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CONTROL OF THE AIRCRAFT LATERAL-DIRECTIONAL MOTION DURING LANDING USING THE H-INF CONTROL AND THE DYNAMIC INVERSION

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In this paper, the phase of aircraft landing associated to the lateral-directional plane is discussed and a new automatic control system is developed; it takes into consideration the crosswind and the sensor errors. H-inf control and the dynamic inversion concept are used because the first mentioned technique handles very well the system uncertainties providing robust stability, while the good precision tracking is assured by the dynamic inversion approach. The new automatic landing system for aircraft control in the lateral plane also includes two reference models and an observer for the estimation of the system state. To test the correctness of the theoretical control system, we software implemented the new designed controller for a light aircraft flight in lateral-directional plane and we analyzed the results as well as the controller robustness with respect to the lateral wind.

Key words: landing, H-inf control, dynamic inversion, observer, reference model.

1. INTRODUCTION

In recent years, beside the conventional control laws (proportional-derivative, proportional-integral or proportional-integral-derivative [1–3]), different command filters, dynamic compensators, or state observers [4–6] useful for the design of the Automatic Landing Systems (ALSs), lots of scientific researchers used in the design process of the ALSs different intelligent concepts such as: 1) the optimal synthesis H_2, H_∞, H_2/H_∞ [7, 8]; 2) the dynamic inversion concept; 3) the adaptive synthesis based on dynamic inversion theory and neural networks theory [9]; 4) fuzzy techniques [10]. From the optimal control approaches' point of view, the most known designed controllers have been obtained by Shue & Agarwal (mixed H_2/H_∞ controller) [11], Ochi & Kanai (H_∞ controller) [12], the drawback of these controllers being that their robustness is not analyzed in the presence of sensor errors and crosswind (lateral wind); this barriers are lifted in our paper. There are only a few papers where both the crosswind and the sensor errors are considered; in [13] a PD-type fuzzy control system is developed by using or not the BPTT (Back-propagation Through Time) concept, the negative point being related to the fact that the wind disturbances are considered only as initial conditions, while the persistent wind disturbance is not considered. In many papers [1, 14], to improve the robustness of the controllers for aircraft landing in lateral-directional plane, there have also been used different techniques including linear quadratic optimal control (LQR/LQG), μ-synthesis, dynamic inversion approach, neural networks, and H-inf control [15]. In these papers, either the aircraft does not track accurately the desired flight path or the obtained algorithms are not tested in the presence of crosswind and/or wind gusts. According to this paper's authors, little progress has been reported for the flight control systems in the lateral-directional plane by using both the H-inf control and the dynamic inversion; this is why, we are motivated to design such a new landing control system taking into consideration the crosswind and the errors of the sensors. The theoretical issues presented in this paper represent an extension of those in [4]; in this paper, the landing in longitudinal plane is analyzed, while, in ours, the aircraft motion in lateral-directional plane is studied. Analyzing the specialty literature, we remark that the system presented in our paper is the only control system based on the dynamic inversion and H-inf control which uses an optimal observer, two reference models (for the calculation of aircraft desired lateral deviation and sideslip angle), and two methods for optimal control law's obtaining. All these new elements increase the generality, applicability, and simplicity of the control design and make our new control system able to reject the sensor measurement noises and crosswind with low intensity, a robustness study with respect to the lateral wind being achieved. The general control law will consist of two components: 1) a guidance component, for the imposed landing trajectory, calculated by using the dynamic inversion approach in two variants; 2) an optimal component which will be designed by means of the H-inf method. The two methods for the calculation of the guidance component make our control law superior to the one in [4] because there the design procedure is more complicated since the controller is separated in two subsystems (a stable and an unstable one) which must be stabilized separately.

2. AIRCRAFT DYNAMICS IN LATERAL PLANE DURING LANDING APPROACH PHASE

If the motion of the aircraft in lateral plane is made without errors (deviation of the aircraft from the runway direction is zero) the landing of aircraft is safer; to increase the safety of this process, before the start of the two landing main stages in longitudinal plane (the glide slope phase and the flare phase), the pilot must cancel the aircraft lateral deviation with respect to the runway. In order to achieve this, one can use different control systems for the flight direction control with radio navigation subsystem and equipment for the measurement of the distance between the aircraft and the radio markers [9, 16] or the system which will be designed in the next sections of this paper.

Using the small disturbances' method with respect to an equilibrium trajectory, the linearization of an aircraft nonlinear dynamics can be achieved. The lateral-directional dynamics (A) used in this paper belongs to a light airplane (Charlie airplane) [5]:

$$\begin{gathered}\dot{\beta} = a_{11}\beta + a_{12}p + a_{13}r + a_{14}\varphi + b_{11}\delta_a + b_{12}\delta_r + \frac{a_{11}}{V_0}V_{vy},\ \dot{p} = a_{21}\beta + a_{22}p + a_{23}r + b_{21}\delta_a + b_{22}\delta_r + \frac{a_{21}}{V_0}V_{vy},\\ \dot{r} = a_{31}\beta + a_{32}p + a_{33}r + b_{31}\delta_a + b_{32}\delta_r + \frac{a_{31}}{V_0}V_{vy},\ \dot{\varphi} = p,\ \dot{\psi} = r,\ \dot{Y} = -V_0\beta + V_0\psi + V_{vy},\\ \dot{\delta}_a = -\frac{1}{T_a}\delta_a + \frac{1}{T_a}\delta_{a_c},\ \dot{\delta}_r = -\frac{1}{T_r}\delta_r + \frac{1}{T_r}\delta_{r_c},\end{gathered} \tag{1}$$

