Full-Order Observer Design for Linear Systems with Unknown Inputs

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(Received 6 April 2012)

In this paper, a full order observer without unknown inputs reconstruction is suggested in order to achieve finite-time reconstruction of the state vector for a class of linear systems with unknown inputs. The observer is a simple one, its derivation being direct and easy. It will be shown that the problem of full order observers for linear systems with unknown inputs can be reduced in this case to a standard one (the unknown input vector will not interfere in the observer equations). The effectiveness of the suggested design algorithm is illustrated by a numerical example (aircraft longitudinal motion), and, for the same aircraft dynamics, we make a comparison between our new observer and other already existing observers from the existence conditions and dynamic characteristics' point of view; the superiority of the new designed observer is demonstrated.

Keywords: Observer, Unknown input, Algorithm, Aircraft motion

1. Introduction

A. Antecedents and motivations

The plant input and output signals are used to estimate the plant state, which is then employed to close the control loop [1]. The aim of observers is to augment or replace sensors in a control system. Starting from the first observers, introduced by Luenberger [2, 3], the observers for plants with both known and unknown inputs have been developed resulting the so-called unknown input observer (UIO) architectures, such as, for example, those in [4-9].

The state estimation problem for linear-time invariant (LTI) multivariable system, subjected to unknown inputs, has received considerable attention in the last two decades [7, 10-17]. The method presented in [10] increases the dimension of the observer considerably, more interesting being the approach developed by Wang et al. [11], which propose a procedure to design reduced-order observers without any knowledge of these inputs; existence conditions for this observer have been provided by Kudva et al. [13]. The generalized inverse matrix [14], the Silverman's inverse method [15] or the singular value decomposition [16] are useful mathematical tools for the design of the linear observer [7].

Hui and Zak [1] proposed two design procedures for two different classes of observers for systems with unknown inputs. In the first approach, the state of the observed system is decomposed into known and unknown components, the unknown component being a projection of the whole state along the subspace in which the available state component resides. A dynamical system to estimate the unknown component is constructed and, combining the output of the dynamical system (which estimates the unknown state component) with the available state information, they obtained an observer that estimates the

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whole state [1]. The second design algorithm uses both the sliding modes and the Lyapunov second method.

The state estimators with unknown inputs are very important in robust model-based fault detection [18-20]. Making the difference between the actual system outputs and the estimated outputs, the residuals of the system result. When a fault occurs, the residuals become greater than a pre-specified threshold [1]. This is why, when a system is affected by disturbances or unknown inputs, their effect has to be decoupled from the residuals to avoid false alarms [1]

Other interesting observer design approach was suggested by Hou [8]; his reduced-order observer for linear systems with unknown inputs decomposes the state equation of the system into two subsystems: the first one depends on the unknown inputs, while, in the second one, the unknown inputs may be dropped. One of the assumptions of this approach is that the state of the second subsystem may be obtained through the measurement equation. The designed observer has good results but there are many situations in which this hypothesis does not hold [8]. Maquin, Gaddouna, and Ragot made some modifications to the observer of Hou and a straightforward treatment allowing the unknown input estimation is proposed in [8].

New conditions for the existence of reduced-order linear functional state observers for linear systems with unknown inputs were presented in [21]. Systematic procedures for the synthesis of reduced-order functional observers have been given, the attractive feature of the proposed observer being the simplicity with which the design process can be accomplished [21].

Strict conditions must always be met in order to reconstruct the entire state vector in the presence of unknown inputs. In [22], Valcher designed full-order observers for linear systems with unknown inputs, along with necessary and sufficient conditions for the existence of such observers. The conditions introduced in [22] have been relaxed in [23] allowing delays in the observer, but no design procedure was provided. In [24], the authors introduced a delayed observer by means of a higher dimensional system which incorporated the delayed states into the new state vector. Some geometric conditions were given for the existence of an observer for this augmented system, but the inconvenience of this method is that the observer dimension is much larger than the system dimension [24, 25]. The state estimation for linear systems with unknown inputs is made by using observers with delay in [25], the advantage of this kind of observers being the ease in establish necessary conditions for existence of unknown input observers with zero-delay [25]. The approach in [25] develops a parameterization of the observer gain that decouples the unknown inputs from the estimation error and uses the remaining freedom to ensure stability of the error dynamics. The algorithm in [25] generalizes the design procedure proposed by Valcher [22] to the case of reduced-order delayed observers, and treats the full-order observer as a special case of a reduced order observer where the dynamic portion reconstructs the entire state vector. The approach is quite general and it encompasses the design of full-order observers via appropriate choices of design matrices [25]; the algorithm is better than the one in [24] from the observer dimension point of view (the dimension of the observer with delay [25] is not greater than the original system dimension).

The observers producing delayed estimates of the system state can be very useful in many applications because it may be possible to use the delayed state estimate in feedback control by using the techniques described in [26]. Such unknown input observers have multiple applications. For example these observers are useful in the domain of communication systems based on the principle of chaotic synchronization [27]. For these systems, the transmitter is a dynamic system which takes the message as an input, and the receiver is an unknown input observer that synchronizes its state with that of the transmitter in order to recover the message. Thus, a delay in state reconstruction can certainly be tolerated [25].

Another application for unknown input observers is in the area of fault detection and identification [28], where a delay in detecting/reconstructing the fault is generally not a problem. The conclusion is that observers with the smallest possible delay must be designed [25].

For the majority of the existing approaches, the number of unknown inputs must be less than the number of outputs, and, moreover, additional structural requirements on the system to be observed are met [29, 30]. Those conditions turn out to be rather restrictive because, for instance, they cannot cover the simplest class of mechanical systems with unknown inputs wherein only the position is measurable [31]. Another disadvantage of observers is that only asymptotic convergence to zero of the observation and error is guaranteed [32]. However, for instance, for hybrid systems the finite time exact observation is quite important because it is necessary to ensure that the observation convergence time is less than the dwell time (for example, in the case of walking robots [25, 33, 34]).

