for the imposed landing trajectory by using the dynamic inversion approach in two variants: a) by using the pitch angle and pitch angular rate; b) by using the estimated state vector. The other component of the control law $\boldsymbol{u}_\infty$ will be determined by using the H-inf method. The two calculation methods for $\bar{\boldsymbol{u}}$ differ from the ones presented in [7] especially by a greater degree of generality, applicability, and simplicity; the control law presented in [7] consists of separating the controller in two subsystems: a stable one and an unstable one which must be stabilized separately, this leading to a more complicated design procedure. The designed ALS must assures the convergences $\boldsymbol{x} \to \hat{\boldsymbol{x}} \to \bar{\boldsymbol{x}}, y \to \bar{y}, z \to \bar{z}$.

The paper is organized as follows: aircraft dynamics in longitudinal plane during landing is given in the second section; design of the H-inf control is presented in section III; in the fourth section of the paper, complex simulations to validate the proposed automatic landing system have been performed and analyzed; finally, some conclusions are shared in section V.

II. Aircraft Dynamics in Longitudinal Plane during Landing

A. Longitudinal linear model of the airplane

The linearization of an aircraft nonlinear dynamics is generally based on the small disturbances method with respect to an equilibrium trajectory, usually associated with the sea level; the dynamics used in this paper belongs to the flight of a Boeing 747. The linear model of the aircraft motion (**A**), in longitudinal plane, is described by the state equation [2], [3]:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} + G\boldsymbol{u}_w , \tag{1}$$

with $\boldsymbol{x} \in R^{5\times1} -$ the state vector, $\boldsymbol{x} = [u \;\; w \;\; q \;\; \theta \;\; H]^T, \boldsymbol{u} \in R^{2\times1} -$ the command vector, $\boldsymbol{u} = [\delta_e \;\; \delta_T]^T$, while $\boldsymbol{u}_w \in R^{2\times1}, \boldsymbol{u}_w = [V_{vx} \;\; V_{vz}]^T$, is the vector of disturbances V_{vx} and V_{vz} (the components of the wind velocity along the longitudinal and vertical axes of the aircraft [7]); in equation (1), u is the aircraft longitudinal velocity, w - aircraft vertical velocity, q – aircraft pitch angular rate, θ - aircraft pitch angle, H – aircraft altitude, while δ_e and δ_T are the elevator deflection and the engine command, respectively. Taking into account that the vertical velocity w is much smaller than u, the aircraft velocity in longitudinal plane can be approximated as follows:

$$V = \sqrt{u^2 + w^2} \cong u ; \tag{2}$$

thus, the nominal value of V is considered to be $V_0 \cong u(0) = u_0$. The matrices $A \in R^{5\times5}, B \in R^{5\times2}$ and $G \in R^{5\times2}$ are, respectively [8]:

$$A=\begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & a_{52} & 0 & a_{54} & 0 \end{bmatrix}, B=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, G=\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{3}$$

while the equations of the actuators are:

$$\dot{\delta}_e = -\frac{1}{T_e}\delta_e + \frac{1}{T_e}\delta_{ec}, \dot{\delta}_T = -\frac{1}{T_T}\delta_T + \frac{1}{T_t}\delta_{Tc}; \tag{4}$$

δ_{ec} and δ_{Tc} are the commands applied to elevator engine, respectively. If one considers δ_e and δ_T as new states, $\boldsymbol{x}$ and $\boldsymbol{u}$ become:

$$\boldsymbol{x} = [u \;\; w \;\; q \;\; \theta \;\; H \;\; \delta_e \;\; \delta_T]^T, \boldsymbol{u} = [\delta_{ec} \;\; \delta_{Tc}]^T, \tag{5}$$

while the new matrices $A \in R^{7\times 7}, B \in R^{7\times 2}$, and $G \in R^{7\times 2}$ are:

$$A=\begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} & 0 & b_{11} & b_{12} \\ a_{21} & a_{22} & a_{23} & 0 & 0 & b_{21} & b_{22} \\ a_{31} & a_{32} & a_{33} & 0 & 0 & b_{31} & b_{32} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & a_{52} & 0 & a_{54} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1/T_e & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/T_T \end{bmatrix}, B=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/T_e & 0 \\ 0 & 1/T_T \end{bmatrix}, G=\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{6}$$

B. Wind shear model

The column vector $\boldsymbol{u}_w$ is associated to a stochastic process defined by means of the velocities' spectrum; thus, the velocities may be modeled by using generator filters [6] with white noise input type. In this paper, the disturbances are the wind shears, their model being described by the equations [7]:

$$V_{vx} = -V_{vx_0}\sin(\omega_0 t), \; V_{vz} = -V_{vz_0}[1-\cos(\omega_0 t)], \omega_0 = 2\pi/T_0, \tag{7}$$

where T_0 is the flight time period inside the wind shear, while V_{vx_0} and V_{vz_0} are the maximum absolute values of the wind velocities with respect to aircraft longitudinal and vertical axes, respectively; the aircraft faces head and rear wind combined with vertical wind.

For the calculation of the matrix *G*, one starts by replacing $\boldsymbol{u}_w = 0$ in the state equation (1) and, after that, replaces *u* with $(u - V_{vx})$ and *w* with $(w - V_{vz})$; the coefficients of the velocities V_{vx}, V_{vz} and, thus, the elements of the matrix *G* are obtained as follows:

$$g_{11} = -a_{11}, g_{12} = -a_{12}, g_{21} = -a_{21}, g_{22} = -a_{22}, g_{31} = -a_{31}, g_{32} = -a_{32}. \tag{8}$$

III. Design of the H-inf Control

A. Design of the first component (guidance component) of the control law $\boldsymbol{u}$

Consider the vector $z = [H \;\; u]^T = C'\boldsymbol{x}$ that contains the system controllable output variables, while the vector $\bar{z} = [\bar{H} \;\; \bar{u}]^T$ contains the reference variables (the imposed values of the flight altitude and longitudinal velocity). The system output vector is y, chosen of the following form:

$$y = [H \;\; \dot{H} \;\; u \;\; \dot{u} \;\; \theta \;\; q]^T = C\boldsymbol{x}. \tag{9}$$

Taking into account the differential equations of the states H and u, obtained from (1), by using $\boldsymbol{x}$ and $\boldsymbol{u}$ having the forms (5) and the matrices A, B, G - equations (6), one yields:

$$C' = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T, C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & a_{52} & 0 & a_{54} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{11} & a_{12} & 0 & a_{14} & 0 & b_{11} & b_{12} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{10}$$