where β is the aircraft sideslip angle, φ and ψ are the roll angle and the yaw angle, respectively, p – the aircraft roll angular rate, r – the aircraft yaw angular rate, Y – the deviation of the aircraft with respect to the runway direction, δ_a and δ_r – the deflection angles of the ailerons and rudder, respectively, δ_{a_c} and δ_{r_c} – the roll and the yaw commands (commands applied to the actuators), V_{vy} – the component of the wind velocity along the lateral axis of the aircraft, V_0 – aircraft nominal velocity, T_a and T_r – the effectors' time delay constants of the ailerons and rudder, respectively. Aircraft lateral motion may be written under the state equation:

$$\dot{x} = Ax + Bu + Gw, \tag{2}$$

where x is the state vector, u – the command vector, while the crosswind $w = V_{vy}$ is the system's disturbances – estimated by the equipment from aircraft's navigation system; $x = [\beta\ p\ r\ \varphi\ \psi\ Y\ \delta_a\ \delta_r]^T$, $u = [\delta_{a_c}\ \delta_{r_c}]^T$. By identification of equations (1) and (2), there are obtained the matrices $A \in R^{8\times 8}, B \in R^{8\times 2}$, and $G \in R^{8\times 1}$;

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & 0 & 0 & b_{11} & b_{12} \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 & b_{21} & b_{22} \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 0 & b_{31} & b_{32} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -V_0 & 0 & 0 & 0 & V_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_a} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_r} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{T_a} & 0 \\ 0 & \frac{1}{T_r} \end{bmatrix}, G = \begin{bmatrix} \frac{a_{11}}{V_0} \\ \frac{a_{21}}{V_0} \\ \frac{a_{31}}{V_0} \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}. \tag{3}$$

3. DESIGN OF THE H-INF CONTROL

A. Problem's formulation. For the design of the control law, one considers the vector containing the system controllable output variables $z = \begin{bmatrix} Y & \beta \end{bmatrix}^T = C'x$ and the vector $\bar{z} = \begin{bmatrix} \bar{Y} & \bar{\beta} \end{bmatrix}^T$ which contains the system's reference variables (the imposed values of the lateral deviation and aircraft sideslip angle, respectively). The system output vector is *y*, chosen of the following form: $y = \begin{bmatrix} Y & \dot{Y} & \beta & \varphi & p & \psi & r \end{bmatrix}^T = Cx$; the sensors' errors have been not taken into account here. The matrices $C \in R^{7\times 8}$ and $C' \in R^{2\times 8}$ are respectively:

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -V_0 & 0 & 0 & 0 & V_0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, C' = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{4}$$

By means of the equations: $\dot{\bar{x}} = A\bar{x} + B\bar{u}, \bar{z} = C'\bar{x}, \bar{y} = C\bar{x}$, of the dynamic inversion and by using the vector $\bar{z}$, we will calculate $\bar{x}, \bar{u}$ and $\bar{y}$; a line above a vector denotes that this vector is the desired (imposed) one. The general control law (lateral plane) will consist of two components: 1) a guidance component $(\bar{u})$, for the imposed landing trajectory, calculated by using the dynamic inversion; 2) an optimal component (u_∞) which will be designed by means of the H-inf method. The formula of the general control law is: $u = \bar{u} + u_\infty$.

B. Design of the first component (guidance component) of the control law *u*. To obtain the control law $\bar{u}$, a coordinates' change is achieved using the transformation matrix $T \in R^{8\times 8}$;

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = Tx, x = T^{-1}\begin{bmatrix} \xi \\ \eta \end{bmatrix}, \tag{5}$$

where ξ is a state vector consisting of the controlled variables and their derivatives, *i.e.* [4]: $\xi = \begin{bmatrix} z_1 & \dot{z}_1 & \cdots & z_1^{(r_1-1)} & z_2 & \dot{z}_2 & \cdots & z_2^{(r_2-1)} & \cdots & z_p & \dot{z}_p & \cdots & z_p^{(r_p-1)} \end{bmatrix}^T$, with $z_i^{(r_i-1)}$ – the (r_i-1) order derivative of z_i; for the aircraft dynamics in lateral-directional plane, $z_1 = Y, z_2 = z_p = \beta$. The states which are not included in the vector ξ can be found in the second state vector (η); considering that *n* is the dimension of the square matrix *T*, the dimension of vector η is $n - \tilde{r} = n - \sum_{i=1}^{p} r_i$; the values of r_i are deduced below.

To obtain the values of the relative degrees r_1 and $r_2 = r_p$, we derivate with respect to time the equations associated to z_1 and z_2 until the components of the command vector *u*, i.e. δ_{a_c} and δ_{r_c}, are obtained; the following equations have resulted:

$$\begin{aligned} \dddot{Y} &= a_{y\beta}\beta + a_{yp}p + a_{yr}r + a_{y\varphi}\varphi + b_{ya}\delta_a + b_{yr}\delta_r - \frac{V_0 b_{11}}{T_a}\delta_{a_c} - \frac{V_0}{T_r}b_{12}\delta_{r_c} + \left(-\bar{a}_{11} + a_{31}\right)V_{vy} - a_{11}\dot{V}_{vy}, \\ \ddot{\beta} &= \bar{a}_{11}\beta + \bar{a}_{12}p + \bar{a}_{13}r + \bar{a}_{14}\varphi + \bar{b}_{11}\delta_a + \bar{b}_{12}\delta_r + \frac{1}{T_a}b_{11}\delta_{a_c} + \frac{1}{T_r}b_{12}\delta_{r_c} + \frac{\bar{a}_{11}}{V_0}V_{vy} + \frac{a_{11}}{V_0}\dot{V}_{vy}, \end{aligned} \tag{6}$$

where $\bar{a}_{11} = a_{11}^2 + a_{12}a_{21} + a_{13}a_{31}, \bar{a}_{12} = a_{11}a_{12} + a_{12}a_{22} + a_{13}a_{32} + a_{14}, \bar{a}_{13} = a_{11}a_{13} + a_{12}a_{23} + a_{13}a_{33}, \bar{a}_{14} = a_{11}a_{14},$