Sliding mode observers [35-38] are widely used due to their insensitivity, which is stronger than mere robustness with respect to some classes of unknown inputs, and due to their possibility to use the equivalent output injection concept for the unknown inputs identification [31]. The sliding-mode observers [39-42] are based on the possibility to transform the actual system into a block observable form and, after that, by using the concept of equivalent output injection, the sequential estimation of each transformed state is made [31]. The disadvantages of these observers are related to obligatory filtration, which causes an intrinsic error in the observed states that cannot be eliminated; other disadvantages of these observers are related to the system structure which must be such that the transformation to the triangular form can be performed. A new generation of observers based on higher-order sliding-mode differentiators has been recently studied in the literature [43-49]. This kind of observers has the advantage of the first order sliding mode observers, but avoids the filtration process, allowing the finite-time convergence to zero for the estimation error [31]. In the design process of these observers, the unknown inputs were supposed uniformly bounded [50, 51], which allows the stabilization of the observation error observer to a neighborhood of the zero point, by means of a linear observer and the usage of a robust exact differentiator that must ensure the finite time exact reconstruction of the original state [31].

We may conclude that there are a lot of observers for linear systems with unknown inputs. There are 3 important design methods: geometrical methods (introduced first time by Bhattacharyya [13]), algebraic methods (used in observer design by Kudva, Viswadam şi Ramakrishna [12], Hautus [29], Hou & Muller [17], Darouach, Zasadzinski, Xu [7], Yang & Wild [52], O'Reilly [53] and so on), and methods that use the generalized inverse [54]. Each of them has advantages and disadvantages.

B. Main contribution

The classical full order observers are easier to implement from the software point of view. Their disadvantages are related to the important number of constraints (existence conditions). This paper presents a simple full-order observer design, its derivation being direct and easy. It will be shown that the problem of full order observers for linear systems with unknown inputs can be reduced in this case to a standard one (the unknown input vector does not interfere in the observer equations). The existence conditions for the obtained observer are given. The observer is an improved version of the observer designed by O'Reilly [7, 53, 54]; we obtain a new observer with a smaller number of constraints, but with similar or even better performances than those of other observers. In the O'Reilly observer design [7, 53, 54] the technique of pole placement has been used for determining one of the observer matrices. This method has its limitations and it can not always be used because, for instance, we do not know how to choose the eigenvalues of the observer matrix for different cases. That is why an alternative method for the observer design is needed. The approach presented in this paper does not use the pole placement technique but only the partitioning of the matrices and the solving of linear matriceal equation systems. We make a comparison between the O'Reilly observer old version and the O'Reilly observer improved version as well as a comparison between the O'Reilly observer improved version and 3 observers that are representative for the research area of the LTI systems with unknown inputs.

The paper is organized as follows: the design of the O'Reilly full-order observer [7, 53, 54] is given in section 2; an improved version of this observer (the main contribution of the paper), is designed in section 3; in section 4 the new observer is validated by means of a numerical simulation for the case of an aircraft longitudinal motion and a comparison between the 2 design approaches is achieved; in section 5 we compare the O'Reilly observer improved version with 3 observers that are representative for the research area of LTI systems with unknown inputs. Finally, some conclusions are shared in section 6.

2. Design of the O'Reilly Full-Order Observer

The observer in this section has been designed by O'Reilly [7, 53, 54]. The simple method to design a full-order observer for the linear systems with unknown inputs (presented in papers [7, 53, 54]) reduces the design procedure of full-order observers with unknown inputs to a standard one, where all the inputs are known. The existence conditions are also given and it has been shown that these conditions are generally adopted for unknown inputs observer problem [7]. The design of this full-order observer is briefly presented in this section.

Let us consider a LTI system described by [55-57]:

$$\begin{cases} \dot{x} = Ax + Bu + Dd, \\ y = Cx, \end{cases} \tag{1}$$

where $x \in \mathcal{R}^{n\times 1}$ is the system state vector, $u \in \mathcal{R}^{p\times 1}$ – the system known input vector, $d \in \mathcal{R}^{s\times 1}$ – the system unknown input vector, and $y \in \mathcal{R}^{m\times 1}$ – the output vector; the known matrices A, B, C, D have appropriate dimensions $\left(A \in \mathcal{R}^{n\times n}, B \in \mathcal{R}^{n\times p}, C \in \mathcal{R}^{m\times n}, D \in \mathcal{R}^{n\times s}\right)$. We assume $m \geq s, \operatorname{rank}(D) = s,$ $\operatorname{rank}(C) = m$, and the pair (C, A) is observable [7, 54].

Following [53], the full-order observer dynamics of the linear system with unknown inputs (1) is [7]:

$$\begin{cases} \dot{z} = Nz + Ly + Gu, \\ \hat{x} = z - Ey, \end{cases} \tag{2}$$

with $z \in \mathcal{R}^{n\times 1}$; $\hat{x} \in \mathcal{R}^{n\times 1}$ is the system estimated state vector. N, L, G, and E are unknown matrices of appropriate dimensions which must be calculated such that $\hat{x}$ asymptotically converges to x [54].

Considering the observer reconstruction error $e = \hat{x} - x$, the observer error dynamics is:

$$\dot{e} = Ne + \left(NP + LC - PA\right)x + \left(G - PB\right)u - PDd, \tag{3}$$

with $P = I_n + EC$ [54]. For the observer asymptotically convergence, equation (3) must have the homogeneous form $\dot{e} = Ne$; thus, the following convergence conditions result [7, 54]:

$$NP + LC - PA = 0, G - PB = 0, \left(I_n + EC\right)D = 0. \tag{4}$$

In addition, the matrix N must be stable $\left(\sigma(N) \subset C_-\right)$.

From the third equation (4) we easily obtain the expression of matrix E:

$$E = -D(CD)^{+}, \quad (5)$$

where $(CD)^{+}$ is the generalized pseudo-inverse of (CD), given by [58]: $(CD)^{+} = \left[(CD)^{T}(CD)\right]^{-1}(CD)^{T}$. The generalized pseudo-inverse of (CD) can be obtained if and only if (CD) is full column rank, namely $\mathrm{rank}(CD) = s; CD \in \mathcal{R}^{m\times s}$.