Now, by using $\bar{z}$ and the dynamic inversion principle, $\bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{u}}$ are calculated with respect to $\bar{z}$ and, after that, the vector $\bar{y}$ is obtained by means of the equations [7]:

$$\dot{\bar{\boldsymbol{x}}} = A\bar{\boldsymbol{x}} + B\bar{\boldsymbol{u}}, \bar{z} = C'\bar{\boldsymbol{x}}, \bar{y} = C\bar{\boldsymbol{x}}. \tag{11}$$

The command law is calculated by using the formula:

$$\boldsymbol{u} = \bar{\boldsymbol{u}} + \boldsymbol{u}_\infty, \tag{12}$$

where $\boldsymbol{u}_\infty$ is the optimal command that is calculated by means of the H-inf method, while the component $\bar{\boldsymbol{u}}$ is calculated by using the dynamic inversion. Hence, a coordinates' change is achieved by means of the transformation matrix $T \in R^{7\times 7}$ [7]:

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = Tx, x = T^{-1}\begin{bmatrix} \xi \\ \eta \end{bmatrix}, \tag{13}$$

where ξ is a state vector consisting of the controlled variables and their derivates, i.e. [7]

$$\xi = \left[z_1 \;\; \dot{z}_1 \;\; \cdots \;\; z_1^{(r_1-1)} \;\; z_2 \;\; \dot{z}_2 \;\; \cdots \;\; z_2^{(r_2-1)} \;\; \cdots \;\; z_p \;\; \dot{z}_p \;\; \cdots \;\; z_p^{(r_p-1)}\right]^T, \tag{14}$$

with $z_i^{(r_i-1)}$ – the (r_i-1) order derivative of z_i; for the aircraft dynamics in longitudinal plane, $z_1=H$, $z_2=z_p=u$. The second state vector (η) contains the states which are not included in the vector ξ; the dimension of vector η is $n - r = n - \sum_{i=1}^{p} r_i$, where n is the dimension of the square matrix T, while the values of r_i are deduced below.

In order to obtain the values of the relative degrees r_1 and $r_2= r_p$, one derivates with respect to time the equations of $\dot{u}$ and $\dot{w}$ such that, in the expressions of the variables $\ddot{u}$ and $\ddot{w}$ some terms containing the components of the control law $(\delta_{ec}, \delta_{Tc})$ appear; by derivation with respect to time of the variables $\dot{u}$ and $\dot{w}$ (expressed by using (1) and (6)), it results some terms containing the variables $\dot{\delta}_{ec}$ and $\dot{\delta}_{Tc}$; these can be expressed by means of equations (4); one obtains $\ddot{u}$ and $\ddot{w}$ as functions of δ_{ec}, δ_{Tc}, and other states. Thus, the relative degree of the state u is r_2=2. In order to obtain the relative degree of the altitude (H), one derivates the differential equation associated to H, i.e. $\dot{H} = a_{52}w + a_{54}\theta$, and obtains $\ddot{H} = a_{52}\dot{w} + a_{54}\dot{\theta}$. Then, this is done again and $\dddot{H}$ is obtained. Therefore, the relative degree of the altitude is r_1=3. The following equations result:

$$\begin{aligned}
\ddot{u} &= a'_{11}u + a'_{12}w + a'_{13}q + a'_{14}\theta + a'_{16}\delta_e + a'_{17}\delta_T + \frac{b_{11}}{T_e}\delta_{ec} + \frac{b_{12}}{T_T}\delta_{Tc} + g'_{11}V_{vx} + g'_{12}V_{vz} + g_{11}\dot{V}_{vx} + g_{12}\dot{V}_{vz}, \\
\dddot{H} &= a'_{51}u + a'_{52}w + a'_{53}q + a'_{54}\theta + a'_{56}\delta_e + a'_{57}\delta_T + \frac{a_{52}b_{21}}{T_e}\delta_{ec} + \frac{a_{52}b_{22}}{T_T}\delta_{Tc} + g'_{51}V_{vx} + g'_{52}V_{vz} + g''_{51}\dot{V}_{vx} + g''_{52}\dot{V}_{vz},
\end{aligned} \tag{15}$$

with

$$\begin{aligned}
&a'_{11} = a_{11}^2 + a_{12}a_{21}, a'_{12} = a_{11}a_{12} + a_{12}a_{22}, a'_{13} = a_{12}a_{23} + a_{14}, a'_{14} = a_{11}a_{14}, \\
&a'_{16} = a_{11}b_{11} + a_{12}b_{21} - \frac{b_{11}}{T_e}, a'_{17} = a_{11}b_{12} + a_{12}b_{21} - \frac{b_{12}}{T_T}, g'_{11} = a_{11}g_{11} + a_{12}g_{21}, g'_{12} = a_{11}g_{12} + a_{12}g_{22}, \\
&a'_{51} = a_{54}a_{31} + a_{52}(a_{21}a_{11} + a_{21}a_{12} + a_{23}a_{31}), a'_{52} = a_{54}a_{32} + a_{52}(a_{21}a_{12} + a_{22}^2 + a_{23}a_{32}), a'_{53} = a_{54}a_{33}, \\
&a'_{54} = a_{52}(a_{22}a_{23} + a_{22}a_{33} + a_{21}a_{14}), a'_{56} = a_{52}\left(a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} - \frac{1}{T_e}b_{21}\right) + a_{54}b_{31}, \\
&a'_{57} = a_{54}b_{32} + a_{52}\left(a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} - \frac{1}{T_T}b_{22}\right), g'_{51} = a_{54}g_{31} + a_{52}(a_{21}g_{11} + a_{22}g_{21} + a_{23}g_{31}), \\
&g'_{52} = a_{54}g_{32} + a_{52}(a_{21}g_{12} + a_{22}g_{22} + a_{23}g_{33}), g''_{51} = a_{52}g_{21}, g''_{52} = a_{52}g_{22}.
\end{aligned} \tag{16}$$

Thus, the state vectors ξ and η are:

$$\xi = [H\ \dot{H}\ \ddot{H}\ u\ \dot{u}]^T, \eta = [\theta\ q]^T; \tag{17}$$

the vector η, having the dimension n-r=2, contains the main states that must be controlled during landing, other than the ones included in the ξ; the information regarding the other components of the general state vector $(\boldsymbol{x})$, i.e. w, δ_e, and δ_T, is in the variables $\dot{H}, \ddot{H}$, and $\dot{u}$.