$\overline{b}_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} - \frac{1}{T_a}b_{11}$, $\overline{b}_{12} = a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} - \frac{1}{T_r}b_{12}$, $a_{y\beta} = V_0\left(a_{31} - \overline{a}_{11}\right)$, $a_{yp} = V_0\left(a_{32} - \overline{a}_{12}\right)$, $a_{yr} = V_0\left(a_{33} - \overline{a}_{13}\right)$, $a_{y\varphi} = -V_0\overline{a}_{14}$, $b_{ya} = V_0\,(b_{31} - \overline{b}_{11})$, $b_{yr} = V_0\,(b_{32} - \overline{b}_{12})$. Thus, according to equations (6), the relative degrees are $r_1 = 3$ and $r_2 = 2$. Hence, one gets: $\xi = \left[Y\ \dot{Y}\ \ddot{Y}\ \beta\ \dot{\beta}\right]^T$, $\eta = \left[\varphi\ p\ r\right]^T$; the vector η, having the dimension $n - \tilde{r} = 3$, contains the main states that must be controlled during landing, other than the ones included in the vector ξ. The matrix T is determined from the equations (5); we have:

$$\begin{bmatrix} Y \\ \dot{Y} \\ \ddot{Y} \\ \beta \\ \dot{\beta} \\ \varphi \\ p \\ r \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -V_0 & 0 & 0 & 0 & V_0 & 0 & 0 & 0 \\ -V_0a_{11} & -V_0a_{12} & V_0\left(1 - a_{13}\right) & -V_0a_{14} & 0 & 0 & -V_0b_{11} & -V_0b_{12} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{11} & a_{12} & a_{13} & a_{14} & 0 & 0 & b_{11} & b_{12} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{T} \begin{bmatrix} \beta \\ p \\ r \\ \varphi \\ \psi \\ Y \\ \delta_a \\ \delta_r \end{bmatrix}. \tag{7}$$

Considering $w = 0$, by means of equation (5), one transforms the system (2) into the following one: $\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = \hat{A}\begin{bmatrix} \xi \\ \eta \end{bmatrix} + \hat{B}u$; $\hat{A} = TAT^{-1}, \hat{B} = TB$. The next step is to partitionate the matrices $\hat{A}$ and $\hat{B}$ with respect to the dimensions of the vectors ξ and η: $\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix}\begin{bmatrix} \xi \\ \eta \end{bmatrix} + \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \end{bmatrix}u$, where $\hat{A}_{11} \in R^{5\times5}, \hat{A}_{12} \in R^{5\times3}, \hat{A}_{21} \in R^{3\times5}$, $\hat{A}_{22} \in R^{3\times3}$, $\hat{B}_1 \in R^{5\times2}, \hat{B}_2 \in R^{3\times2}$. The previous equation is equivalent with the next ones:

$$\dot{\xi} = \hat{A}_{11}\xi + \hat{A}_{12}\eta + \hat{B}_1u,\ \dot{\eta} = \hat{A}_{21}\xi + \hat{A}_{22}\eta + \hat{B}_2u; \tag{8}$$

imposing $\xi = \overline{\xi}, \dot{\xi} = \dot{\overline{\xi}}$, with $\overline{\xi} = \left[\overline{Y}\ \overline{\dot{Y}}\ \overline{\ddot{Y}}\ \overline{\beta}\ \overline{\dot{\beta}}\right]^T$, $\dot{\overline{\xi}} = \left[\overline{\dot{Y}}\ \overline{\ddot{Y}}\ \overline{\dddot{Y}}\ \overline{\dot{\beta}}\ \overline{\ddot{\beta}}\right]^T$, it results the command vector $\overline{u}$ as follows: $\overline{u} = \hat{B}_1^+\left(\dot{\overline{\xi}} - \hat{A}_{11}\overline{\xi} - \hat{A}_{12}\eta\right)$, $\hat{B}_1^+$ being here the pseudo-inverse of the matrix $\hat{B}_1$.

Replacing the vector $\overline{\xi}$ in the equation of $\overline{u}$, one obtains:

$$\overline{u} = \hat{B}_1^+\left\{\left[\overline{\dot{Y}}\ \overline{\ddot{Y}}\ \overline{\dddot{Y}}\ \overline{\dot{\beta}}\ \overline{\ddot{\beta}}\right]^T - \hat{A}_{11}\overline{\xi} - \hat{A}_{12}\eta\right\} = \hat{B}_1^+\left\{\left[0\ 0\ \overline{\dddot{Y}}\ 0\ \overline{\ddot{\beta}}\right]^T - \hat{A}'_{11}\,\overline{\xi} - \hat{A}_{12}\eta\right\};\ \hat{A}'_{11} = \begin{bmatrix} \hat{a}_{11} & \hat{a}'_{12} & \hat{a}_{13} & \hat{a}_{14} & \hat{a}_{15} \\ \hat{a}_{21} & \hat{a}_{22} & \hat{a}'_{23} & \hat{a}_{24} & \hat{a}_{25} \\ \hat{a}_{31} & \hat{a}_{32} & \hat{a}_{33} & \hat{a}_{34} & \hat{a}_{35} \\ \hat{a}_{41} & \hat{a}_{42} & \hat{a}_{43} & \hat{a}_{44} & \hat{a}'_{45} \\ \hat{a}_{51} & \hat{a}_{52} & \hat{a}_{53} & \hat{a}_{54} & \hat{a}_{55} \end{bmatrix}, \tag{9}$$