After the calculation of matrix E (equation (5)), we obtain $P = I_n + EC$, and, by using the second equation (4), we get [7, 54]:

$$G = PB = (I_n + EC)B = \left(I_n - D(CD)^{+}C\right)B. \quad (6)$$

The determination of the matrices N and L, by means of the first equation (4), is a difficult task because this matriceal equation has 2 unknown matrices. In order to use the well known results obtained for the classical full-order observer without unknown inputs [53], the first equation (4) can be written as [7]:

$$N = PA - KC, \quad (7)$$

where K is calculated by using the pole placement technique such that $\sigma(N) \subset C_{-}$. Next, substituting (7) into the first equation (4) we find [7]:

$$L = -PAE + K(I_m + CE). \quad (8)$$

Then, the full-order observer dynamics of the linear system with unknown inputs becomes:

$$\begin{cases} \dot{z} = (PA - KC)z + \left[K(I_m + CE) - PAE\right]y + PBu, \\ \hat{x} = z - Ey. \end{cases} \quad (9)$$

The full-order observer design problem has been reduced now to finding a matrix E that satisfies the third equation (4) and a matrix K such that $\sigma(N) \subset C_{-}$. This problem is equivalent to the standard problem of the observers' design when all inputs are known. The eigenvalues of $N = PA - KC$ can be arbitrarily located, by choosing the matrix K suitably, if and only if the pair (PA, C) is observable. If the pair (PA, C) is not observable, the calculation of matrix K is made such that the observer is asymptotically stable if and only if (PA, C) is detectable [7].

Necessary and sufficient conditions to design a stable observer specified in [7] are given by the following 2 theorems [7]:

Theorem 1 [7]:

For the linear time-invariant system described by equations (1), the full-order observer exists if and only if:

1) $\mathrm{rank}(CD) = \mathrm{rank}(D) = s$;

2) $\mathrm{rank}\begin{bmatrix} \mathbf{s}P - PA \\ C \end{bmatrix} = n, (\forall)\mathbf{s} \in \mathrm{C}, \mathrm{Re}\{\mathbf{s}\} \geq 0$.

Theorem 2 [7]:

If condition 1) holds and $\mathrm{rank}(P) = n - s$, the following 3 conditions are equivalent:

1) the pair (PA, C) is observable or at least detectable;

2) $\mathrm{rank}\begin{bmatrix} \mathbf{s}P - PA \\ C \end{bmatrix} = n, (\forall)\mathbf{s} \in \mathrm{C}, \mathrm{Re}\{\mathbf{s}\} \geq 0$;

3) $\operatorname{rank}\begin{bmatrix} \mathbf{s}I_n - A & D \\ C & 0 \end{bmatrix} = n + s$, $(\forall)\mathbf{s} \in \mathrm{C}$, $\operatorname{Re}\{\mathbf{s}\} \geq 0$.

The proofs of the two theorems are presented in [7]. We may conclude that, for the design of O'Reilly full-order observer, we made 4 assumptions ($m \geq s, \operatorname{rank}(D) = s, \operatorname{rank}(C) = m$, (C, A) is observable) and, according to the above two theorems, 3 existence conditions (constraints) must be fulfilled ($\operatorname{rank}(CD) = \operatorname{rank}(D) = s$; $\operatorname{rank}(P) = n - s$; the pair (PA, C) is observable or at least detectable).

3. Design of the O'Reilly Full-Order Observer Improved Version

The disadvantages of the O'Reilly full-order observer design are the 3 existence conditions and, most important, the calculation of the matrix L by means of the matrix K which has been obtained by using the technique of pole placement. The eigenvalues of $N = PA - KC$ have to be located arbitrarily, by choosing a suitably K matrix, if and only if the pair (PA, C) is observable. If the pair (PA, C) is not observable, K is determined such that the observer is asymptotically stable if and only if (PA, C) is detectable [7]. The choice of the eigenvalues for the matrix $N = PA - KC$ has a great influence on the poles of the observer and, accordingly, on the observer convergence speed. The pole placement technique is easy to implement in the case of observers' design, but it has some disadvantages: 1) the technique becomes difficult to be used for systems with big order or for poorly controlled systems; 2) if we choose fast poles for the observer, the advantage is that the observer estimation error decays rapidly, but the disadvantage is that the system needs perfect sensors and/or noise free environment; 3) if we choose slow poles for the observer, the advantage is that the system is less sensitive to process disturbances and measurement noise, but the disadvantage is that the observer estimation error decays slowly.

So, in the case of O'Reilly observer, the choice of the eigenvalues is one of the most important tasks; the design approach [7, 53, 54] does not present a specific method for the choice of the eigenvalues. This is why, to improve the approach, we must either find a way to optimize the choice of the eigenvalues or find other method to determine the matrices N and L as solutions of the first equation (4).

In this section of the paper we design a new approach for the state estimation problem in the case of LTI multivariable system, subjected to unknown inputs. The new approach must make the design problem equivalent with the standard problem of the observers' design when all inputs are known. Moreover, the pole placement technique will not be used and, accordingly, the most difficult task of the approach is now the determination of the matrices N and L, by solving one equation (first equation (4)) with two unknown variables (matrices N and L). The new observer convergence speed must be less than the convergence speed of the O'Reilly observer old version. Because the O'Reilly observer [7, 53, 54] has a direct and easy derivation, in order to obtain a better observer, we will also focus on the observer design procedure which also must be direct and simple and lead to as simple observer as the O'Reilly observer. The last concern regarding the O'Reilly observer is related to the 3 existence conditions. Two of these conditions arise from the need of (PA, C) observability, or at least detectability. Our new observer will have only one existence condition because we will calculate the matrices N and L, not by using the pole placement technique.

The new observer will have the same form with the one designed by O'Reilly (equation (2)). The design of the observer involves, in this case too, the determination of the 4 unknown matrices $N, L, G,$ and E from equation (2). The same 4 assumptions are made.

To calculate the matrices N and L, first we partitionate the matrices $N, P, L, C,$ and A as follows:

$$N = \begin{bmatrix} N_1 & N_2 \\ N_3 & N_4 \end{bmatrix}, P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}, L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, C^T = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \tag{10}$$

where

$$\begin{aligned} &N \in \mathcal{R}^{n\times n}, N_1 \in \mathcal{R}^{(n-m)\times(n-m)}, N_2 \in \mathcal{R}^{(n-m)\times m}, N_3 \in \mathcal{R}^{m\times(n-m)}, N_4 \in \mathcal{R}^{m\times m}, \\ &P \in \mathcal{R}^{n\times n}, P_1 \in \mathcal{R}^{(n-m)\times(n-m)}, P_2 \in \mathcal{R}^{(n-m)\times m}, P_3 \in \mathcal{R}^{m\times(n-m)}, P_4 \in \mathcal{R}^{m\times m}, \\ &L \in \mathcal{R}^{n\times m}, L_1 \in \mathcal{R}^{(n-m)\times m}, L_2 \in \mathcal{R}^{m\times m}, C \in \mathcal{R}^{m\times n}, C_1 \in \mathcal{R}^{m\times(n-m)}, C_2 \in \mathcal{R}^{m\times m}, \\ &A \in \mathcal{R}^{n\times n}, A_1 \in \mathcal{R}^{(n-m)\times(n-m)}, A_2 \in \mathcal{R}^{(n-m)\times m}, A_3 \in \mathcal{R}^{m\times(n-m)}, A_4 \in \mathcal{R}^{m\times m}. \end{aligned} \tag{11}$$

