

Estimation of Aircraft State during Landing by means of Multiple Observers

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Abstract - The paper presents three multiple observers which are useful for the state reconstruction in the case of nonlinear systems characterized by unknown inputs. In the observers' design process, the Lyapunov and linear matrix inequality theories are used. The paper innovation is related to the use of Takagi-Sugeno multiple-model and multiple observers to the estimation of an aircraft state during the landing process. The validation of the three observers' design algorithms is achieved through complex numerical simulations in Matlab/Simulink; the states and the unknown input vectors of the Takagi-Sugeno multiple-models are estimated and it is proved the proper functioning of the multiple observers as well as the very good estimation of the system's states.

Keywords - Multiple observer; Multiple-model; Aircraft landing

I. INTRODUCTION

The most important role of an observer is to replace or augment sensors in a system. The observers for plants with both known and unknown inputs are called unknown input observers [1-3]. Because of the environment noises, measurements' uncertainty, faults of sensors and actuators, a physical process can be affected by disturbances; because the process normal behavior can be affected, one must estimate these disturbances such that their effects are minimized. These disturbances (unknown inputs) may affect the inputs of the system, making difficult the estimation of the states [4].

The observers can be designed both for linear and nonlinear dynamics, but the design process in the second case is more difficult; that's why, the linearization method can be used to transform a nonlinear system into a linear one; in [4] there has been proved that any nonlinear system can be brought to the general form of multiple-models; this means a sum of linear models, each of them characterizing the system in a specific operating regime. Usually, the obtaining of a multiple-model is achieved by using the linearization around an operating point [5]; in this case, each local model is a linear, affine, and time invariant system due to the presence of some linearization constants. If the multiple-model is available, the estimation of states and unknown inputs can be achieved by means of multiple observers; in practice, there are many situations where some of the system inputs are inaccessible and, in these cases, an unknown input observer is necessary for the estimation of the system's state; the suggested technique consists in associating to each local model a local unknown input observer. The multiple observer (global observer) is the sum of the local observers weighted by the activation functions associated to the local models [6].

The motivation of this work rises owing to the fact that it is often difficult to design a model which takes into account all the complexity of the studied system; on the other hand, the landing

of aircraft is one of the most difficult stage of flight; in our paper different multiple observers are presented, software implemented, and validated for the motion of aircraft during landing. There is the first time when the estimation of an aircraft state is achieved during landing in longitudinal plane by using Takagi-Sugeno multiple-model and multiple observers.

II. THE STRUCTURE OF THE MULTIPLE-MODEL

The observers to be presented in this paper are dedicated to nonlinear systems having unknown inputs; these unknown inputs influence both the system's state and the output vector. The aim of the observer is to estimate the state and the unknown inputs of the system. This leads to complex nonlinear systems and there are many approaches to deal with such systems. A possible approach is, for example, the linearization of the system; in this case some linear models result; by interpolation of these linear models, a multiple-model general form results [7]:

$$E\dot{x}(t) = \sum_{i=1}^M \mu_i(\xi(t)) [A_i x(t) + B_i u(t) + D_i v(t)], y(t) = Cx(t) + Gv(t), \quad (1)$$

where $x(t) \in \mathcal{R}^n$ is the system state vector, $u(t) \in \mathcal{R}^m$ – the system known input vector, $v(t) \in \mathcal{R}^q$ – the system unknown input vector, $y(t) \in \mathcal{R}^p$ – the output vector, while $C \in \mathcal{R}^{p \times n}$ is the output matrix of the system. For the local model “ i ”, $A_i \in \mathcal{R}^{n \times n}$ is the matrix associated to the state, $B_i \in \mathcal{R}^{n \times m}$ – the input matrix; the matrices E, A_i, B_i, D_i, C and G are considered real and known; the matrix E may be nonsingular. The activation functions $\mu_i(\xi(t)), i = \overline{1, M}$, have the properties: $\sum_{i=1}^M \mu_i(\xi(t)) = 1$, $0 \leq \mu_i(\xi(t)) \leq 1, (\forall) i = \overline{1, M}$; the so-called decision vector $\xi(t)$ depends on the inputs and/or measurable variables. The number of local models (M) depends on the precision of modeling, the complexity of the nonlinear system to be approximated by means of a multiple-model, and the structure of the activation functions. For the ease of notations, in the approaches presented below, the time dependence (t) is omitted.