where $\hat{a}_{ij}, i, j = \overline{1,5}$, are the elements of the matrix $\hat{A}_{11}$, and $\hat{a}'_{12} = \hat{a}_{12} - 1, \hat{a}'_{23} = \hat{a}_{23} - 1, \hat{a}'_{45} = \hat{a}_{45} - 1$. Replacing (9) in the second equation (8), with $\xi = \overline{\xi}$ and $u = \overline{u}$, one successively gets:

$$\dot{\eta} = \hat{A}_{21}\overline{\xi} + \hat{A}_{22}\eta +$$
$$+ \hat{B}_2\hat{B}_1^+\left\{\left[0\ 0\ \overline{\dddot{Y}}\ 0\ \overline{\ddot{\beta}}\right]^T - \hat{A}'_{11}\,\overline{\xi} - \hat{A}_{12}\eta\right\} = \left(\hat{A}_{22} - \hat{B}_2\hat{B}_1^+\hat{A}_{12}\right)\eta + \left(\hat{A}_{21} - \hat{B}_2\hat{B}_1^+\hat{A}'_{11}\right)\overline{\xi} + \hat{B}_2\hat{B}_1^+\left[0\ 0\ \overline{\dddot{Y}}\ 0\ \overline{\ddot{\beta}}\right]^T,$$

equation that can be put under one of the two equivalent forms:

$$\dot{\eta} = \hat{A}_\eta\,\eta + \hat{B}_z\overline{z}^{(r)} + \hat{A}_\xi\overline{\xi} \Leftrightarrow \dot{\eta} = \hat{A}_\eta\,\eta + \hat{B}_y\overline{Z}, \tag{10}$$

with $\hat{A}_\eta = \hat{A}_{22} - \hat{B}_2\hat{B}_1^+\hat{A}_{12}$, $\hat{A}_\xi = \hat{A}_{21} - \hat{B}_2\hat{B}_1^+\hat{A}'_{11}$, $\hat{B}_z\bar{z}^{(r)} = \hat{B}_2\hat{B}_1^+\left[0\ 0\ \dddot{\bar{Y}}\ 0\ \ddot{\bar{\beta}}\right]^T$, $\bar{z}^{(r)} = \left[\bar{z}_1^{(r_1)}\ \bar{z}_2^{(r_2)}\right]^T = \left[\dddot{\bar{Y}}\ \ddot{\bar{\beta}}\right]^T$, $\hat{B}_y = \left[\hat{B}_z\ \hat{A}_\xi\right]$, $\bar{Z} = \left[\bar{z}^{(r)}\ \xi\right]^T = \left[\dddot{\bar{Y}}\ \ddot{\bar{\beta}}\ \bar{Y}\ \dot{\bar{Y}}\ \ddot{\bar{Y}}\ \bar{\beta}\ \dot{\bar{\beta}}\right]^T$; the previous matrices' dimensions are: $\hat{A}_\eta \in R^{3\times3}$, $\hat{A}_\xi \in R^{3\times5}$, $\hat{B}_z \in R^{3\times2}$, $\hat{B}_y \in R^{3\times7}$. If $\hat{B}_2\hat{B}_1^+ = \begin{bmatrix}\hat{b}_{11} & \hat{b}_{12} & \hat{b}_{13} & \hat{b}_{14} & \hat{b}_{15}\\ \hat{b}_{21} & \hat{b}_{22} & \hat{b}_{23} & \hat{b}_{24} & \hat{b}_{25}\\ \hat{b}_{31} & \hat{b}_{32} & \hat{b}_{33} & \hat{b}_{34} & \hat{b}_{35}\end{bmatrix}$, then $\hat{B}_z = \begin{bmatrix}\hat{b}_{13} & \hat{b}_{15}\\ \hat{b}_{23} & \hat{b}_{25}\\ \hat{b}_{33} & \hat{b}_{35}\end{bmatrix}$. Thus, to obtain the command vector $\bar{u}$, one determines the vector η by solving the second equation (10) and then uses equation (9). From the expression of $\hat{B}_z\bar{z}^{(r)}$, it results: $\hat{B}_1^+\left[0\ 0\ \dddot{\bar{Y}}\ 0\ \ddot{\bar{\beta}}\right]^T = \hat{B}_2^+\hat{B}_z\bar{z}^{(r)}$, which, replaced in equation (9) having the form: $\bar{u} = \hat{B}_1^+\left[0\ 0\ \dddot{\bar{Y}}\ 0\ \ddot{\bar{\beta}}\right]^T - \hat{B}_1^+\hat{A}'_{11}\,\bar{\xi} - \hat{B}_1^+\hat{A}_{12}\eta$, leads to the following expression:

$$\bar{u} = \hat{B}_u^{-1}\left(\bar{z}^{(r)} - \hat{B}_\xi\bar{\xi} - \hat{B}_\eta\eta\right), \tag{11}$$

where $\hat{B}_u^{-1} = \hat{B}_2^+\hat{B}_z$, $\hat{B}_\xi = \hat{B}_u\hat{B}_1^+\hat{A}'_{11}$, $\hat{B}_\eta = \hat{B}_u\hat{B}_1^+\hat{A}_{12}$; $\hat{B}_u \in R^{2\times2}$, $\hat{B}_\xi \in R^{2\times5}$, $\hat{B}_\eta \in R^{2\times3}$. Taking into account all the issues presented above, one concludes that $\bar{u}$ can be obtained by using either equation (9) or equation (11).