The observer design is concentrated into the following theorem:

Theorem 3:

Consider the LTI multivariable system (1); using the 4 assumptions $m \geq s$, $\text{rank}(D) = s, \text{rank}(C) = m$, and (C, A) is an observable pair of matrices, we can design a convergent observer for LTI systems with unknown inputs described by equations (2), where the 4 unknown matrices $N, L, G,$ and E have the following forms:

$$\begin{aligned} &N = \begin{bmatrix} N_1 & \left(\tilde{A}_2 - \left(\tilde{A}_1 - \tilde{A}_2 P_4^+ P_3\right)\left(C_1 - C_2 P_4^+ P_3\right)^+ C_2\right)P_4^+ \\ N_3 & \left(\tilde{A}_4 - \left(\tilde{A}_3 - \tilde{A}_4 P_4^+ P_3\right)\left(C_1 - C_2 P_4^+ P_3\right)^+ C_2\right)P_4^+ \end{bmatrix}, \\ &L = \begin{bmatrix} \left(\tilde{A}_1 - \tilde{A}_2 P_4^+ P_3\right)\left(C_1 - C_2 P_4^+ P_3\right)^+ \\ \left(\tilde{A}_3 - \tilde{A}_4 P_4^+ P_3\right)\left(C_1 - C_2 P_4^+ P_3\right)^+ \end{bmatrix}, \\ &G = PB = (I_n + EC)B = \left[I_n - D(CD)^+ C\right]B, \\ &E = -D(CD)^+, \end{aligned} \tag{12}$$

where

$$\begin{aligned} &\tilde{A}_1 = P_1 A_1 + P_2 A_3 - N_1 P_1, \tilde{A}_2 = P_1 A_2 + P_2 A_4 - N_1 P_2, \\ &\tilde{A}_3 = P_3 A_1 + P_4 A_3 - N_3 P_1, \tilde{A}_4 = P_3 A_2 + P_4 A_4 - N_3 P_2, \end{aligned} \tag{13}$$

and N_1, N_3 are random matrices.

Proof:

Substituting (10) into the first equation (4) we find:

$$\begin{cases} N_1 P_1 + N_2 P_3 + L_1 C_1 - P_1 A_1 - P_2 A_3 = 0_{(n-m)\times(n-m)}, \\ N_1 P_2 + N_2 P_4 + L_1 C_2 - P_1 A_2 - P_2 A_4 = 0_{(n-m)\times m}, \\ N_3 P_1 + N_4 P_3 + L_2 C_1 - P_3 A_1 - P_4 A_3 = 0_{m\times(n-m)}, \\ N_3 P_2 + N_4 P_4 + L_2 C_2 - P_3 A_2 - P_4 A_4 = 0_{m\times m}. \end{cases} \tag{14}$$

The previous matriceal equations system has 6 unknown variables (the matrices $N_1, N_2, N_3, N_4, L_1, L_2$) and only 4 equations. The system can be solved if only the number of unknown variables is equal or less than the number of equations. Therefore, to transform the equations system (14) into a compatible determined system, we choose two of the six matrices as random matrices. We choose $N_1 \in \mathcal{R}^{(n-m)\times m}$ and $N_3 \in \mathcal{R}^{m\times(n-m)}$ randomly, but the approach works well even if, instead of matrices N_1 and N_3, from the 4 matrices $N_i, i = \overline{1,4}$, any two matrices are chosen randomly. The other two matrices, together with L_1 and L_2 are now the 4 unknown variables of the system (14); matriceal equations system (14) became a compatible determined system, its solving being not a difficult task.

Using the notations (13), the system (14) is divided into the two following systems:

$$\begin{cases} N_2 P_3 + L_1 C_1 = \tilde{A}_1 , \\ N_2 P_4 + L_1 C_2 = \tilde{A}_2 , \end{cases} \tag{15}$$

$$\begin{cases} N_4 P_3 + L_2 C_1 = \tilde{A}_3 , \\ N_4 P_4 + L_2 C_2 = \tilde{A}_4 . \end{cases} \tag{16}$$

To solve the first system, from the second equation (15), we get $N_2 = \left(\tilde{A}_2 - L_1 C_2\right)P_4^+$ and we replace it into the first equation (15); we obtain $L_1 = \left(\tilde{A}_1 - \tilde{A}_2 P_4^+ P_3\right)\left(C_1 - C_2 P_4^+ P_3\right)^+$. Thus, the solutions of system (15) are:

$$\begin{aligned} N_2 &= \left(\tilde{A}_2 - \left(\tilde{A}_1 - \tilde{A}_2 P_4^+ P_3\right)\left(C_1 - C_2 P_4^+ P_3\right)^+ C_2\right)P_4^+ , \\ L_1 &= \left(\tilde{A}_1 - \tilde{A}_2 P_4^+ P_3\right)\left(C_1 - C_2 P_4^+ P_3\right)^+ . \end{aligned} \tag{17}$$

The system (16) is solved similarly and we obtain:

$$\begin{aligned} N_4 &= \left(\tilde{A}_4 - \left(\tilde{A}_3 - \tilde{A}_4 P_4^+ P_3\right)\left(C_1 - C_2 P_4^+ P_3\right)^+ C_2\right)P_4^+ , \\ L_2 &= \left(\tilde{A}_3 - \tilde{A}_4 P_4^+ P_3\right)\left(C_1 - C_2 P_4^+ P_3\right)^+ . \end{aligned} \tag{18}$$

Now, substituting (17) and (18) into (10), we obtain the first two expressions (12) with N_1 and N_3 – random matrices. The matrices E and G are the solutions of the last two equations (4). Their calculation is made, as in the O'Reilly approach, by using the equations (5) and (6). Because the 3 conditions (4) hold, equation (3) has the homogeneous form $\dot{e} = Ne$; e (the observer error) converges asymptotically to zero and the asymptotic state reconstruction occurs if and only if the matrix N is stable (all its eigenvalues are placed in the left-hand side of the complex plane). The theorem 3 is now demonstrated, but we are not sure if the matrix N is stable. That is why we check if $\sigma(N) \subset C_-$; if the condition is fulfilled, N has been obtained properly, the dynamics of the observer error has the homogeneous form $\dot{e} = Ne,$ and, as a consequence, the error e converges asymptotically to zero ($\hat{x}$ asymptotically converge to x); otherwise, other random matrices N_1 and N_3 are chosen and the systems (15) and (16) are again solved until the matrix N is stable. In the software program of our new algorithm, the fulfillment of the condition $\sigma(N) \subset C_-$ is made in a "while" loop.