Using now the coordinates' change (13) [7], considering $\boldsymbol{u}_w = 0$ and $\boldsymbol{u} = \bar{\boldsymbol{u}}$, the system (1) gets the form:

$$\begin{bmatrix}\dot{\xi} \\ \dot{\eta}\end{bmatrix} = \hat{A}\begin{bmatrix}\xi \\ \eta\end{bmatrix} + \hat{B}\bar{\boldsymbol{u}}; \hat{A} = TAT^{-1}, \hat{B} = TB. \tag{18}$$

If the matrices $\hat{A}$ and $\hat{B}$ are partitioned with respect to the dimensions of the vectors ξ and η, it results [7]:

$$\begin{bmatrix}\dot{\xi}\\ \dot{\eta}\end{bmatrix}=\begin{bmatrix}\hat{A}_{11} & \hat{A}_{12}\\ \hat{A}_{21} & \hat{A}_{22}\end{bmatrix}\begin{bmatrix}\xi\\ \eta\end{bmatrix}+\begin{bmatrix}\hat{B}_1\\ \hat{B}_2\end{bmatrix}\bar{\boldsymbol{u}}; \tag{19}$$

the matrices in the above equation have the following dimensions $\hat{A}_{11}\in R^{5\times 5}, \hat{A}_{12}\in R^{5\times 2}, \hat{A}_{21}\in R^{2\times 5}$, $\hat{A}_{22}\in R^{2\times 2}, \hat{B}_1\in R^{5\times 2}, \hat{B}_2\in R^{2\times 2}$. Equation (19) is equivalent with

$$\dot{\xi}=\hat{A}_{11}\xi+\hat{A}_{12}\eta+\hat{B}_1\bar{\boldsymbol{u}}, \tag{20}$$

$$\dot{\eta}=\hat{A}_{21}\xi+\hat{A}_{22}\eta+\hat{B}_2\bar{\boldsymbol{u}}. \tag{21}$$

Imposing $\xi=\bar{\xi}$ and $\dot{\xi}=\dot{\bar{\xi}}$, with

$$\bar{\xi}=\begin{bmatrix}\bar{H} & \dot{\bar{H}} & \ddot{\bar{H}} & \bar{u} & \dot{\bar{u}}\end{bmatrix}^T, \dot{\bar{\xi}}=\begin{bmatrix}\dot{\bar{H}} & \ddot{\bar{H}} & \dddot{\bar{H}} & \dot{\bar{u}} & \ddot{\bar{u}}\end{bmatrix}^T, \tag{22}$$

from equation (20) the command vector $\bar{\boldsymbol{u}}$ is obtained as follows:

$$\bar{\boldsymbol{u}}=\hat{B}_1^{+}\left(\dot{\bar{\xi}}-\hat{A}_{11}\bar{\xi}-\hat{A}_{12}\eta\right), \tag{23}$$

with $\hat{B}_1^{+}$ – the pseudo-inverse of the matrix $\hat{B}_1$.

For the obtaining of the matrix *T*, the vectors (17) are replaced in (13) and the following differential equations

$$\begin{aligned}&\dot{H}=a_{52}u+a_{54}\theta, \ddot{H}=a_{52}\dot{w}+a_{54}\dot{\theta}, \dot{\theta}=q,\\ &\dot{u}=a_{11}u+a_{12}w+a_{14}\theta+b_{11}\delta_e+b_{12}\delta_T,\\ &\dot{w}=a_{21}u+a_{22}w+a_{23}q+b_{21}\delta_e+b_{22}\delta_T\end{aligned} \tag{24}$$

are taken into account; one yields:

$$\begin{bmatrix}H\\ \dot{H}\\ \ddot{H}\\ u\\ \dot{u}\\ \theta\\ q\end{bmatrix}=\underbrace{\begin{bmatrix}0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & a_{52} & 0 & a_{54} & 0 & 0 & 0\\ a_{52}a_{21} & a_{52}a_{22} & (a_{54}+a_{52}a_{23}) & 0 & 0 & a_{52}b_{21} & a_{52}b_{22}\\ 1 & 0 & 0 & 0 & 0 & 0 & 0\\ a_{11} & a_{12} & 0 & a_{14} & 0 & b_{11} & b_{12}\\ 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0\end{bmatrix}}_{T}\begin{bmatrix}u\\ w\\ q\\ \theta\\ H\\ \delta_e\\ \delta_T\end{bmatrix}. \tag{25}$$

Replacing (22) in (23), one obtains the equation

$$\bar{\boldsymbol{u}}=\hat{B}_1^{+}\left\{\begin{bmatrix}0\\ 0\\ \dddot{\bar{H}}\\ 0\\ \ddot{\bar{u}}\end{bmatrix}-\underbrace{\begin{bmatrix}\hat{a}_{11}\bar{H}+(\hat{a}_{12}-1)\dot{\bar{H}}+\hat{a}_{13}\ddot{\bar{H}}+\hat{a}_{14}\bar{u}+\hat{a}_{15}\dot{\bar{u}}\\ \hat{a}_{21}\bar{H}+\hat{a}_{22}\dot{\bar{H}}+(\hat{a}_{23}-1)\ddot{\bar{H}}+\hat{a}_{24}\bar{u}+\hat{a}_{25}\dot{\bar{u}}\\ \hat{a}_{31}\bar{H}+\hat{a}_{32}\dot{\bar{H}}+\hat{a}_{33}\ddot{\bar{H}}+\hat{a}_{34}\bar{u}+\hat{a}_{35}\dot{\bar{u}}\\ \hat{a}_{41}\bar{H}+\hat{a}_{42}\dot{\bar{H}}+\hat{a}_{43}\ddot{\bar{H}}+\hat{a}_{44}\bar{u}+(\hat{a}_{45}-1)\dot{\bar{u}}\\ \hat{a}_{51}\bar{H}+\hat{a}_{52}\dot{\bar{H}}+\hat{a}_{53}\ddot{\bar{H}}+\hat{a}_{54}\bar{u}+\hat{a}_{55}\dot{\bar{u}}\end{bmatrix}}_{\hat{A}'_{11}\bar{\xi}}-\hat{A}_{12}\eta\right\} \tag{26}$$

or

$$\bar{\boldsymbol{u}} = \hat{B}_1^+ \left\{ \begin{bmatrix} 0 & 0 & \ddot{\bar{H}} & 0 & \ddot{\bar{u}} \end{bmatrix}^T - \hat{A}'_{11}\bar{\xi} - \hat{A}_{12}\eta \right\}, \tag{27}$$

where $\hat{A}'_{11}$ is calculated from $\hat{A}_{11}$ making the substitutions

$$\hat{a}'_{12} = \hat{a}_{12} - 1, \hat{a}'_{23} = \hat{a}_{23} - 1, \hat{a}'_{45} = \hat{a}_{45} - 1, \tag{28}$$

the other elements of the matrices $\hat{A}_{11}$ and $\hat{A}'_{11}$ being the same; $\hat{a}_{ij}$, i , $j = \overline{1,5}$ are the elements of the matrix $\hat{A}_{11}$.