If one imposes two convergence equations: $\hat{x} \to x$ ($\hat{x}$ is the estimated state) and $z^{(r)} = \left[\dddot{Y}\ \ddot{\beta}\right]^T \to \to \bar{z}^{(r)} = \left[\dddot{\bar{Y}}\ \ddot{\bar{\beta}}\right]^T$, another form of the control law $\bar{u}$ is obtained:

$$\bar{u} = \left[\delta_{a_c}\ \ \delta_{r_c}\right]^T = \hat{B}_u^{-1}\left(\bar{z}^{(r)} - A_x\hat{x} - B_v\left[V_{vy}\ \ \dot{V}_{vy}\right]^T\right), \tag{12}$$

with $\hat{B}_u^{-1} = \begin{bmatrix}-\dfrac{V_0 b_{11}}{T_a} & -\dfrac{V_0 b_{12}}{T_r}\\ \dfrac{b_{11}}{T_a} & \dfrac{b_{12}}{T_r}\end{bmatrix}^{-1}$, $A_x = \begin{bmatrix}a_{y\beta} & a_{yp} & a_{yr} & a_{y\varphi} & 0 & 0 & b_{ya} & b_{yr}\\ \bar{a}_{11} & \bar{a}_{12} & \bar{a}_{13} & \bar{a}_{14} & 0 & 0 & \bar{b}_{11} & \bar{b}_{12}\end{bmatrix}$, $B_v = \begin{bmatrix}\left(\bar{a}_{31} - \bar{a}_{11}\right) & -a_{11}\\ \dfrac{\bar{a}_{11}}{V_0} & \dfrac{a_{11}}{V_0}\end{bmatrix}$.

C. Design of the second component of the control law *u*. To obtain the expression of the optimal component (u_∞), the H-inf method is used; thus, putting together the state equation (2), the equations associated to $z_1 = Y$ and $z_2 = \beta$, as well as the equation of the output vector *y*, one obtains:

$$\begin{bmatrix}\dot{x}\\ z_1\\ z_2\\ y\end{bmatrix} = \begin{bmatrix}A_{(8\times8)} & B_{(8\times2)} & G_{(8\times1)} & 0_{(8\times7)}\\ C_{0\,(1\times8)} & D_{01\,(1\times2)} & 0_{(1\times1)} & 0_{(1\times7)}\\ C_{1\,(1\times8)} & D_{11\,(1\times2)} & 0_{(1\times1)} & 0_{(1\times7)}\\ C_{(7\times8)} & 0_{(7\times2)} & 0_{(7\times1)} & D_{22\,(7\times7)}\end{bmatrix}\begin{bmatrix}x\\ u\\ w\\ e\end{bmatrix}; \tag{13}$$

the matrices A, B, G have the forms (3), while C_0, C_1, D_{01}, D_{02} are, respectively: $C_0 = [0\ 0\ 0\ 0\ 0\ 1\ 0\ 0]$, $C_1 = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$, $D_{01} = [c_1\ \ 0]$, $D_{11} = [0\ \ c_2]$; the matrix *C* has the form (4), while $D_{22} = I_7$ for the vector containing the sensor errors $e = \left[e_Y\ e_{\dot{Y}}\ e_\beta\ e_\varphi\ e_p\ e_\psi\ e_r\right]^T$. It is known that the sensors have sometimes errors; for example, the most important errors of a gyro sensor are: 1) the bias; 2) the scale factor; 3) the calibration error of the scale factor; 4) the noise of the sensor. The bias and the noise are the most severe for the control performance during landing. Usually, on aircraft there are used gyro sensors to measure the angular rates (in our case *p* and *r*); by integration of the obtained values, the roll and yaw angles result. Moreover, on aircraft there are transducers for the attack angle and for the sideslip angle (β). Therefore, in this paper, we considered sensor errors for β, *p*, and *r*. In the ALS's software implementation, we will use the general model of the gyro sensors from [3], but will increase considerably the values of the sensors' errors to study the robustness of the ALS.

To proof that, in steady regime, the forms of $z_1 = Y$ and $z_2 = \beta$ are the same with the ones in equation (13), one has used the expansion of $z^T = \left[z_1^T\ z_2^T\right]^T$ as function of state (*x*) and of the system's command (*u*);

for u_0=0, there have been successively obtained: $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = z(x,u) \cong \underbrace{z(x_0, u_0)}_{z_0} + \left(\frac{\partial z}{\partial x}\right)_{(x_0,0)} \Delta x + \left(\frac{\partial z}{\partial u}\right)_{(x_0,0)} \Delta u \cong$

$$\cong z_0 + \underbrace{\begin{bmatrix} \frac{\partial z_1}{\partial x_1} & \cdots & \frac{\partial z_1}{\partial x_n} \\ \frac{\partial z_2}{\partial x_1} & \cdots & \frac{\partial z_2}{\partial x_n} \end{bmatrix}_{(x_0,0)}}_{\begin{bmatrix} C_0 \\ C_1 \end{bmatrix}} \Delta x + \underbrace{\begin{bmatrix} \frac{\partial z_1}{\partial u_1} & \frac{\partial z_1}{\partial u_2} \\ \frac{\partial z_2}{\partial u_1} & \frac{\partial z_2}{\partial u_2} \end{bmatrix}_{(x_0,0)}}_{\begin{bmatrix} D_{01} \\ D_{11} \end{bmatrix}} \Delta u \Leftrightarrow \Delta z \cong \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} \Delta x + \begin{bmatrix} D_{01} \\ D_{11} \end{bmatrix} \Delta u \Leftrightarrow z \cong \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} x + \begin{bmatrix} D_{01} \\ D_{11} \end{bmatrix} u \cong \overline{C}\, x +$$