The algorithm to design a new observer for the state estimation problem in the case of LTI multivariable systems with unknown inputs has some important advantages with respect to the O'Reilly design algorithm. First of all, our algorithm eliminates the most important disadvantage of the O'Reilly algorithm: the choice of eigenvalues for the matrix N. An alternative method has been used to obtain the matrices $N, L, G,$ and $E,$ without using the pole placement technique. Thus, no condition regarding the observability or the detectability of the pair (PA, C) is needed in our new observer design and, therefore, the only one existence condition of the observer is $\operatorname{rank}(CD) = \operatorname{rank}(D) = s$. Our new observer makes the design problem equivalent with the standard problem of the observers' design when all inputs are known. The comparison between the convergence speed of O'Reilly algorithm and the convergence speed of our new designed observer is made in the next section of this paper.

Remark 1:

Because we can choose which are the outputs of the system, the matrix C in equation (1) can be judiciously chosen; thus, if $C = I_{m\times n}$ (the identity matrix) the only existence condition of our new observer $(\text{rank}(CD) = \text{rank}(D) = s)$ is always fulfilled and the third assumption of the algorithm $(\text{rank}(C) = m)$ is always valid because $n \geq m$. In these circumstances, our algorithm has no existence conditions and only 3 assumptions have to be made. Thus, the main advantage of O'Reilly full-order observer improved version is the lack of apriori restrictions on the class of systems that can be considered.

The algorithm associated to the improved version of the O'Reilly observer is summarized below:

Step 1: Working under the 4 assumptions $m \geq s, \text{rank}(D) = s, \text{rank}(C) = m, (C, A)-$ observable pair of matrices, we check if the only constraint of the approach $(\text{rank}(CD) = \text{rank}(D) = s)$ is valid;

Step 2: We calculate the matrices E and G (solutions of the last two equations (4)) by means of equations (5) and (6);

Step 3: We partitionate the matrices $N, P, L, C,$ and A by using equations (10) and (11);

Step 4: We choose the random matrices N_1 , N_3 and we calculate the matrices $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4$;

Step 5: We solve the systems (15) and (16) and we obtain the matrices L_1, L_2, N_2, N_4 – equations (17) and (18);

Step 6: We build the matrices N and L and we check if $\sigma(N) \subset C_-$; if the condition is fulfilled, N has been obtained properly and the error e converges asymptotically to zero ($\hat{x}$ asymptotically converge to x); otherwise, we return to step 4 and repeat the steps $4 \div 6$ until N is stable;

Step 7: The observer described by equations (2) is completely designed and we can obtain now the state and estimated state time history.

Remark 2:

The old version of the O'Reilly algorithm uses the pole placement technique to calculate the matrix N and, of course, the chosen eigenvalues have to be placed in the left-hand side of the complex plane; thus the use of the pole placement technique is an apriori guarantee that the result matrix N will be stable. For our algorithm, there is no apriori guarantee that $\sigma(N) \subset C_-$ after the first browsing of the steps $1 \div 6$, but the stability of the matrix N is provided by the "while" loop which includes steps $4 \div 6$. There in no risk for an infinit "while" loop and, that's why, the improved version of the O'Reilly algorithm will always provide a Hurwitz matrix N and, as a consequence, the observer will always be convergent.

4. Comparison between the O'Reilly observer Old Version and the O'Reilly Observer Improved Version

The validation of our new algorithm for a full-order observer design is made, in this section, for the case of longitudinal motion of a Charlie aircraft [59]. The validation of the algorithm is performed in Matlab/Simulink environment. Consider the state equation associated to the longitudinal motion of a Charlie aircraft [59] having the form (1) with:

$$x=\left[\Delta u\ \ \Delta\alpha\ \ \Delta\theta\ \ \Delta q\right]^T,\ u=\delta_e,$$
$$A=\begin{bmatrix}-0.007 & 0.012 & -9.81 & 0\\ -0.128 & -0.54 & 0 & 1\\ 0 & 0 & 0 & 1\\ 0.065 & 0.96 & 0 & -0.99\end{bmatrix},\ B=D=\begin{bmatrix}0\\ -0.04\\ 0\\ -12.5\end{bmatrix},\ C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}; \tag{19}$$

u is the aircraft longitudinal velocity, $\alpha -$ the aircraft attack angle, $\theta -$ the aircraft pitch angle, $q -$ the aircraft pitch angular rate, δ_e is the elevator deflection, while Δ is associated with the perturbation of the variables from their nominal values; $d \in \mathcal{R}^{s\times 1}$ is considered a random signal. The input signal of the system can be chosen as a step signal, a sinusoidal signal or any random signal. In this simulation, we calculated, by using the ALGLX algorithm [59], the gain matrix $\overline{K}$ (feedback of the closed loop system) and we considered that the input vector of the system is $u=\delta_e=-\overline{K}\hat{x}$.

For the improved version of the O'Reilly observer we obtained the graphic characteristics in Fig.1 (the 4 states $x_i, i=\overline{1,4}-$ solid line and the 4 estimated states $\hat{x}_i, i=\overline{1,4}-$ dashed line); the graphics of the system states are superposed over the graphics of the estimated states.

Using the same flight data (same matrices) and the same unknown input vector, we also implemented the O'Reilly observer old version in Matlab/Simulink in order to compare the results. Thus, in Fig.2 we represent the time histories of the state estimation errors $e_i=x_i-\hat{x}_i, i=\overline{1,4}$ for the two versions of the O'Reilly observer (O'Reilly observer old version – dashed line and O'Reilly observer improved version – solid line).

