The Estimation of an Aircraft Motions by using the Bass-Gura Full-Order Observer

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Abstract— **This paper deals with the study of a full-order observer for the state estimation of the linear systems. The observer is based on the Bass-Gura formula for the determination of the observer gain matrix. The obtained algorithm will be structured into 4 steps; the Bass-Gura algorithm replaces the well known pole placement method. The only disadvantage of the designed full-order observer is related to the choosing of the observer eigenvalues, the Bass-Gura formula representing a procedure which is similar to the pole placement technique. In this paper, the full-order observer is designed for the longitudinal and lateral motions of a light aircraft. The obtained observer is a simple one, its derivation being direct and easy. It will be shown that the only constraint (existence condition) of the Bass-Gura algorithm is that the original system (observable system) must have only one output; the main advantage of the Bass-Gura observer is its simplicity and ease of software implementation. The effectiveness of the suggested design algorithm is illustrated by two Matlab/Simulink numerical simulations for the longitudinal and lateral motions of a small aircraft. For both examples, it will be shown that the Bass-Gura algorithm is convergent (the components of the estimation error vector tend to zero).**

I. INTRODUCTION

The state estimation problem is widely presented in many scientific papers. In most cases only the input (inputs) and the output (outputs) of a system are measurable and, therefore, the estimation of the variables plays an important role in the control process [1], [2]. By measuring some of the state vector components we reduce the number of the sensors, especially in the case of difficult to measured state variables (for example the case of elastic deformations [3], [4], [5]).

One of the most important applications of the observers is related to the implementation of the closed-loop control algorithms [6]. The concept of observers for dynamic processes was introduced by Luenberger in 1966 [7], [8], but the so called "Luenberger observer" appeared several years after the Kalman filter, which is an observer optimized for the noise present in the system state equations. The observers are used both for systems with known inputs and for systems with unknown inputs. The recent observers are designed for systems with unknown inputs or for systems with subsystems having unknown dynamics. For such systems the reduced order observers are used. For the state estimation in the case of stochastic systems, the most performing and used observer is still the Kalman filter [9], [10], [11], while the adaptive observers with neural networks [12], [13], [14], [15] are used for nonlinear systems with unknown inputs or systems with unknown subsystems. Recent research involves the design of observers for systems with internal delay [16], [17], [18] based on the Lyapunov theory; in this case the observers' design is based on the solving of some matriceal inequalities.

There is a connection between the controller pole placement problem and the observer pole placement problem. In order to determine a gain matrix such that the system's poles have the desired locations, we use the duality principle and we transform the observer pole placement problem into a controller pole placement problem; after that, an algorithm for the controller pole placement is used to obtain the gain matrix and finally we transform back the gain matrix into the observer context [19].

In the next sections of the paper a full-order observer is designed for the longitudinal and lateral motions of a light aircraft. The observer is based on the Bass-Gura formula for the determination of the observer gain matrix. The obtained algorithm will be structured into 4 steps and its effectiveness is proved by means of Matlab/Simulink numerical simulations. The paper is organized as follows: the design of the Bass-Gura full-order observer is presented in section II; in section III we validate this observer by means of numerical simulations for the longitudinal and lateral motions of a light aircraft. Finally, some conclusions are shared in section IV.

II. THE DESIGN OF THE BASS-GURA OBSERVER

Let us consider the general class of linear systems described by [20]:

$$\begin{cases} \dot{x} = Ax + Bu, \\ y = Cx; \end{cases} \quad (1)$$

u, y are the input vector and the output vector, respectively, $x \in R^n$ – the state vector, while A, B, C are constant matrices.

Using the notation $\hat{x}$ for the estimated state vector, we design the observer described by the equation [6]:

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + Ly(t) + Hu(t). \quad (2)$$

The matrices $\hat{A}, L$, and H are calculated such that, in stationary regime, $\hat{x}(t) \to x(t)$. We consider the observer error $e(t) = x(t) - \hat{x}(t)$ and we calculate:

$$\dot{e}(t) = \hat{A}e(t) + \left(-\hat{A} + A - LC\right)x(t) + (B - H)u(t); \quad (3)$$

L is the gain matrix of the observer. Because we want $e(t) \to 0$, our aim is to bring the equation (3) to the form

$\dot{e}(t) = \hat{A}e(t)$ with a stable matrix $\hat{A}$; therefore, the following conditions must be fulfilled [6]:

$$B = H,\quad \hat{A} = A - LC. \tag{4}$$

Taking into account the equation (4), (2) gets the form:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}y(t)), \tag{5}$$

which is the general form of the Luenberger observer.