III. DESIGN OF THE MULTIPLE OBSERVERS

In this section, there are presented three multiple observers, their design being borrowed from [7]. Paper [7] presents a method for state-estimation of Takagi-Sugeno descriptor systems affected by unknown inputs. Sufficient existence conditions of the unknown input observers are given and strict linear matrix inequalities are solved to obtain the observers' gain matrices. The proposed observers will be then used in a numerical example.

A. Design of the first multiple observer

The first multiple observer has been designed in [7] and its equations are:

$$\dot{z} = \sum_{i=1}^M \mu_i(\xi) [N_i z + M_i u + L_i y], \hat{x} = z + T_2 y, \quad (2)$$

where $\hat{x} \in \mathcal{R}^n$ is the system estimated state vector. N_i, M_i, L_i and T_2 are unknown matrices of appropriate dimensions to be calculated such that \hat{x} asymptotically converges to x . Considering the observer reconstruction error $e = \hat{x} - x = T_1 E x - z$ with T_1 and T_2 – matrices satisfying the constraints: $T_1 E + T_2 C = I_n$ and $T_2 G = 0$, the observer error dynamics is [7]:

$$\dot{e} = \sum_{i=1}^M \mu_i(\xi) [N_i e + (T_1 A_i - N_i T_1 E - L_i C)x + (T_1 B_i - M_i)u + (T_1 D_i - L_i G)v].$$

For the observer asymptotically convergence, its error dynamics must have the homogeneous form $\dot{e} = \sum_{i=1}^M \mu_i(\xi) N_i e$; thus, the following convergence conditions result:

$$\begin{aligned} N_i &= T_1 A_i + K_i C, T_1 E + T_2 C = I_n, T_2 G = 0, \\ T_1 D_i + K_i G &= 0, M_i = T_1 B_i, L_i = N_i T_2 - K_i, \end{aligned} \quad (3)$$

where the notation $K_i = N_i T_2 - L_i$ has been used.

The first four equations can be used to determine a matrix $\Omega \in \mathcal{R}^{n \times (n+p(M+1))}$ such that $\Omega R = S$ and $N_i = \Omega S_i$, where

$$\begin{aligned} \Omega &= [\tilde{\Omega}_1 \quad \tilde{\Omega}_2], R = \begin{bmatrix} \tilde{R}_1 & \tilde{R}_2 \\ \tilde{R}_3 & \tilde{R}_4 \end{bmatrix}, S = [\tilde{S}_1 \quad \tilde{S}_2], S_i = \begin{bmatrix} \tilde{S}_{i,1} \\ \tilde{S}_{i,2} \end{bmatrix}, \\ \tilde{\Omega}_1 &= [T_1 \quad T_2], \tilde{\Omega}_2 = [K_1 \quad K_2 \quad \dots \quad K_M], \tilde{R}_1 = \begin{bmatrix} E & 0_{n \times q} \\ C & G \end{bmatrix}, \\ \tilde{R}_2 &= \begin{bmatrix} D_1 & \dots & D_M \\ 0_{p \times q} & \dots & 0_{p \times q} \end{bmatrix}, \tilde{R}_3 = [0_{pM \times n} \quad 0_{pM \times q}], \tilde{R}_4 = I_M \otimes G, \\ \tilde{S}_1 &= [I_n \quad 0_{n \times q}], \tilde{S}_2 = 0_{n \times Mq}, \tilde{S}_{i,1} = [A_i^T \quad 0_{n \times p}]^T, \tilde{S}_{i,2} = e_i \otimes C; \end{aligned} \quad (4)$$

$e_i \in \mathcal{R}^{M \times 1}$ are column vectors whose elements are null except the component “ i ” which is 1. The solving of the equation $\Omega R = S$ is possible if the following condition is fulfilled [7]:

$\text{rank}(R) = \text{rank} \begin{bmatrix} D_1 & \dots & D_M \\ I_M \otimes G \end{bmatrix} + n + \text{rank}(G)$. In this case, we get $\Omega = SR^+ + TR^\perp$, where “ $+$ ” is the pseudo-inverse symbol, $R^\perp = I - RR^+$, while $T \in \mathcal{R}^{n \times (n+p(M+1))}$ is the solution of the linear matrix inequality (LMI) [7]:

$$\begin{bmatrix} \Phi_i & (R^\perp S_i)^T \bar{T}^T \\ \bar{T} (R^\perp S_i) & -P \end{bmatrix} < 0, \quad (5)$$

with $\Phi_i = (R^\perp S_i)^T \bar{T}^T (SR^+ S_i) + (SR^+ S_i)^T \bar{T} (R^\perp S_i) + (SR^+ S_i)^T P (SR^+ S_i) - P$ and $\bar{T} = PT \in \mathcal{R}^{n \times (n+p(M+1))}$.

The block diagram associated to the equations (1) and (2) is

presented in Fig. 1.

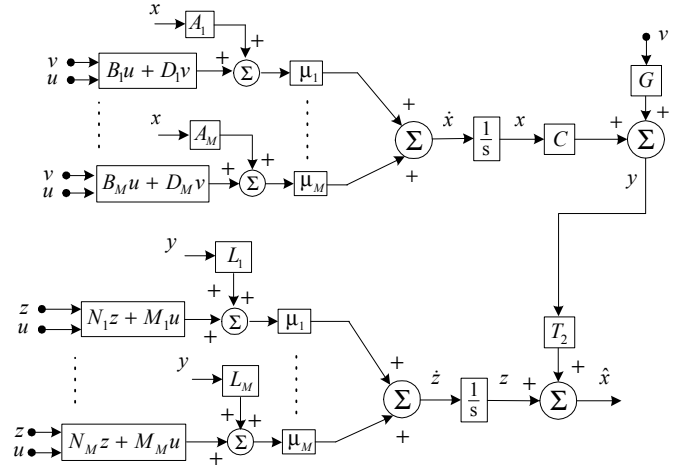


Figure 1. Block diagram of the first multiple observer

B. Design of the second multiple observer

The second multiple observer is also designed in [7] to estimate the state of a Takagi-Sugeno descriptor system as well as to minimize the negative influence of the unknown inputs on the estimating process, when the decoupling of these unknown inputs from the multiple-model is not possible; with other words, it is not possible to use the multiple observer (2) because of the presence of the unknown inputs' vector v .

In [8] it is proved that the descriptor dynamics:

$$\dot{x}(t) = \sum_{i=1}^M \mu_i(\xi) [A_i x + B_i u], y(t) = \sum_{i=1}^M \mu_i(\xi) C_i x, \quad (6)$$

is stable and verifies the inequality $\|y\|_2 < \gamma \|u\|_2$ if there exists a symmetric and positive defined matrix $P \in \mathcal{R}^{n \times n}$ verifying the LMI: $\begin{bmatrix} A_i^T P + P A_i + C_i^T C_i & P B_i \\ B_i^T P & -\gamma^2 I \end{bmatrix} < 0$; γ is a positive constant.

In this case, a multiple observer for the attenuation of the unknown inputs' influence can be designed if the estimation error e and the unknown input vector v verify the condition [7]: $\|e\|_2 < \gamma \|v\|_2$. In this situation, the design of the multiple observer is concentrated into the following theorem:

Theorem 1 [7]:

A multiple observer having the form (2) to attenuate the unknown inputs' influence can be designed for the Takagi-Sugeno system (1) if the condition:

$$\text{rank} \begin{bmatrix} E & 0 \\ C & G \end{bmatrix} = n + \text{rank}(G) \quad (7)$$

is fulfilled and, moreover, if there exist a symmetric and positive defined matrix $P \in \mathcal{R}^{n \times n}$ and the matrices $\bar{T} \in \mathcal{R}^{n \times (n+p)}$ and $\bar{K}_i \in \mathcal{R}^{n \times p}$ which verify the linear matrix inequality:

$$\begin{bmatrix} \Psi_{i,1} & \Psi_{i,2} \\ \Psi_{i,2}^T & -\gamma^2 I_q \end{bmatrix} < 0, \quad (8)$$