Replacing (27) in (21), with $\xi = \bar{\xi}$, one successively obtains

$$\begin{aligned} \dot{\eta} &= \hat{A}_{21}\bar{\xi} + \hat{A}_{22}\eta + \hat{B}_2\hat{B}_1^+ \left\{ \begin{bmatrix} 0 & 0 & \ddot{\bar{H}} & 0 & \ddot{\bar{u}} \end{bmatrix}^T - \hat{A}'_{11}\bar{\xi} - \hat{A}_{12}\eta \right\} = \\ &= \left(\hat{A}_{22} - \hat{B}_2\hat{B}_1^+\hat{A}_{12}\right)\eta + \left(\hat{A}_{21} - \hat{B}_2\hat{B}_1^+\hat{A}'_{11}\right)\bar{\xi} + \hat{B}_2\hat{B}_1^+ \begin{bmatrix} 0 & 0 & \ddot{\bar{H}} & 0 & \ddot{\bar{u}} \end{bmatrix}^T, \end{aligned} \tag{29}$$

equation that can be expressed under the form:

$$\dot{\eta} = \hat{A}_\eta\,\eta + \hat{B}_z\bar{z}^{(r)} + \hat{A}_\xi\bar{\xi} \tag{30}$$

or

$$\dot{\eta} = \hat{A}_\eta\,\eta + \hat{B}_y\bar{Z}, \tag{31}$$

where

$$\begin{aligned} &\hat{A}_\eta = \hat{A}_{22} - \hat{B}_2\hat{B}_1^+\hat{A}_{12}, \hat{A}_\xi = \hat{A}_{21} - \hat{B}_2\hat{B}_1^+\hat{A}'_{11}, \hat{B}_z\bar{z}^{(r)} = \hat{B}_2\hat{B}_1^+ \begin{bmatrix} 0 & 0 & \ddot{\bar{H}} & 0 & \ddot{\bar{u}} \end{bmatrix}^T, \\ &\bar{z}^{(r)} = \begin{bmatrix} \bar{z}_1^{(r_1)} & \bar{z}_2^{(r_2)} \end{bmatrix}^T = \begin{bmatrix} \ddot{\bar{H}} & \ddot{\bar{u}} \end{bmatrix}^T, \hat{B}_y = \begin{bmatrix} \hat{B}_z & \hat{A}_\xi \end{bmatrix}, \bar{Z} = \begin{bmatrix} \bar{z}^{(r)} & \bar{\xi} \end{bmatrix}^T = \begin{bmatrix} \ddot{\bar{H}} & \ddot{\bar{u}} & \bar{H} & \dot{\bar{H}} & \ddot{\bar{H}} & \bar{u} & \dot{\bar{u}} \end{bmatrix}^T; \end{aligned} \tag{32}$$

the dimensions of the above matrices are, respectively: $\hat{A}_\eta \in R^{2\times2}, \hat{A}_\xi \in R^{2\times5}, \hat{B}_z \in R^{2\times2}, \hat{B}_y \in R^{2\times7}$. If $\hat{B}_2\hat{B}_1^+ = \begin{bmatrix} \hat{b}_{11} & \hat{b}_{12} & \hat{b}_{13} & \hat{b}_{14} & \hat{b}_{15} \\ \hat{b}_{21} & \hat{b}_{22} & \hat{b}_{23} & \hat{b}_{24} & \hat{b}_{25} \end{bmatrix}$, then $\hat{B}_z = \begin{bmatrix} \hat{b}_{13} & \hat{b}_{15} \\ \hat{b}_{23} & \hat{b}_{25} \end{bmatrix}$. Thus, for the calculation of the command vector $\bar{\boldsymbol{u}}$, first one solves equation (31) and obtains the vector η and then uses equation (27). From the expression of $\hat{B}_z\bar{z}^{(r)}$ (equation (32)), it results:

$$\hat{B}_1^+ \begin{bmatrix} 0 & 0 & \ddot{\bar{H}} & 0 & \ddot{\bar{u}} \end{bmatrix}^T = \hat{B}_2^+\hat{B}_z\bar{z}^{(r)}, \tag{33}$$

which, replaced in (27), leads to the following one:

$$\bar{\boldsymbol{u}} = \hat{B}_u^{-1}\left(\bar{z}^{(r)} - \hat{B}_\xi\bar{\xi} - \hat{B}_\eta\eta\right), \tag{34}$$

with

$$\hat{B}_u^{-1} = \hat{B}_2^+\hat{B}_z, \hat{B}_\xi = \hat{B}_u\hat{B}_1^+\hat{A}'_{11}, \hat{B}_\eta = \hat{B}_u\hat{B}_1^+\hat{A}_{12}; \tag{35}$$

these matrices have the following dimensions: $\hat{B}_u \in R^{2\times 2}, \hat{B}_\xi \in R^{2\times 5}, \hat{B}_\eta \in R^{2\times 2}$. Therefore, $\bar{\boldsymbol{u}}$ can be obtained by means of equation (27) or by using equation (34).

Another form of the command law $\bar{\boldsymbol{u}}$ results from (15) if one imposes the convergence of $z^{(r)} = \left[\dddot{H} \;\; \ddot{u}\right]^T$ to $\bar{z}^{(r)} = \left[\bar{\dddot{H}} \;\; \bar{\ddot{u}}\right]^T$ and the convergence of the system estimated state $(\hat{\boldsymbol{x}})$ to $\boldsymbol{x}$; in these conditions, it yields:

$$\bar{\boldsymbol{u}} = \begin{bmatrix} \bar{\delta}_{ec} \\ \bar{\delta}_{Tc} \end{bmatrix} = \hat{B}_u^{-1}\left(\bar{z}^{(r)} - A_x \hat{\boldsymbol{x}} - G' \boldsymbol{u}_w - G'' \dot{\boldsymbol{u}}_w\right), \tag{36}$$

with $\hat{B}_u = \begin{bmatrix} \dfrac{a_{52}b_{21}}{T_e} & \dfrac{a_{52}b_{22}}{T_T} \\ \dfrac{b_{11}}{T_e} & \dfrac{b_{12}}{T_T} \end{bmatrix}, A_x = \begin{bmatrix} a'_{51} & a'_{52} & a'_{53} & a'_{54} & 0 & a'_{56} & a'_{57} \\ a'_{11} & a'_{12} & a'_{13} & a'_{14} & 0 & a'_{16} & a'_{17} \end{bmatrix}, G' = \begin{bmatrix} g'_{51} & g'_{52} \\ g'_{11} & g'_{12} \end{bmatrix}, G'' = \begin{bmatrix} g''_{51} & g''_{52} \\ g''_{11} & g''_{12} \end{bmatrix}.$

B. The structure of the new automatic landing system

The structure of the new automatic landing system, using dynamic inversion and H-inf method, is presented in Fig. 1.