From the simulation results' point of view, the comparison between the two variants of the O'Reilly algorithm leads to the following conclusions: 1) Both variants of the O'Reilly observer are convergent (the 4 components of the state estimation error tend to zero); 2) The O'Reilly observer improved version is characterized by a convergence speed of 2 seconds, while, for the same numerical example, the convergence speed associated to the O'Reilly observer old version is 4 seconds; this means an advantage of our new observer from the convergence speed point of view (a decrease of 50%); 3) The oscillations' amplitudes of the state estimation errors are smaller for the O'Reilly observer improved version than the ones of the O'Reilly observer old version.

5. Comparison between Our Observer and Other Observers for LTI Systems with Unknown Inputs

In this section, we compare our new observer with other already existing observers for LTI systems with unknown inputs. We briefly present 3 important observers (observers that are representative for the research area of LTI systems with unknown inputs) from the specialty literature and we compare these 3 observers with our new designed observer.

The first observer has been designed by Hui & Zak [60] and its design is concentrated into the following theorem:

Theorem 4 [60]:

Consider the LTI multivariable system (1); using the assumption (C, A) is an observable pair of matrices, we design the convergent observer for LTI systems with unknown inputs described by the equation [60]:

$$\hat{x}=T\begin{bmatrix}I_{n-s}\\ 0_{s\times(n-s)}\end{bmatrix}\tilde{q}+My, \tag{20}$$

where $\hat{x} \in \mathcal{R}^{n\times1}$ is the system estimated state, $y \in \mathcal{R}^{m\times1}$ – the system output vector, $M = D(CD)^{+}$, T – the solution of equation $T^{-1}(I_s - MC)T = \begin{bmatrix} I_{n-s} & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{q}$ – the solution of equation [60]:

$$\dot{\tilde{q}} = \left(\tilde{A}_{11} - \tilde{L}_1\tilde{C}_1\right)\tilde{q} + \left[I_{n-s}\, 0_{(n-s)\times s}\right] T^{-1}\left(\tilde{G}y + Bu\right), \tag{21}$$

with $\tilde{A} = T^{-1}AT = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}, \tilde{C} = CT = \left[\tilde{C}_1 \;\; \tilde{C}_2\right], \tilde{A}_{11} \in \mathcal{R}^{(n-s)\times(n-s)}, \tilde{A}_{12} \in \mathcal{R}^{(n-s)\times s}, \tilde{A}_{21} \in \mathcal{R}^{s\times(n-s)},$ $\tilde{A}_{22} \in \mathcal{R}^{s\times s}, \tilde{C}_1 \in \mathcal{R}^{m\times(n-s)}, \tilde{C}_2 \in \mathcal{R}^{m\times s}$; the matrix $\tilde{L}_1$ is calculated such that $\sigma\left(\tilde{A}_{11} - \tilde{L}_1\tilde{C}_1\right) \subset C_{-}$, while $\tilde{G}$ is $\tilde{G} = AM + T\tilde{L}(I_m - CM)$, where $\tilde{L} = \left[\tilde{L}_1 \;\; 0_{s\times m}\right]$.

The proof of the theorem 4 is presented in [60]. The disadvantages of this Hui & Zak observer are related to the number of existence conditions: 1) $\mathrm{rank}(CD) = \mathrm{rank}(D)$; 2) (CD) is full column rank, namely $\mathrm{rank}(CD) = s$; 3) the pair $\left(\tilde{A}_{11}, \tilde{C}_1\right)$ is observable or at least detectable. The first two observer constraints lead to only one existence condition: $\mathrm{rank}(CD) = \mathrm{rank}(D) = s$; thus this Hui & Zak observer has 2 existence conditions, while our new designed observer has only one constraint $(\mathrm{rank}(CD) = \mathrm{rank}(D) = s)$, but if we choose $C = I_{m\times n}$, our algorithm has no existence conditions. The most important disadvantage of this Hui & Zak observer is the use of the pole placement technique to calculate the matrix $\tilde{L}_1$; the choice of the eigenvalues for the matrix $\tilde{A}_{11} - \tilde{L}_1\tilde{C}_1$ has a great influence on the poles of the observer and, accordingly, on the observer convergence speed.

The second observer presented in this section belongs to Hui & Zak too [60]. Under the assumptions (C, A) is an observable pair of matrices and $\mathrm{rank}(CD) = \mathrm{rank}(D) = r$, the observer design is based on the nonsingular transformation matrices T and S (matrices associated to a coordinates change) such that [60]:

$$\hat{A} = TAT^{-1} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix},\; TD = \begin{bmatrix} D_2 \\ 0 \end{bmatrix}, \hat{C} = SCT^{-1} = \begin{bmatrix} I_r & 0 \\ 0 & \hat{C}_{22} \end{bmatrix}; \tag{22}$$

$\hat{A}_{11} \in \mathcal{R}^{r\times r}, \hat{A}_{12} \in \mathcal{R}^{r\times(n-r)}, \hat{A}_{21} \in \mathcal{R}^{(n-r)\times r}, \hat{A}_{22} \in \mathcal{R}^{(n-r)\times(n-r)}, D_2 \in \mathcal{R}^{r\times s}; \mathrm{rank}(D_2) = r, \hat{C}_{22} \in \mathcal{R}^{(m-r)\times(n-r)}$. From these equations, considering A, C, D – known matrices and r – known positive constant, we determine $D_2, \hat{C}_{22},$ and the nonsingular matrices T and S. The second Hui & Zak observer design [60] is concentrated into the following theorem:

Theorem 5 [60]:

Consider the LTI multivariable system (1); we design the convergent observer for LTI systems with unknown inputs described by the equation [60]:

$$\dot{\hat{x}} = (A - LC)\hat{x} + Bu - Df(\hat{y}, y, \eta), \tag{23}$$

where $y = Cx, \hat{y} = C\hat{x}, L = T^{-1}\hat{L}S, \hat{L} = \begin{bmatrix} kI_r & 0 \\ 0 & \hat{L}_{22} \end{bmatrix}; \hat{L}_{22}$ is calculated such that $\sigma\left(\hat{A}_{22} - \hat{L}_{22}\hat{C}_{22}\right) \subset C_{-}$, k is a constant chosen as follows [60]:

$$2k > \mathrm{Re}\left\{\sigma_{\max}\left[\hat{A}_{11}^T + \hat{A}_{11} + \left(\hat{A}_{21}^T\hat{P}_{22} + \hat{A}_{12}\right)\hat{Q}_{22}^{-1}\left(\hat{A}_{12}^T + \hat{P}_{22}\hat{A}_{21}\right)\right]\right\};$$