Using the notation $r(t) = y(t) - \hat{y}(t) = y(t) - C\hat{x}(t)$, equation (5) becomes:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + Lr(t), \tag{6}$$

where $r(t)$ is called the observer residual.

There are a lot of methods to determine the gain matrix of the observer (L). Because the above observer has the form of the Kalman filter, we can obtain L by solving an optimal problem. Another method is to use the pole placement method. The method Bass–Gura is an alternative to the pole placement method and we will use it to calculate the gain matrix of the observer. That's why this observer is called the Bass-Gura observer [6].

To present the Bass-Gura method, we consider $a(s)$ and $\hat{a}(s)$ – the characteristic polynomials associated to the matrices A and $\hat{A}$, respectively;

$$a(s) = \det(sI\text{-}A),\ \hat{a}(s) = \det(sI\text{-}\hat{A}). \tag{7}$$

The two polynomials may be written:

$$a(s) = s^n + \sum_{i=1}^{n} a_i s^{n-i},\ \hat{a}(s) = s^n + \sum_{i=1}^{n} \hat{a}_i s^{n-i}, \tag{8}$$

where n is the lines and columns number for the matrix A. The dependency between the two characteristic polynomials is [6]:

$$\hat{a}(s) - a(s) = a(s)C(sI_n - A)^{-1}L \tag{9}$$

and, by using the Leverreim–Souriau formula:

$$(sI_n - A)^{-1}a(s) = s^{n-1}I_n + s^{n-2}(A + a_1I_n) + \\ + s^{n-3}(A^2 + a_1A + a_2I_n) + \cdots + (A^{n-1} + a_1A^{n-2} + \cdots + a_{n-1}I_n), \tag{10}$$

equation (9) leads to [6]:

$$\begin{cases} \hat{a}_1 - a_1 = CL, \\ \hat{a}_2 - a_2 = CAL + a_1CL, \\ \hat{a}_3 - a_3 = CA^2L + a_1CAL + a_2CL, \\ \vdots \\ \hat{a}_n - a_n = CA^{n-1}L + a_1CA^{n-2}L + \cdots + a_{n-1}CL. \end{cases} \tag{11}$$

Considering $a = [a_1\ a_2\ \cdots\ a_n]^T$, $\hat{a} = [\hat{a}_1\ \hat{a}_2\ \cdots\ \hat{a}_n]^T$, equation (11) gets the form:

$$\hat{a} - a = T_aO_{A,C}L, \tag{12}$$

with $O_{A,C}$ – the observability matrix of the system (1) and T_a – a lower triangular matrix having the form:

$$T_a = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ a_1 & 1 & 0 & \cdots & 0 \\ a_2 & a_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n-1} & a_{n-2} & a_{n-3} & \cdots & 1 \end{bmatrix}. \tag{13}$$

The observer gain matrix is calculated by using (12) as follows [6]:

$$L = (T_aO_{A,C})^{-1}(\hat{a} - a); \tag{14}$$

the matrices $O_{A,C}$ and T_a has been calculated first. Because T_a is a lower triangular matrix, we may remark that $\det(T_a) = 1$ and, as a consequence, it is nonsingular. In order to apply the formula (14), $O_{A,C}$ must be nonsingular $(rank(O_{A,C}) = n)$; this means that the system (1) must be observable. $\hat{a}$ is calculated by imposing n desired eigenvalues for the observer. Considering that $A \in \mathcal{M}^{n\times n}$ and that the system (1) has s outputs $(C \in \mathcal{M}^{s\times n})$, the dimension of the matrices T_a and $O_{A,C}$ are $T_a \in \mathcal{M}^{n\times n}$ and $O_{A,C} \in \mathcal{M}^{(s-n)\times n}$, respectively. Thus, one yields $s \cdot n = n \Leftrightarrow s = 1$ (the system must have only one output). This is an existence condition of the Bass-Gura observer and a disadvantage of this design method.