If the control law (36) is used, the subsystem for the calculation of the command $\bar{\boldsymbol{u}}$ and of the vector $\bar{y}$ is the one represented in fig. 1 with dashed line. Taking into account all the above equations, it can be concluded that the automatic control of the aircraft in longitudinal plane, during landing, is mainly based on the dynamic inversion and H-inf method. The vector $\bar{Z}$ may be calculated by means of two reference models, the former being a three order reference model, while the latter is a second order reference model (Fig. 2) [8].

Aircraft landing is simplified if the motion of the aircraft in lateral plane is made without errors (deviation of the aircraft from the runway direction is zero). This is why the systems for the automatic control of the flight direction are very important [14], [36-38]. Before the start of the two landing main stages (glide slope phase and flare phase), the pilot must cancel aircraft lateral deviation with respect to the runway. This is made by means of automatic control systems for the flight direction control with radio navigation system and equipment for the measurement of the distance between the aircraft and the radio marker [16], [25], [39-41].

C. Landing procedure

There are three phases in a typical landing procedure: initial approach, glide slope, and flare [42]. During initial approach, the pilot descends from the cruise altitude to an altitude of approximately 400 m above the ground for heavy aircraft or less than 400 m for light aircraft. The pilot then positions the airplane so that the airplane is on a heading towards the runway centreline. When the aircraft approaches the outer airport marker, which is about 4 nautical miles from the runway, the glide slope path signal is intercepted [34]. As the airplane descends along the glide slope path, its pitch, attitude, and speed must be controlled; the aircraft maintains a constant speed along the flight path. The descent rate, for a Boeing 747, must be about 3 m/s and the pitch angle is between -5 to 5 degrees. As the airplane descends 7 to 30 m above the ground (the maximum value is for Boeing 747), the slope angle control system is disengaged and a flare maneuver is executed. The vertical descent rate is slightly decreased so that the landing gear may be able to dissipate the energy of the impact at landing. The pitch angle of the airplane is then adjusted, between 0 to 5 degrees for most aircraft, which allows a soft touchdown on the runway surface [34].

The equation associated to the glide slope phase ($H \geq \overline{H}_0$, $\overline{H}_0 -$ the altitude at which the glide slope phase ends and the second landing phase begins) is:

$$\overline{H} = \left(X - X_{p0}\right)\tan\left(\gamma_c\right), \tag{37}$$

where X is the horizontal distance covered during landing, X_{p_0} is the coordinate, horizontally measured, of the point where the glide slope intersects the horizontal axis, $\overline{H} -$ the imposed altitude, while γ_c is the imposed slope angle of the aircraft trajectory during landing. The components of the imposed vector $\overline{Z}$ are: $\overline{H}, \dot{\overline{H}} = u \cdot \gamma_c$, $\ddot{\overline{H}} = 0, \dddot{\overline{H}} = 0, \overline{u} = u_0 \cong V_0, \dot{\overline{u}} = 0,$ and $\ddot{\overline{u}} = 0$. The equation associated to the flare phase $\left(H < \overline{H}_0\right)$ is:

$$\overline{H} = \overline{H}_0 \exp\left(-t/\tau\right), \tag{38}$$

with τ - the time constant that defines the exponential curvature (flare landing phase); as a consequence,

$$\dot{\overline{H}} = -\frac{1}{\tau}\overline{H}, \ddot{\overline{H}} = \frac{1}{\tau^2}\overline{H}, \dddot{\overline{H}} = -\frac{1}{\tau^3}\overline{H}, \overline{u} = u_0 \cong V_0, \dot{\overline{u}} = 0, \ddot{\overline{u}} = 0. \tag{39}$$

If the reference models in Fig. 2 are used, the roles of the variables $\overline{H}$ and $\overline{u}$ from (37), (38), and (39) are played by the variables H_c and u_c , respectively; the variables $\dddot{\overline{H}}, \ddot{\overline{H}}, \dot{\overline{H}}, \overline{H}, \ddot{\overline{u}}, \dot{\overline{u}}$, and $\overline{u}$ are provided by the reference models.

*D. Design of the second component of the control law **u***

Now, the state equation (1), the equations associated to $z_1 = H$ and $z_2 = u$, as well as the equation of the output vector y, may be combined into the following equation:

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ z_1 \\ z_2 \\ y \end{bmatrix} = \begin{bmatrix} A_{(7\times7)} & B_{(7\times2)} & G_{(7\times2)} & 0_{(7\times6)} \\ C_{0(1\times7)} & D_{01(1\times2)} & 0_{(1\times2)} & 0_{(1\times6)} \\ C_{1(1\times7)} & D_{11(1\times2)} & 0_{(1\times2)} & 0_{(1\times6)} \\ C_{(6\times7)} & 0_{(6\times2)} & 0_{(6\times2)} & D_{22(6\times6)} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{u} \\ \boldsymbol{u}_w \\ e \end{bmatrix}; \tag{40}$$

the matrices A, B, G have the forms (3),

$$C_0 = [0\ 0\ 0\ 0\ 1\ 0\ 0], C_1 = [1\ 0\ 0\ 0\ 0\ 0\ 0], D_{01} = [c_1\ \ 0], D_{11} = [c_2\ \ 0], \tag{41}$$

the matrix C has the form (10), while $D_{22} = I_6$ for the vector containing the sensor errors:

$$e = \left[e_H\ \ e_{\dot{H}}\ \ e_u\ \ e_{\dot{u}}\ \ e_\theta\ \ e_q\right]^T. \tag{42}$$

The optimal control law has the form [35]:

$$\boldsymbol{u}_\infty = -K_\infty(\hat{\boldsymbol{x}} - \bar{\boldsymbol{x}}), K_\infty = R_1^{-1}B^T P_\infty, R_1 = D_{11}^T D_{11}; \tag{43}$$

$\boldsymbol{u}_\infty$ minimizes the cost functional:

$$J = \frac{1}{2}\int_0^\infty z_2^T z_2 \mathrm{d}t = \frac{1}{2}\int_0^\infty \left[\boldsymbol{x}^T \underbrace{\left(C_1^T C_1\right)}_{Q_1} \boldsymbol{x} + \boldsymbol{u}_\infty^T \underbrace{\left(D_{11}^T D_{11}\right)}_{R_1} \boldsymbol{u}_\infty\right] \mathrm{d}t. \tag{44}$$