$+ \overline{D} u$, where $C_0 = \begin{bmatrix} \frac{\partial z_1}{\partial x_1} & \cdots & \frac{\partial z_1}{\partial x_n} \end{bmatrix}_{(x_0,0)}$, $C_1 = \begin{bmatrix} \frac{\partial z_2}{\partial x_1} & \cdots & \frac{\partial z_2}{\partial x_n} \end{bmatrix}_{(x_0,0)}$, $x_i \left(i = \overline{1,n}\right)$ are the system's states (n=8), $D_{01} = \begin{bmatrix} \frac{\partial Y}{\partial \delta_{a_c}} & \frac{\partial Y}{\partial \delta_{r_c}} \end{bmatrix}_{(x_0,0)} = [c_1 \;\; 0]$, $D_{11} = \begin{bmatrix} \frac{\partial z_2}{\partial u_1} & \frac{\partial z_2}{\partial u_2} \end{bmatrix}_{(x_0,0)} = \begin{bmatrix} \frac{\partial \beta}{\partial \delta_{a_c}} & \frac{\partial \beta}{\partial \delta_{r_c}} \end{bmatrix}_{(x_0,0)} = [0 \;\; c_2]$, $\overline{C} = \begin{bmatrix} C_0 \\ C_1 \end{bmatrix}$, $\overline{D} = \begin{bmatrix} D_{01} \\ D_{11} \end{bmatrix}$. Here, c_1 and c_2 have small positive values; in steady regime (u = 0), one obtains $z_1 = Y$ and $z_2 = \beta$.

The optimal control law has the form [5]: $u_\infty = -K_\infty(\hat{x} - \overline{x})$, $K_\infty = R_1^{-1} B^T P_\infty$, $R_1 = \overline{D}^T \overline{D}$; u_∞ minimizes the cost functional: $J = \frac{1}{2}\int_0^\infty z^T z \mathrm{d}t = \frac{1}{2}\int_0^\infty \left[x^T \underbrace{\left(\overline{C}^T \overline{C}\right)}_{Q_1} x + u_\infty^T \underbrace{\left(\overline{D}^T \overline{D}\right)}_{R_1} u_\infty \right] \mathrm{d}t$. $P_\infty \in R^{8\times 8}$, a symmetric and positive defined matrix, is the solution of the Riccati matriceal equation [4]:

$$A^T P_\infty + P_\infty A - P_\infty \left(B R_1^{-1} B^T - \mu_1^{-2} G G^T\right) P_\infty + Q_1 = 0. \tag{14}$$

Here, R_1 must be a positive matrix, while μ_1 is a small enough positive scalar for which the Riccati equation (14) has a stabilizing solution. To obtain the estimated state $\hat{x}$ and $\Delta\hat{x} = \hat{x} - \overline{x}$, one borrowed the observer presented in [5], i.e.: $\Delta\dot{\hat{x}} = A\Delta\hat{x} + Bu + L_\infty(\Delta y - C\Delta\hat{x})$. The observer gain matrix $L_\infty \in R^{8\times 7}$ is obtained by using the formula [11]: $L_\infty = P_\infty^* C^T \left(D_{22}^T D_{22}\right)^{-1}$, with P_∞^* – the stabilizing solution of the Riccati matriceal equation: $A P_\infty^* + P_\infty^* A^T - P_\infty^* \left(C^T C - \mu_2^{-2} \overline{C}^T \overline{C}\right) P_\infty^* + G G^T = 0$; μ_2 has the same significance with the one associated to μ_1.

D. The structure of the new automatic landing system. The structure of the new automatic system for the aircraft guidance during landing approach phase, using the H-inf control technique, the dynamic inversion method, two reference models, and an optimal observer is presented in Fig. 1. The subsystem for the calculation of the command $\overline{u}$ and of the vector $\overline{y}$ is represented in Fig. 1 with dashed line if the control law (12) is used.