The Bass-Gura algorithm for the state observer design is summarized below:

Step 1: By using the known matrix of the system (matrix A) we determine the characteristic polynomial $a(s)$ associated to this matrix;

Step 2: We choose desired eigenvalues for the observer and we obtain the characteristic polynomial $\hat{a}(s)$ associated to the matrix $\hat{A} = A - LC$;

Step 3: We calculate the matrix T_a by using (13) and the observability matrix of the system $O_{A,C}$ with the form

$$O_{A,C} = [C\ CA\ CA^2\ \cdots\ CA^{n-1}]^T; \tag{15}$$

Step 4: We calculate the gain matrix of the observer by means of equation (14) and we design the observer described by equation (5).

The only disadvantage of the designed observer is related to the choosing of the observer eigenvalues, the Bass-Gura formula representing a procedure which is similar to the pole placement technique. This technique is easy to implement in the case of observers' design, but it has some disadvantages:

1) The technique becomes difficult to be used for systems with big order or for poorly controlled systems;

2) If we choose fast poles for the observer, the advantage is that the observer estimation error decays rapidly, but the disadvantage is that the system needs perfect sensors and/or noise free environment;

3) If we choose slow poles for the observer, the advantage is that the system is less sensitive to process disturbances and measurement noise, but the disadvantage is that the observer estimation error decays slowly.

Considering that the original system is observable, the only constraint (existence condition) of the above algorithm is that the system (1) must have only one output. The main advantage of the Bass-Gura observer is its simplicity and ease of software implementation.

III. The Use of the Observer to the State Estimation of an Aircraft Motions

The validation of Bass-Gura algorithm for a full-order observer design is made, in this section, for the case of longitudinal and lateral motions of a Charlie aircraft [21]. The validation of the algorithm is performed in Matlab/ Simulink environment.

Aircraft longitudinal motion

Let us consider the state equation associated to the longitudinal motion of a Charlie aircraft having the form [21]:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\alpha} \\ \Delta\dot{\theta} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} -0.007 & 0.012 & -9.81 & 0 \\ -0.128 & -0.54 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.065 & 0.96 & 0 & -0.99 \end{bmatrix} \cdot \begin{bmatrix} \Delta u \\ \Delta\alpha \\ \Delta\theta \\ \Delta q \end{bmatrix} + \begin{bmatrix} 0 \\ -0.04 \\ 0 \\ -12.5 \end{bmatrix} \delta_e ;$$

u is the aircraft longitudinal velocity, $\alpha -$ the aircraft attack angle, $\theta -$ the aircraft pitch angle, $q -$ the aircraft pitch angular rate, δ_e is the elevator deflection, while Δ is associated with the perturbation of the variables from their nominal values. The output equation of the system is chosen as $y = Cx$ with $C = [0\ 0\ 1\ 0]$. As the input signal of the system (u) we may consider a unitary step signal, a sinusoidal signal or any random signal. In this simulation, we calculated the gain matrix $\overline{K}$ (feedback of the closed loop system) by using the ALGLX algorithm [21], and we considered, as the input vector of the system, the vector $u = \delta_e = -\overline{K}\hat{x}$.

The Matlab/Simulink model for the Bass-Gura observer is presented in Fig.1. It is used to obtain the graphic characteristics in Fig.2 (the time dependencies of the state estimation errors $e_i = x_i - \hat{x}_i,\ i = \overline{1,4}$).

The Simulink model is used in a Matlab program (the software implementation of the Bass-Gura design algorithm) which is presented in the Appendix of this paper. This soft calculates the observer gain matrix and the gain matrix of the system controller by using the ALGLX algorithm [21].

For the observer (longitudinal motion) we have chosen the eigenvalues -1, -2, -3, and -4. According to the above presented algorithm we successively obtained:

$$a(s) = s^4 + 1.537s^3 - 0.413s^2 + 0.635s - 0.861,$$
$$\hat{a}(s) = s^4 + 10s^3 + 69s^2 + 50s + 24,$$
$$T_a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1.537 & 1 & 0 & 0 \\ -0.413 & 1.537 & 1 & 0 \\ 0.635 & -0.413 & 1.537 & 1 \end{bmatrix}, \qquad (16)$$
$$O_{A,C} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0.065 & 0.96 & 0 & -0.99 \\ -0.187 & -1.468 & -0.637 & 1.94 \end{bmatrix},$$
$$L = [-276.462\ \ 41.643\ \ 8.463\ \ 56.405]^T .$$

In Fig. 3 we represent the four components of the state vector $x_i, i = \overline{1,4} -$ solid line and the four components of the estimated state vector $\hat{x}_i, i = \overline{1,4} -$ dashed line. The graphics of the state variables are superposed over the graphics of the estimated state variables.