The symmetric and positive defined matrix $P_\infty \in R^{7\times 7}$ is the stabilizing solution of the matrix Riccati equation [23]

$$A^T P_\infty + P_\infty A - P_\infty\left(BR_1^{-1}B^T - \mu_1^{-2}GG^T\right)P_\infty + Q_1 = 0. \tag{45}$$

Here, Q_1 and R_1 are positive matrices, while μ_1 is a small enough positive scalar for which the Riccati equation (45) has a stabilizing solution; the determination of the controller gain matrix (K_∞) is the so-called H-inf control problem. The plant inputs are classified as control inputs and disturbances. The control input ($\boldsymbol{u}$) is the output of the controller, which become the input to the actuators driving the system (aircraft). The disturbances ($\boldsymbol{u}_w$ and e) are called exogenous inputs; the main distinction between the control input and the exogenous inputs is that the controller can not manipulate exogenous inputs. The plant outputs are also characterized into two groups; the first group is represented by signals that are measured and become inputs to the controller (y); the second group is represented here by the performance outputs (z_1 and z_2). In this paper, one will not insist on this whole theory since it is well presented in many papers [23], [24], [35]. It can only be noted that the H-inf control problem means to find a controller for the generalized plant such that Infinity norm of the transfer function relating exogenous inputs to performance outputs is minimum. The controller gain matrix (K_∞) has the general form (43) which is typical for the optimal control theory. The optimal control law $\boldsymbol{u}_\infty$ depends on $\Delta\hat{\boldsymbol{x}} = \hat{\boldsymbol{x}} - \bar{\boldsymbol{x}}$, as one can see in (43). To obtain this signal, an ordinary observer must be used; here, in order to obtain the estimated state vector $(\hat{\boldsymbol{x}})$ and the signal $\Delta\hat{\boldsymbol{x}} = \hat{\boldsymbol{x}} - \bar{\boldsymbol{x}}$, one borrowed the observer presented in [35], i.e.:

$$\Delta\dot{\hat{\boldsymbol{x}}} = A\Delta\hat{\boldsymbol{x}} + B\boldsymbol{u} + G\boldsymbol{u}_w + L_\infty(\Delta y - C\Delta\hat{\boldsymbol{x}}). \tag{46}$$

$\boldsymbol{u}_w$ is calculated with (7), using the values of V_{vx_0}, V_{vz_0}, and T_0 whose values are estimated by means of the equipment from the navigation system. The observer gain matrix $L_\infty \in R^{7\times 6}$ is calculated by using the formula:

$$L_\infty = P_\infty^* C^T \left(D_{22}^T D_{22}\right)^{-1}, \tag{47}$$

with P_∞^* – the stabilizing solution of the matrix Riccati equation [23]

$$AP_\infty^* + P_\infty^* A^T - P_\infty^*\left(C^T C - \mu_2^{-2} Q_1\right) P_\infty^* + GG^T = 0; \tag{48}$$

μ_2 is a small enough positive scalar for which the Riccati equation (48) has a stabilizing solution.

E. Design control algorithm

To design the automatic control system in Fig. 1, the following algorithm can be used:

Step 1: The calculation of the transformation matrix T – equation (25);

Step 2: The calculation of the matrices $\hat{A}$ and $\hat{B}$ – equation (18);

Step 3: The selection of the matrices $\hat{A}_{11}, \hat{A}_{12}, \hat{A}_{21}, \hat{A}_{22}, \hat{B}_1, \hat{B}_2$ by using (19);

Step 4: The calculation of the matrix $\hat{A}'_{11}$ by means of (26);

Step 5: The calculation of the matrices $\hat{A}_\eta, \hat{A}_\xi, \hat{B}_z, \hat{B}_y$ – equations (32);

Step 6: The calculation of the matrices $\hat{B}_u^{-1}, \hat{B}_\xi, \hat{B}_\eta$ by using (35);

Step 7: The solving of the Riccati equations (45) and (48); it results P_∞ and P_∞^*;

Step 8: The calculation of the gain matrices K_∞ and L_∞ by means of (43) and (47), respectively;

Step 9: Software implementation of the block diagram in Fig. 1 and the obtaining of the variables time history.

The desired landing trajectory of aircraft involves two variables' control: the forward speed (u) and the altitude (H). According to the landing requirements, the aircraft must descend from cruising altitude to a lower altitude around 500 m. Meanwhile, the aircraft speed also reduces from the cruising speed to an approach value and, after that, it remains constant. So, when one designs the desired trajectory, one designs the desired forward speed u first. The H-inf control system associated to aircraft flight during landing (longitudinal plane), based on dynamic inversion, assures the following convergences: $\Delta y \to 0\left(y = C\boldsymbol{x} \to \bar{y} = C\overline{\boldsymbol{x}}, \boldsymbol{x} \to \overline{\boldsymbol{x}}\right), \Delta z \to 0$, $\left(z = C'\boldsymbol{x} \to \bar{z} = C'\overline{\boldsymbol{x}}\right), \Delta\hat{\boldsymbol{x}} \to 0\left(\hat{\boldsymbol{x}} \to \boldsymbol{x} \to \overline{\boldsymbol{x}}\right), \boldsymbol{u}_\infty \to 0$, $\overline{\boldsymbol{u}} \to 0, \boldsymbol{u} \to 0$.