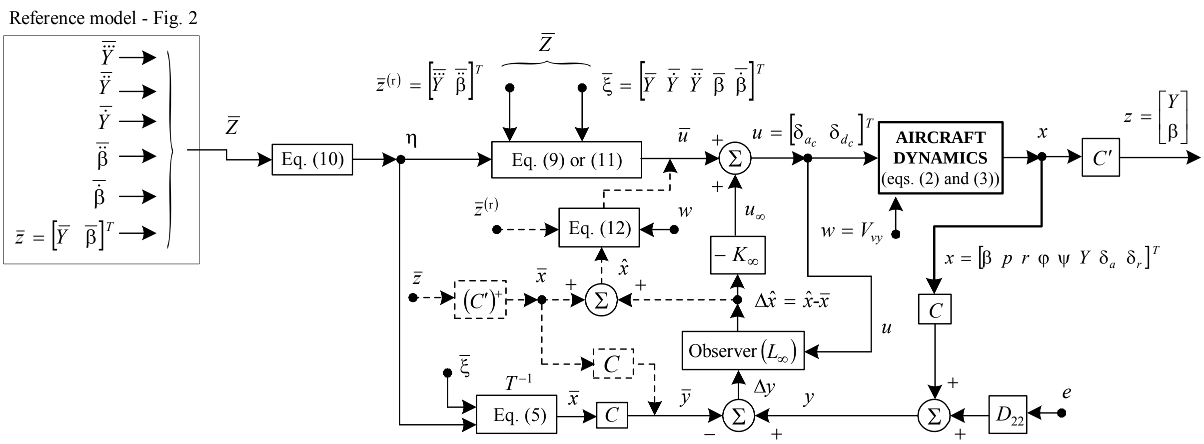

Fig. 1 – ALS for the aircraft control in lateral plane using the H-inf control and the dynamic inversion.

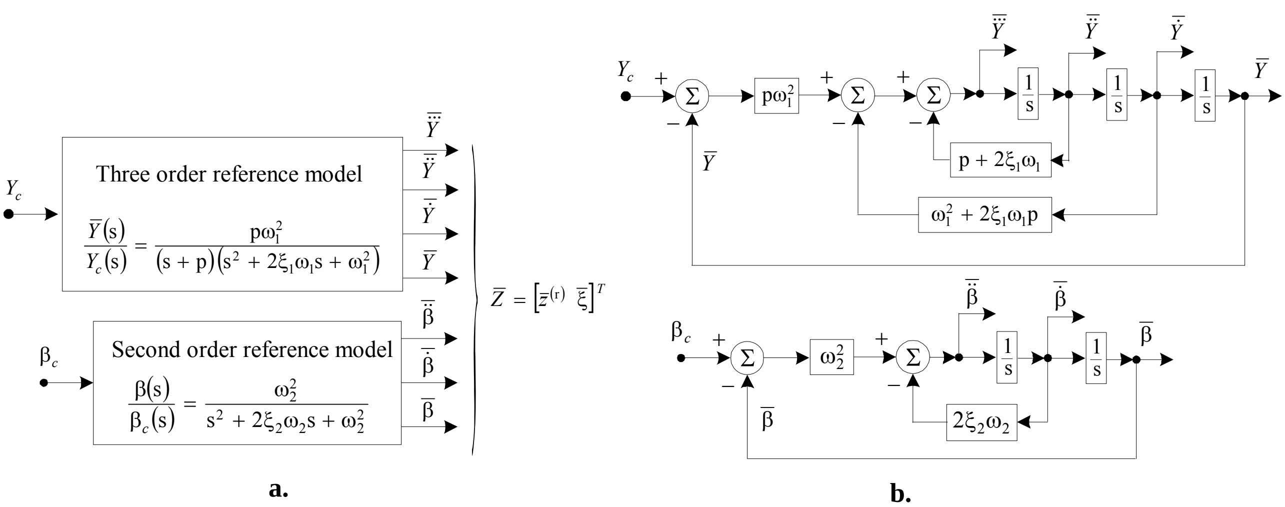

Fig. 2 – Block diagrams of the three order and second order reference models: a) simplified block diagrams; b) detailed block diagrams.

The automatic control of the aircraft in lateral-directional plane, during landing approach phase, is mainly based on the dynamic inversion and H-inf method. The vector $\overline{Z}$ may be calculated by means of two reference models (Fig. 2), the former being a three order reference model (associated to *Y*), while the latter, a second order reference model, is associated to the angle β [5].

The desired landing trajectory of aircraft involves two variables' control: the sideslip angle (β) and the aircraft lateral deviation with respect to the runway (*Y*). The optimal control system associated to aircraft flight during landing (lateral-directional plane), based on dynamic inversion and H-inf method, assures the convergences: $\Delta y \to 0\left(y \to \overline{y}, x \to \overline{x}\right), \Delta z \to 0, \left(z = C'x \to \overline{z} = C'\overline{x}\right), \Delta\hat{x} \to 0\left(\hat{x} \to x \to \overline{x}\right), u_\infty \to 0, \overline{u} \to 0, u \to 0$.

4. NUMERICAL SIMULATION RESULTS

To study the performances of the new obtained automatic landing system (lateral-directional plane), we consider the landing approach phase of a Charlie airplane. Complex simulations in Matlab/Simulink environment have been performed; thus, one designed the optimal observer, the H-inf controller, and, after that, validated the proposed automatic landing system in lateral-directional plane using real flight data.