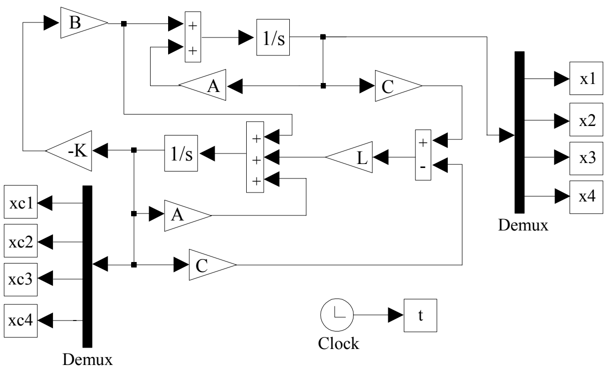

Fig. 1. Matlab/Simulink model of the ensemble system-observer

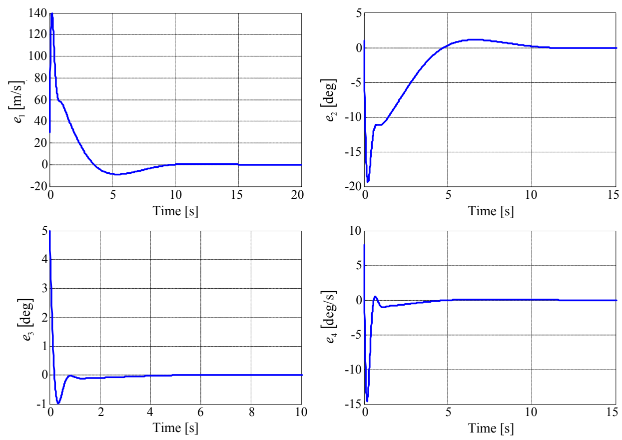

Fig. 2. State estimation errors (aircraft longitudinal motion)

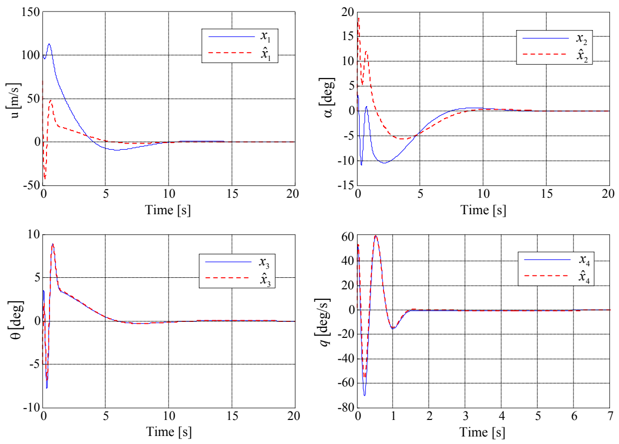

Fig. 3. State variables and the estimated state variables (aircraft longitudinal motion)

Aircraft lateral motion

Let us consider now the lateral motion of the same Charlie aircraft having the form [21]:

$$\begin{bmatrix}\Delta\dot{\beta}\\ \Delta\dot{r}\\ \Delta\dot{p}\\ \Delta\dot{\varphi}\end{bmatrix}=\begin{bmatrix}-0.05 & -0.99 & 0.08 & 0.04\\ 0.59 & -0.11 & -0.03 & 0\\ 0.30 & 0.38 & -0.46 & 0\\ 0 & 0.08 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta\beta\\ \Delta r\\ \Delta p\\ \Delta\varphi\end{bmatrix}+\begin{bmatrix}0.01 & 0\\ -0.4 & 0.12\\ 0.15 & 1.06\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_r\\ \delta_a\end{bmatrix}.$$

β is the aircraft sideslip angle, $r-$ the aircraft yaw angular rate, $p-$ the aircraft roll angular rate, $\varphi-$ the aircraft roll angle, δ_r is the rudder deflection, δ_a is the ailerons deflection, while Δ is again associated with the perturbation of the variables from their nominal values.