IV. Numerical Simulation Results

A. Numerical simulation setup

To study the performances of the new obtained automatic landing system, one considers the landing of a Boeing 747. Complex simulations in Matlab/Simulink environment have been performed; thus, one designed the optimal observer, the H-inf controller, and, after that, validated the proposed automatic landing system. The values of the coefficients for Boeing 747 dynamics have been borrowed from [7]: $a_{11} = -0.021$, $a_{12} = 0.122$, $a_{14} = -0.322$, $a_{21} = -0.209$, $a_{22} = -0.53$, $a_{23} = 2.21$, $a_{31} = 0.017$, $a_{32} = -0.164$, $a_{33} = -0.412$, $a_{52} = -1$, $a_{54} = = V_0 = 70\,\mathrm{m/s}$, $b_{11} = 0.01$, $b_{12} = 1$, $b_{21} = -0.064$, $b_{22} = -0.044$, $b_{31} = -0.378$, $b_{32} = 0.544$, $T_e = 0.3\,\mathrm{s}$, $T_T = 2\,\mathrm{s}$, $\bar{u} = V_0$, $V_{vx0} = 1\,\mathrm{m/s}$, $V_{vz0} = 1\,\mathrm{m/s}$, $T_0 = 30\,\mathrm{s}$. By means of equation (8), the elements of matrix G have been calculated; the vector (42) is $e = [0.2\,\mathrm{m} \quad 0.2\,\mathrm{deg/s} \quad 0.2\,\mathrm{m} \quad 0\,\mathrm{m/s^2} \quad 0.2\,\mathrm{deg} \quad 0.1\,\mathrm{deg/s}]^T$, while, for the reference models, one has chosen: $p = 25$, $\xi_1 = \xi_2 = 0.7$, $\omega_1 = \omega_2 = 2\,\mathrm{rad/s}$. For the first landing phase, the following values have been used: $\bar{H}_p = \bar{H}(0) = 420\,\mathrm{m}$, $X(0) = 0$, $X_{p0} = -\bar{H}_p / \tan(\gamma_c)$, $\gamma_c = -2.5\,\mathrm{deg}$. The other components of vector $\bar{\xi}$ are provided by the reference models, $\eta(0) = [-4.2\,\mathrm{deg} \quad -0.46\,\mathrm{deg/s}]^T$, $\mu_1 = 50$, $\mu_2 = 1$, while the initial value of the state is $x(0) = [72\,\mathrm{m/s} \quad -3.1\,\mathrm{m/s} \quad 0\,\mathrm{grd/s} \quad -0.6\,\mathrm{deg} \quad 420\,\mathrm{m} \quad 0\,\mathrm{deg} \quad 0\,\mathrm{deg}]^T$.

B. Results and discussion

Using the above presented algorithm, for both stages of the landing, the following matrices resulted:

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 70 & 0 & 0 & 0 \\ 0.209 & 0.53 & 67.79 & 0 & 0 & 0.064 & 0.044 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.021 & 0.122 & 0 & -0.322 & 0 & 0.01 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & -2.512 & -5.01 & 1.27 & 4.94 & 177.42 & 222.92 \\ 0 & 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & -0.15 & -0.28 & 0.069 & 0.111 & 10.598 & 12.835 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \\ 0 & -0.031 & -0.063 & 0.016 & 0.086 & 2.204 & 2.771 \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 14.72 & 0.17 \\ 0 & 0 \\ 2 & 0.33 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \hat{A}'_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -25.11 & -50.11 & 12.72 & 49.41 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1.508 & -2.88 & 0.69 & 1.117 \end{bmatrix}, \quad \hat{A}_\eta = \begin{bmatrix} 0 & 1 \\ 22.044 & 27.715 \end{bmatrix}, \quad \hat{A}_\xi = \begin{bmatrix} 0 & 0 \\ 0 & -0.31 \\ 0 & -0.63 \\ 0 & 0.17 \\ 0 & 0.86 \end{bmatrix}^T, \quad \hat{B}_y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -0.31 \\ 0 & -0.63 \\ 0 & 0.17 \\ 0 & 0.86 \end{bmatrix}^T,$$

$$\hat{B}_z = 0_{2\times 2}, \quad P_\infty = \begin{bmatrix} 6.1 & -0.6 & 4.1 & 9 & 0.1 & 0 & 9.9 \\ -0.6 & 65.9 & -126.9 & -462 & -7.4 & 1.9 & -16.6 \\ 4.1 & -126.9 & 316.4 & 909.4 & 11.3 & -4.9 & 58.9 \\ 9 & -462 & 909.4 & 3308.6 & 52.6 & -13.4 & 109.5 \\ 0.1 & -7.4 & 11.3 & 52.6 & 1.2 & -0.2 & 1 \\ 0 & 1.9 & -4.9 & 13.4 & -0.2 & 0.1 & -0.9 \\ 9.9 & -16.6 & 58.9 & 109.5 & 1 & -0.9 & 44.8 \end{bmatrix}, \quad P_\infty^* = \begin{bmatrix} 0.081 & -0.084 & 0.001 & -0.013 & 0.005 & 0 & 0 \\ -0.084 & 0.422 & 0.050 & 0.072 & 0.015 & 0 & 0 \\ 0.001 & 0.050 & 0.011 & 0.007 & -0.005 & 0 & 0 \\ -0.013 & 0.072 & 0.007 & 0.013 & 0.014 & 0 & 0 \\ 0.005 & 0.015 & -0.005 & 0.014 & 0.986 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$K_\infty = \begin{bmatrix} -0.076 & 0.330 \\ 3.755 & -0.554 \\ -9.796 & 1.964 \\ -26.887 & 3.649 \\ -0.314 & 0.032 \\ 0.201 & -0.030 \\ -1.850 & 1.493 \end{bmatrix}^T, \quad L_\infty = \begin{bmatrix} 2.113 & 6.147 & -2.198 & 5.796 & 394.647 & 0 & 0 \\ 19.296 & 89.383 & -12.863 & 49.145 & 335.087 & 0 & 0 \\ 32.384 & -33.891 & 0.535 & -5.292 & 2.113 & 0 & 0 \\ -2.463 & 11.303 & 1.446 & 1.786 & -1.118 & 0 & 0 \\ -5.292 & 29.048 & 3.114 & 5.474 & 5.796 & 0 & 0 \\ 0.535 & 20.077 & 4.609 & 3.114 & -2.198 & 0 & 0 \end{bmatrix}^T.$$

In Fig. 3 the observer estimation errors are presented for the two phases of landing; the optimal observer is convergent - all the 7 components of the state $\Delta\hat{\boldsymbol{x}} = \hat{\boldsymbol{x}} - \bar{\boldsymbol{x}}$ tend to zero in about 5 seconds. The results are the same for the cases when the sensor errors are taken or not into consideration.

In Fig. 4 and Fig. 5 there are represented the time characteristics for the glide slope landing phase and flare landing phase, respectively; the characteristics have been represented for the ALS affected by wind shears in the presence or in the absence of sensor errors (the sensors are used for the measurement of the states). The last four mini-graphics in Figs. 4 and 5 represent the deviations of the forward speed (*u*), sink rate $(\dot{H})$, slope angle (γ), and altitude (*H*), with respect to their nominal values, i.e. $u-\bar{u}$, $\dot{H}-\bar{\dot{H}}$, $\gamma-\gamma_c$, $H-\bar{H}$. The presence of the sensor errors is not visible – the curves with solid line (obtained for the ALS without sensor errors) overlap almost perfectly over the curves plotted with dashed line (obtained for the ALS with sensor errors). The time origin for the flare trajectory is chosen zero when the altitude is $H=H_0=30$ m (the altitude at which the glide slope phase ends).