$\hat{Q}_{22} \in \mathcal{R}^{(n-r)\times(n-r)}$ is a positive defined matrix, while $\hat{P}_{22}$ is the solution of the Lyapunov equation [60]:

$$\left(\hat{A}_{22} - \hat{L}_{22}\hat{C}_{22}\right)^T \hat{P}_{22} + \hat{P}_{22}\left(\hat{A}_{22} - \hat{L}_{22}\hat{C}_{22}\right) = -\hat{Q}_{22}\,. \tag{24}$$

The function $f(\hat{y}, y, \eta)$ is:

$$f(\hat{y}, y, \eta) = \begin{cases} \eta \dfrac{F(y-\hat{y})}{\left\|F(y-\hat{y})\right\|_2}, & F(y-\hat{y}) \neq 0, \\ rR, & F(y-\hat{y}) = 0, \end{cases} \tag{25}$$

with $R^T = [1\ 1], \eta -$ positive constant, and $F = \left[D_2^T\ \ 0\right]S$.

The proof of the theorem 5 is presented in [60]. The second Hui & Zak observer has 2 existence conditions: 1) $\operatorname{rank}(CD) = \operatorname{rank}(D)$; 2) the pair $\left(\hat{A}_{22}, \hat{C}_{22}\right)$ is observable or at least detectable. The second existence condition must be fulfilled in order to calculate the matrices $\hat{L}_{22}$ by choosing desired eigenvalues for the matrix $\hat{A}_{22} - \hat{L}_{22}\hat{C}_{22}$.

The third observer (ALGOOR observer) presented in this section belongs to Lungu et al. [61] and its design is concentrated into the following theorem:

Theorem 6 [61]:

Under the assumptions (C, A) is an observable pair of matrices, $\operatorname{rank}(C) = m, \operatorname{rank}(D) = s$, and $m + s = n$, we can design a convergent observer for the LTI systems with unknown inputs described by the equations [61]:

$$M\dot{z} = Fz + Gu + Hy, \hat{x} = Pz + Qy, \tag{26}$$

with $M, F, G, H, P, Q -$ the solutions of the equations:

$$G = MNB, HC = MNA - FN, MND = 0\,, \tag{27}$$

namely:

1) $M = \begin{bmatrix} M_1 & 0_{(n-m)\times(n-m)} \\ M_3 & 0_{(n-s)\times(n-m)} \end{bmatrix}$, where $M_1 \in \mathcal{R}^{(n-m)\times(n-s)}$ and $M_3 \in \mathcal{R}^{(n-s)\times(n-s)}$ are the solutions of the equations:

$$M_1 D_2 = 0, M_3 D_2 = 0\,. \tag{28}$$

with $D^T = [D_1\ \ D_2], D_1 \in \mathcal{R}^{(n-m)\times s}, D_2 \in \mathcal{R}^{(n-s)\times s}$;

2) $G = MNB,$ with $N = \begin{bmatrix} N_1 & N_2 \\ N_3 & N_4 \end{bmatrix} = \begin{bmatrix} 0_{(n-s)\times(n-m)} & I_{(n-s)\times m} \\ 0_{(n-m)\times(n-m)} & 0_{(n-m)\times m} \end{bmatrix}$;

3) $H = \begin{bmatrix} M_1 A_3 C_1^+ \\ M_3 A_3 C_1^+ \end{bmatrix}, F = \begin{bmatrix} M_1 A_4 - M_1 A_3 C_1^+ C_2 & 0_{(n-m)\times(n-m)} \\ M_3 A_4 - M_3 A_3 C_1^+ C_2 & 0_{(n-s)\times(n-m)} \end{bmatrix}$, where $A = \begin{bmatrix} A_{1(n-m)\times(n-m)} & A_{2(n-m)\times m} \\ A_{3m\times(n-m)} & A_{4m\times m} \end{bmatrix}$ and $C = \left[C_{1m\times(n-m)}\ \ C_{2m\times(n-s)}\right]$;

4) $P = \begin{bmatrix} 0_{(n-m)\times(n-s)} & P_{2(n-m)\times(n-m)} \\ 0_{(n-s)\times(n-s)} & -C_2^+ C_1 \end{bmatrix}$, $Q = \begin{bmatrix} Q_{1(n-m)\times m} \\ C_2^+ \end{bmatrix}$, with P_2 and $Q_1 -$ the solutions of the equations:

$$P_2 + Q_1 C_1 = I_{n-m}\,, Q_1 C_2 = 0_{(n-m)\times(n-s)}\,. \tag{29}$$

The proof of theorem 6 is presented in [61]. This observer has 4 existence conditions: 1) $\operatorname{rank}(C) = m$ (m is the number of the system outputs); 2) $\operatorname{rank}(D) = s$ (s is the number of the system unknown inputs); 3) $m + s = n$ (n is the number of the system states); 4) the matrices C_1 and C_2 must be full column ranks in order to calculate their pseudo-inverses C_1^+ and C_2^+, respectively. Despite the large number of constraints, the ALGOOR observer has 2 important advantages: 1) the observer design algorithm does not use nonsingular transformations like other algorithms [60, 62, 63]; 2) the ALGOOR observer does not use the technique of pole placement for the calculation of the observer matrices. Compared with O'Reilly observer improved version, the ALGOOR observer for LTI systems with unknown inputs has the same advantages, but the number of existence conditions is larger and its design is more difficult. That is why, from the constraints and design point of view, the new version of the O'Reilly observer is better and, beside the observer convergence, the main advantage of the O'Reilly observer improved version is the lack of apriori restrictions on the class of systems that can be considered.

Taking into account the design of the above 3 observers, we conclude the superiority of our new observer from the existence conditions' point of view; because our new observer has only one constraint, it has a greater generality.

Now, we want to make a comparison between our new observer and the 3 observers summarized in this section from the dynamic characteristics' point of view; therefore, we implemented in Matlab/Simulink the 4 observers using the same aircraft dynamics, same flight data, and same unknown input vector in order to compare the convergence speeds (transient regime periods) and the oscillations' amplitudes.