For the observer (lateral motion) we have chosen the same eigenvalues as the ones associated to the aircraft lateral motion and we obtained:

$$a(s)=s^4+0.635s^3+0.669s^2+0.234s-0.0127,$$
$$\hat{a}(s)=s^4+10s^3+69s^2+50s+24,$$
$$T_a=\begin{bmatrix}1 & 0 & 0 & 0\\ 0.635 & 1 & 0 & 0\\ 0.669 & 0.635 & 1 & 0\\ 0.234 & 0.669 & 0.635 & 1\end{bmatrix}, \qquad (17)$$
$$O_{A,C}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0.065 & 0.96 & 0 & -0.99\\ -0.187 & -1.468 & -0.637 & 1.94\end{bmatrix},$$
$$L=[17.73 \quad 64.215 \quad 57.207 \quad 9.364]^T.$$

The Matlab/Simulink model for the Bass-Gura observer (lateral motion) is again the one in Fig.1. The software (Matlab program) used in this case is similar with the one presented in the Appendix.

It is used to obtain the time dependencies of the state estimation errors $e_i = x_i - \hat{x}_i$, $i=\overline{1,4}$ (Fig.4). In Fig. 5 we represent the four components of the state vector $x_i, i=\overline{1,4}-$ solid line and the four components of the estimated state vector $\hat{x}_i, i=\overline{1,4}-$ dashed line. The graphics of the state variables are again superposed over the graphics of the estimated state variables.

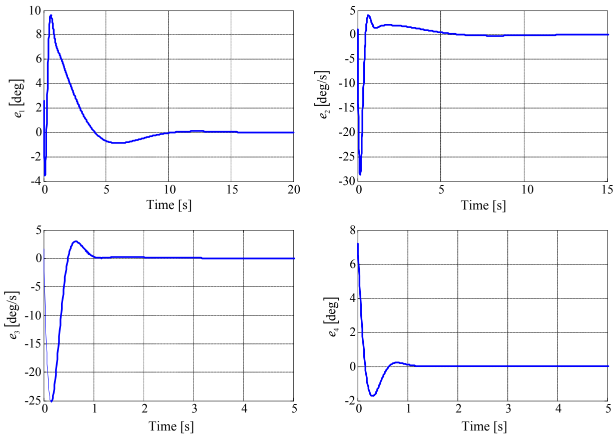

Fig. 4. State estimation errors (aircraft lateral motion)

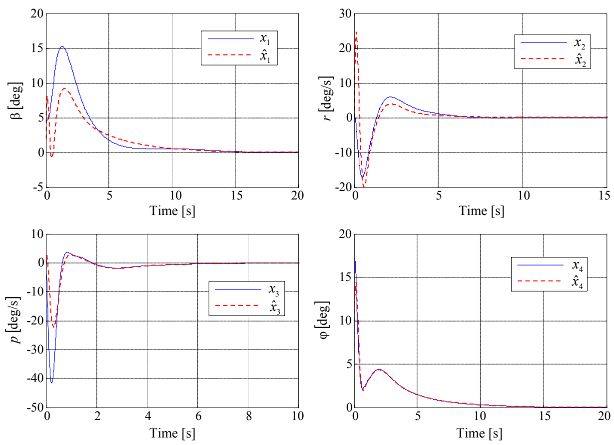

Fig. 5. State variables and the estimated state variables (lateral motion)

We can remark that for both longitudinal and lateral motions of the aircraft, the Bass-Gura algorithm is convergent (the four components of the estimation error vector tend to zero).

IV. Conclusion

This paper presents a simple full-order observer design, its derivation being direct and easy. The observer is based on the Bass-Gura formula for the determination of the observer gain matrix. The obtained algorithm is structured into 4 steps and its effectiveness is proved by means of numerical simulations for the longitudinal and lateral motions of a light aircraft. For both longitudinal and lateral motions of the aircraft, the Bass-Gura algorithm is convergent (the components of the estimation error vector tend to zero).