The coefficients' values for the dynamics of a Charlie light airplane landing have been borrowed from [5]: $a_{11} = -0.0013, a_{12} = 0, a_{13} = -1, a_{14} = 0.15, a_{21} = -1.33, a_{22} = -0.98, a_{23} = 0.33, a_{31} = 0.17, a_{32} = -0.17, a_{33} = -0.217$, $b_{11} = 0, b_{12} = 0.015, b_{12} = 0.015, b_{21} = 0.23$, $b_{22} = 0.06, b_{31} = 0.026, b_{32} = -0.15, V_0 = 67\,\text{m/s}, T_a = 0.1\,\text{s}, T_r = 0.1\,\text{s}$, $\mu_1 = 10, \mu_2 = 10, c_1 = 1.05, c_2 = 0.72, \overline{Y} = 0\,\text{m}, \overline{\beta} = 0\,\text{deg}, V_{vy} = 10\,\text{m/s}, \eta(0) = 0_{3\times 1}$; the vector containing the sensor errors is $e = \left[0\text{ m} \;\; 0\text{ m/s} \;\; 1\text{ deg} \;\; 0\text{ deg} \;\; 1\text{ deg/ s} \;\; 0\text{ deg} \;\; 1\text{ deg/ s}\right]^T$, matrix *G* has been obtained with equation (3), while the initial state is $x(0) = \left[0.1\text{ deg} \;\; 0\text{ deg/ s} \;\; -1\text{ deg/ s} \;\; 0\text{ deg} \;\; 2\text{ deg} \;\; 15\text{ m} \;\; 0\text{ deg} \;\; 0\text{ deg}\right]^T$; for the reference models, we have chosen: $\text{p} = 25, \xi_1 = \xi_2 = 0.7, \omega_1 = \omega_2 = 2\,\text{rad/s}$. The sensor errors' values have been chosen very large because it is important to use strong disturbances instead of small ones when designing a robust ALS.

In Fig. 3 we represent the time characteristics for the flight direction control system (Fig. 1); before the start of the two landing main stages in longitudinal plane, the pilot must cancel the aircraft lateral deviation with respect to the runway. The characteristics have been represented for the ALS affected by crosswind in the presence or in the absence of sensor errors (the sensors are used for the measurement of the states). The presence of the sensor errors is not visible – the curves with solid line (obtained for the ALS without sensor errors) overlap almost perfectly over the curves plotted with dashed line (obtained for the ALS with sensor errors).

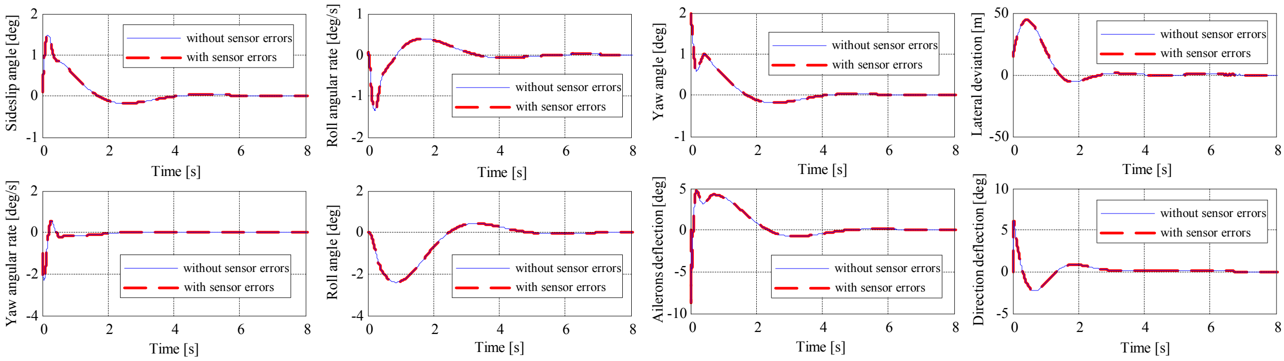

Fig. 3 – Time characteristics of the lateral-directional control system, with or without sensor errors.

The landing approach begins at 67 m/s nominal speed; this should be maintained constant. To test the robustness of the designed ALS, in simulations, we have taken into consideration the crosswind, because low-altitude crosswind can be a serious threat to the safety of aircraft in landing. From the 4th mini-graphic in Fig. 3, we can see that the stationary value of the aircraft lateral deviation is very close to zero; this error is very good if we analyze the Federal Aviation Administration (FAA) accuracy requirements for Category III (best category) [17]; according to FAA Category III accuracy requirements, the lateral error (lateral deviation with respect to the runway center line) must be less than 4.1 m. If the lateral error is between 4.1 m and 4.6 m, the aircraft meets the Category II Precision standards, while, if the lateral error is between 4.6 m and 9.1 m, it meets Category I Precision standards. The reason that our design meets the best requirement for lateral-directional motion during landing and achieves the design goal is that we used the H-inf robust control technique; this method can handle the plant with measurement noise (sensor errors) and crosswind. If the crosswind is stronger than its maximum accepted value, the pilot must avoid having the aircraft enter into this wind shear. From the sideslip angle point of view, the errors are less than 0.1 deg.; we conclude that $\beta \to \beta_c = 0$ deg.

5. CONCLUSIONS

The purpose of this study was to design a new ALS using the H-inf control and the dynamic inversion techniques. The H-inf method can handle both robustness and exact tracking problem, being very suited for the ALSs' design. The new obtained system represents an extension of the ALS designed in [4] where only the landing in longitudinal plane is analyzed; here, the aircraft motion in lateral-directional plane is studied. In the specialty literature, the system presented here is the only control system based on the dynamic inversion, H-inf control, which takes into consideration the crosswind, the sensor errors and uses an optimal observer, two reference models, and two methods for the obtaining of the optimal control law. The two calculation methods for the optimal control law give the ALS a greater degree of generality, applicability, and simplicity. The simulation results are promising and show the robustness of the new landing architecture with respect to the crosswind; moreover, the very good errors meet the FAA accuracy requirements for Category III. On the other hand, the designed control law has the ability to reject the measurement noise produced by the sensors and the crosswind. Compared to other existing approaches in the field of ALSs, our method achieved higher tracking precision. The use of the dynamic inversion makes our control system more general and, therefore, it can be used both for the case when aircraft dynamics is nonlinear and for the case when aircraft dynamics is linear; thus, the dynamic inversion's usage increases the generality character of our new automatic landing system.

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