Remark 1

From the theoretical part of this paper, one retained the mandatory values of the slope angle (the difference between the pitch angle and the attack angle): -2.5 degrees in the first landing phase and 0 degrees in the second phase, respectively. By analyzing Figs. 4 and 5, it is remarked the correctness of the simulation data. During the glide slope, the aircraft must have a linear descendent trajectory (8^{th} graphic in Fig. 4) and, as a consequence, the pitch angle must be negative; as one can see in Fig. 4, the pitch angle is -2.65 degrees, while the attack angle is slightly negative $(\cong -0.15\,\text{deg})$; it results the desired slope angle (-2.5 degrees). In the flare phase, the aircraft must describe a parabolic trajectory (8^{th} graphic in Fig. 5) with a null slope angle; as one can see in Fig. 5, the pitch angle is slightly positive (0.15 degrees), while the attack angle is slightly positive too (0.15 deg); it results the desired null slope angle.

Remark 2

The landing begins at a longitudinal speed initially exceeding the nominal speed by 2m/s (see the first graphic in Fig. 4). The speed should be reduced to the normal speed (70 m/s) and then kept at this value; this landing process begins at 420 m (see the 8^{th} graphic in Fig. 5). To test the robustness of the designed ALS, in all simulations, one has taken into consideration the wind shears, because low-altitude wind shear is always a serious threat to the safety of aircraft in landing. From last graphics in Figs. 4 and 5, it can be seen that the largest error between the desired path and the actual path is less than 0.3 m during the glide slope phase and less than

0.2 m during flare. These errors are very good if the Federal Aviation Administration (FAA) accuracy requirements for Category III (the best category) [43] are analyzed; according to FAA Category III accuracy requirements, the vertical error (altitude deviation with respect to its nominal value) must be less than 0.5 m, the lateral deviation must be less than 4.1 m, while the final altitude at the end of flare must be 0 m. The reason that the design in this paper meets the requirement and achieves the design goal is that the H-inf robust control technique has been used; this method can handle the plant with measurement noise (sensor errors) and wind shears. If the wind shear is stronger than its maximum accepted value, the pilot must avoid having the aircraft enter into wind shear. From the forward speed and sink rate point of view, the errors are less than 0.1m/s (9th and 10th graphics in Figs. 4 and 5); because the deviation of the slope angle with respect to its nominal value (γ-γ_c) tends to zero (11th graphic in Figs. 4 and 5) one concludes $\gamma \rightarrow \gamma_c = -2.5\,\text{deg}$, for the first landing phase, and $\gamma \rightarrow \gamma_c = 0\,\text{deg}$ for the second landing phase.

In Fig. 6, the characteristics $H(X)$ and $\overline{H}(X)$ are represented. The optimal control system associated to aircraft flight during landing (longitudinal plane), based on dynamic inversion and H-inf method, assures the convergence $H \rightarrow \overline{H}$, for both cases when the sensor errors are taken or not into consideration. A little difference appears during the first landing phase (between 420 m and 150 m), but this error is canceled and it is null from this point by the end of the first landing phase; this error meets the FAA Category III accuracy requirements (see the 12th graphic in Fig. 4). During the second phase of landing, the trajectory corresponds to a desired exponential curve and, moreover, it is smoother than the desired one; because the point where the airplane landing gear touches the ground is approximately the same with the desired point (situated to approximately 11100 m horizontal distance from the point associated to the start of the landing), our ALS has an advantage especially for civil airplanes (smoother descend means better comfort to passengers).

Remark 3

For the Boeing 747 (the aircraft chosen in this paper for the validation of our new ALS), the first phase of the landing process (glide slope phase) takes approximately 128 seconds (see the 8th graphic in Fig. 4), while the second phase of the landing (flare phase) takes approximately 25 seconds (see the 8th graphic in Fig. 5); in the same time, the steady values of aircraft longitudinal velocity and aircraft vertical velocity are $u \cong 70\,\text{m/s}$, $w \cong -3.05\,\text{m/s}$ (glide slope – second graphic in Fig. 4) and $u \cong 70\,\text{m/s}, w \cong -1.2\,\text{m/s}$ (flare – second graphic in Fig. 5), respectively. By using this information, a short analysis regarding the correctness of the numerical simulation data can be made: 1) the horizontal distance covered by the aircraft in the first landing phase must be approximately $X = 70\,\text{m/s} \cdot 153\,\text{s} = 10710\,\text{m}$ – value confirmed by Fig. 6; 2) the vertical distance covered by the aircraft in the first landing phase must be approximately $3.05\,\text{m/s} \cdot 128\,\text{s} = 390.4\,\text{m}$, while the vertical distance covered in the second landing phase must be approximately $1.2\,\text{m/s} \cdot 25\,\text{s} = 30\,\text{m}$. These values are again confirmed by Figs. 4 and 5: the glide slope phase means a 390 m descent for the aircraft center of gravity, while the flare phase means a 30 m descend.

V. Conclusions

Recently, most aircraft have installed automatic landing systems. These systems provide outstanding flight control performance. They provide a smoother landing and increase the passengers' comfort. But there are limits: these systems work only within a specified operational safety envelope. The purpose of this study was to design a robust ALS using the H-inf control and the dynamic inversion techniques. The H-inf method can handle both robustness and exact tracking problem, and thus it is very suited for the design of automatic landing systems.

The ALS designed in this paper represents an improved version of the automatic landing system designed in [7] and it differs from other similar automatic landing systems from the specialty literature; our ALS is designed for the control of landing in the longitudinal (vertical) plane but it can also be applied to the lateral-directional motion of the aircraft during landing or other flight trajectories. Our new automatic landing system has some additional elements with respect to the one presented in [7]: an optimal observer and two reference models which provide the desired altitude, velocity on the landing curve, and their derivatives up to relative degrees of the system. The two calculation methods for the optimal control law give the ALS a greater degree of generality, applicability, and simplicity. The simulation results are promising and show the robustness of the algorithm even in the presence of wind shears and sensor errors; moreover, the very good errors meet the FAA accuracy requirements for Category III. On the other hand, the designed control law has the ability to reject the measurement noise from sensors, wind gust, and wind shears having low intensity. Compared with other existing approaches in the field of automatic landing systems, our method achieved much higher tracking precision. The use of the dynamic inversion makes our control system more general and, therefore, it can be used both for the case when aircraft dynamics is nonlinear and for the case when aircraft dynamics is linear; thus, the use of the dynamic inversion principle increases the generality character of our new automatic landing system.

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