Appendix

```
% Design of the Bass-Gura observer for aircraft longitudinal motion
clear all;close all;
% The calculation of the controller gain matrix
A=[-0.007 0.012 -9.81 0;-0.128 -0.54 0 1;0 0 0 1;0.065 0.96 0 -0.99];
B=[0;-0.04;0;-12.5];Q=[10 0 0 0;0 10 0 0;0 0 100 0;0 0 0 1];R=[2];
```

```
[K,P,E] = LQR(A,B,Q,R);I2=[1 0;0 1];C=[0 0 1 0];
% Define the matrix T
N3=randn(4,3);contor=1;T(:,1)=B(:,1);
for i=1:4
   for j=1:3
      T(i,j+1)=N3(i,j);
   end
end
Ab=(inv(T))*A*T;
Bb=(inv(T)*B);
Kb=place(Ab,Bb,E);
e=eig(Ab-Bb*Kb);
k1=Kb(1);k21=Kb(2);k22=Kb(3);k23=Kb(4);
r1=5;Rb=[r1];R=Rb;
Pb=r1*[k1 k21 k22 k23;k21 1 0 0;k22 0 1 0;k23 0 0 1];
ee=eig(Rb);Qb=-(Pb*Ab+(transpose(Ab))*Pb-Pb*Bb*Kb);
PPP=transpose(inv(T))*Pb*inv(T);
KKK=inv(R)*transpose(B)*PPP;
G=eig(A-B*KKK);m=rank(T);
while real(G(1))>0 | real(G(2))>0 | real(G(3))>0 | real(G(4))>0 | m<4
N3=randn(4,3);contor=contor+1;T(:,1)=B(:,1);
for i=1:4
   for j=1:3
      T(i,j+1)=N3(i,j);
   end
end
Ab=(inv(T))*A*T;
Bb=(inv(T)*B);
Kb=place(Ab,Bb,E);
e=eig(Ab-Bb*Kb);k1=Kb(1);k21=Kb(2);k22=Kb(3);k23=Kb(4);
r1=1;Rb=[r1];R=Rb;
Pb=r1*[k1 k21 k22 k23;k21 1 0 0;k22 0 1 0;k23 0 0 1];
ee=eig(Rb);
Qb=-(Pb*Ab+(transpose(Ab))*Pb-Pb*Bb*Kb);
end    % End of while
Q=transpose(inv(T))*Qb*inv(T);R=Rb;
[KK,PP,EE]= LQR(A,B,Q,R);K=KK;
% The design of the Bass-Gura observer
n=size(A,1);m=size(B,2);s=size(C,1);
x0=[100;1;0;10];        % Initial value of the state vector (x0)
xc0=[70;0;-5;2];        % Initial value of the estimated state vector (xc0)
q1=-1;q2=-2;q3=-3;q4=-4;    % Desired eigenvalues of the observer
OB=obsv(A,C);           % Calculation of the observability matrix
% Calculation of the polynomial for matrices A and A_hat (Ac)
a=(poly(A))';
ac=[1   -(q1+q2+q3+q4)   (q1*q2+q1*q3+q1*q4+q2*q3*q2*q4+q3*q4)
(-q3*q4*(q1+q2)-q1*q2*(q3+q4)) (q1*q2*q3*q4)]';
Dif=ac(2:length(ac))-a(2:length(a));      % Dif=ac-a
% Ccalculation of the observer gain matrix
Ta=[1 0 0 0;a(2) 1 0 0;a(3) a(2) 1 0;a(4) a(3) a(2) 1]
L=(inv(Ta*OB))*Dif;
Aobs=A-L*C;eig(Aobs)
% The graphics of the estimation errors (K=0)
K=zeros(m,n);sim('Sch_Bass_Gura');
subplot(221);plot(t,x1-xc1);grid;
subplot(222);plot(t,x2-xc2);grid;
subplot(223);plot(t,x3-xc3);grid;
subplot(224);plot(t,x4-xc4);grid;
% The graphics of the state variables (K=K)
K=KK;sim('Sch_Bass_Gura');h=figure;
subplot(221);plot(t,x1,'b',t,xc1,'r--');grid;
subplot(222);plot(t,x2,'b',t,xc2,'r--');grid;
subplot(223);plot(t,x3,'b',t,xc3,'r--');grid;
subplot(224);plot(t,x4,'b',t,xc4,'r--');grid;
```


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