In Fig.3 we represent the time dependencies of the 4 state estimation errors for the O'Reilly observer improved version (solid line) and the for the 3 observers summarized in this section (first Hui & Zak observer, second Hui & Zak observer, and the ALGOOR observer).

From the dynamic characteristics' point of view, the comparison between the 4 observers for systems with unknown inputs leads to the following conclusions: 1) All the 4 observers are convergent (the 4 components of the state estimation error tend to zero); 2) The O'Reilly observer improved version is characterized by a convergence speed of 2 seconds, while, for the same aircraft dynamics, the convergence speed associated to other 3 observers is about $3 \div 4$ seconds; this means an advantage of our new observer from the convergence speed point of view (a decrease of $33.33 \div 50$ % of the convergence speed); 3) The oscillations' amplitudes of the state estimation errors are smaller for the O'Reilly observer improved version than the ones of the 3 observers summarized in this section.

Taking into account all the above conclusions, we remark the superiority of our new observer from the dynamic characteristics' point of view. Our observer convergence speed (2 seconds) represents a very good convergence speed in the research area of the LTI systems with unknown inputs and, therefore, we conclude that it can be used with very good results to the state estimation in the case of LTI systems with unknown inputs.

6. Conclusions

This paper presents a simple full-order observer design, its derivation being direct and easy. It has been shown that the problem of full order observers for linear systems with unknown inputs can be reduced in this case to a standard one (the unknown input vector does not interfere in the observer equations). The effectiveness of the suggested design algorithm is illustrated by a numerical example (aircraft longitudinal motion), and, for the same aircraft dynamics, we made a comparison between our new observer and other already existing observers; the superiority of the new designed observer has been demonstrated from the dynamic characteristics' point of view (the O'Reilly observer improved version has a better convergence speed and the state estimation errors are characterized by smaller oscillations' amplitudes) and from the constraints' point of view (our new algorithm has only one existence condition which can be eliminated in the case of a judicious choice of the system outpus).

The designed observer is a full order observer without unknown inputs reconstruction. In the future we intend to extend the work from this paper and to obtain a new observer with unknown inputs reconstruction; it can be very useful as a subsystem (fault detection/ diagnosis scheme) of a typical fault-tolerant control system.

Acknowledgments

This work was supported by the strategic grant POSDRU/89/1.5/S/61968 (2009), co-financed by the European Social Fund within the Sectorial Operational Program Human Resources Development 2007-2013.

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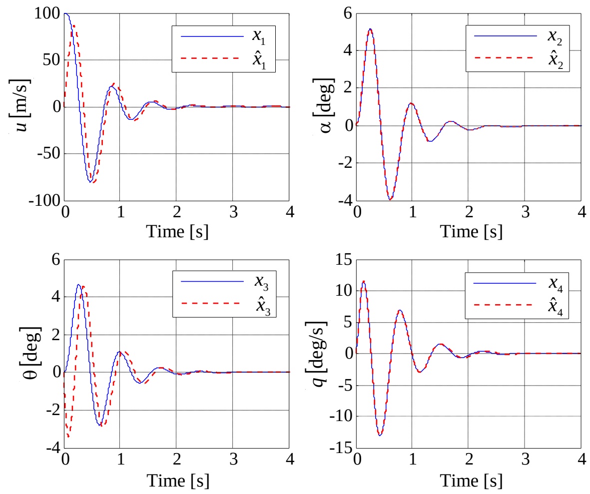

Fig. 1 States and estimated states time history (O'Reilly observer improved version)

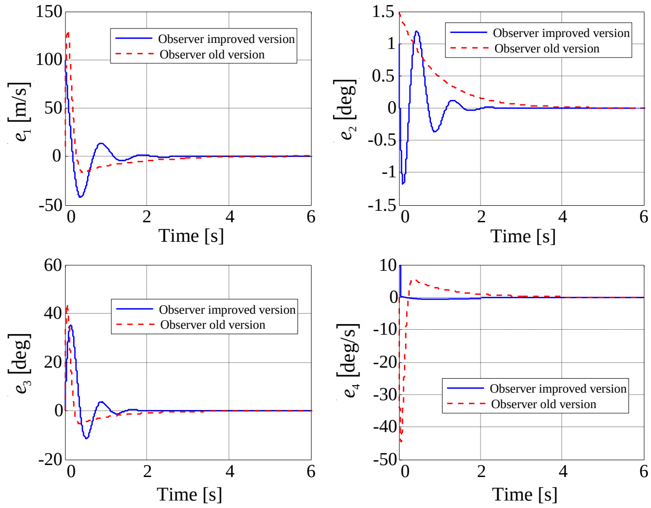

Fig. 2 State estimation errors by using the two versions of the O'Reilly full order observer

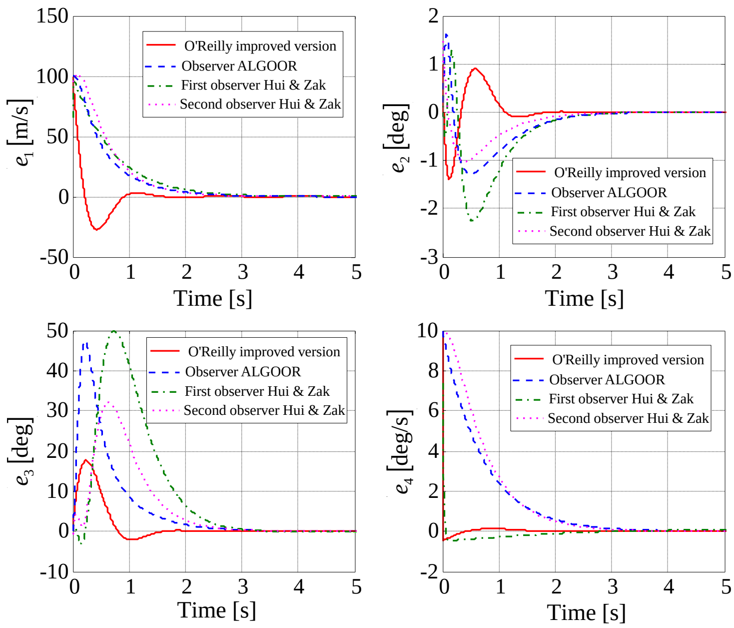

Fig. 3 Comparison between O'Reilly observer improved version and other 3 observers for LTI systems with unknown inputs