where $\Psi_{i,1} = PSR_i^+ A_i + \bar{T}R_i^+ A_i + \bar{K}_i C + (PSR_i^+ A_i + \bar{T}R_i^+ A_i + \bar{K}_i C)^T + I_n$,
 $\Psi_{i,2} = PSR_i^+ D_i + \bar{T}R_i^+ D_i + \bar{K}_i G, (\forall i = \bar{1}, \bar{M}; R_i^+ \in \mathcal{R}^{(n+q) \times n}, R_2^+ \in \mathcal{R}^{(n+q) \times p}$,
 $R_1^+ \in \mathcal{R}^{(n+p) \times n}, R_2^+ \in \mathcal{R}^{(n+p) \times p}$ are defined by using the equations:

$$\begin{bmatrix} E & 0 \\ C & G \end{bmatrix}^+ = \begin{bmatrix} R_1^+ & R_2^+ \\ C & G \end{bmatrix} \begin{bmatrix} E & 0 \\ C & G \end{bmatrix}^+ - I_{n+p} = \begin{bmatrix} R_1^+ & R_2^+ \end{bmatrix}, \quad (9)$$

while $\bar{K}_i = PK_i$ and $\bar{T} = PT$. \blacksquare

In [7] it is proved that if the condition (7) is fulfilled, there can be determined the matrices T_1 and T_2 such that $\begin{bmatrix} T_1 & T_2 \end{bmatrix} R = S$,

where $R = \begin{bmatrix} E & 0 \\ C & G \end{bmatrix}, S = \begin{bmatrix} I_n & 0_{n \times q} \end{bmatrix}$; the matrices T_1 and T_2 are

obtained as follows: $T_1 = SR_1^+ + \bar{T}R_1^+, T_2 = SR_2^+ + \bar{T}R_2^+$, where $T = P^{-1}\bar{T}, K_i = P^{-1}\bar{K}_i$, while P, \bar{T} , and \bar{K}_i are the solutions of the LMI (8); to calculate N_i, M_i and L_i we use the equations (3).

C. Design of the third multiple observer

The third presented observer has been also designed in [7] and it is used to estimate the state of Takagi-Sugeno descriptor type systems when the unknown inputs have been decoupled and attenuated. If there are too many unknown inputs and their decoupling and attenuation is not possible, a compromise can be done in order to obtain less restrictive conditions. Thus, in a first design stage, the estimation of the state vector is independent (decoupled) on the unknown input vector, while, in the second stage of the observer design, the norm of the estimation error is minimized, the robustness of the observer being improved.

The vector of unknown inputs is divided into v and \bar{v} , the matrices associated to these vectors being D_i and \bar{D}_i , respectively; the equations (1) become:

$$E\dot{x} = \sum_{i=1}^M \mu_i(\xi) [A_i x + B_i u + D_i v + \bar{D}_i \bar{v}], y = Cx + Gv + \bar{G}\bar{v}, \quad (10)$$

with $v \in \mathcal{R}^{q \times 1}$ and $\bar{v} \in \mathcal{R}^{\bar{q} \times 1}$. The necessary conditions which must be satisfied in order to design the new observer are presented in the following theorem:

Theorem 2 [7]:

A multiple observer of form (2) assuring the decoupling of the vector v and maximum robustness with respect to \bar{v} exists if $\text{rank}(\bar{R}) = n + \text{rank}[G \ \bar{G}] + \text{rank}\begin{bmatrix} D_1 & \dots & D_M \\ I_M \otimes G \end{bmatrix}$ and if there exists a symmetric and positive defined matrix $P \in \mathcal{R}^{n \times n}$ and the matrix $\bar{T} \in \mathcal{R}^{n \times (n+(M+1)p)}$ – solutions of the LMI:

$$\begin{bmatrix} \bar{\Psi}_{i,1} & \bar{\Psi}_{i,2} \\ \bar{\Psi}_{i,2}^T & -\gamma^2 I \end{bmatrix} < 0 \quad (11)$$

and of the minimizing process of the constant γ ; the above used matrices are defined the equations:

$$\bar{\Psi}_{i,1} = P\bar{S}\bar{R}^+ S_i + \bar{T}\bar{R}^+ S_i + (\bar{S}\bar{R}^+ S_i)^T P + (\bar{R}^+ S_i)^T \bar{T}^T + I_n,$$

$$\bar{\Psi}_{i,2} = P\bar{S}\bar{R}^+ \bar{S}_i + \bar{T}\bar{R}^+ \bar{S}_i, \bar{R} = \begin{bmatrix} \bar{R}_1 & \bar{R}_2 \\ \bar{R}_3 & \bar{R}_4 \end{bmatrix}, \bar{S} = \begin{bmatrix} \bar{S}_1 & \bar{S}_2 \end{bmatrix}, \bar{S}_i = \begin{bmatrix} \bar{S}_{i,1} \\ \bar{S}_{i,2} \end{bmatrix},$$

$$S_i = \begin{bmatrix} \bar{S}_{i,1} \\ \bar{S}_{i,2} \end{bmatrix}, \bar{R}_1 = \begin{bmatrix} E & 0_{n \times q} & 0_{n \times \bar{q}} \\ C & G & \bar{G} \end{bmatrix}, \bar{R}_2 = \begin{bmatrix} D_1 & \dots & D_M \\ 0_{p \times q} & \dots & 0_{p \times \bar{q}} \end{bmatrix}, \quad (12)$$

$$\bar{R}_3 = \begin{bmatrix} 0_{pM \times n} & 0_{pM \times q} & 0_{pM \times \bar{q}} \end{bmatrix}, \bar{R}_4 = I_M \otimes G,$$

$$\bar{S}_1 = \begin{bmatrix} I_n & 0_{n \times (q+\bar{q})} \end{bmatrix}, \bar{S}_2 = 0_{n \times Mq}, \bar{S}_{i,1} = \begin{bmatrix} D_i^T & 0_{\bar{q} \times p} \end{bmatrix}^T,$$

$$\bar{S}_{i,2} = e_i \otimes \bar{G}, \bar{S}_{i,1} = \begin{bmatrix} A_i^T & 0_{n \times p} \end{bmatrix}^T, \bar{S}_{i,2} = e_i \otimes C;$$

$e_i \in \mathcal{R}^{M \times 1}$ have the same meaning as above. \blacksquare

The condition $\text{rank}(\bar{R}) = n + \text{rank}[G \ \bar{G}] + \text{rank}\begin{bmatrix} D_1 & \dots & D_M \\ I_M \otimes G \end{bmatrix}$ is

less restrictive than $\text{rank}(R) = \text{rank}\begin{bmatrix} D_1 & \dots & D_M \\ I_M \otimes G \end{bmatrix} + n + \text{rank}(G)$.

To obtain the total decoupling of all the unknown inputs in (10), the matrices D_i and G must be replaced with $\begin{bmatrix} D_i & \bar{D}_i \end{bmatrix}$ and $\begin{bmatrix} G & \bar{G} \end{bmatrix}$, respectively; a constraint easier to fulfill is obtained [7].

IV. NUMERICAL SIMULATION RESULTS

The above presented multiple observers were software implemented in Matlab/Simulink environment, for the case of an aircraft landing in vertical plane. The landing procedure involves three phases: the initial approach, the glide slope, and the flare [9]. During initial approach, the pilot descends from the cruise altitude to 420 m above the ground (for Boeing 747). At 4 nautical miles from the runway, the glide slope path signal is intercepted; the pitch, attitude and speed must be controlled while aircraft maintains a constant speed; for Boeing 747 the pitch angle is between -5 and 5 deg. At 30 m above the ground, the slope angle control system is disengaged and a flare maneuver is executed. The two main phases of landing (glide slope and flare) are presented in Fig. 2. The dynamics of aircraft during landing (borrowed from [9]) has the form (1) where the following matrices have been used:

$$A_1 = \begin{bmatrix} -0.007 & 0.012 & -9.81 & 0 \\ -0.128 & -0.54 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.065 & 0.96 & 0 & -0.99 \end{bmatrix}, A_2 = \begin{bmatrix} -0.01 & 0.01 & -9.81 & 0 \\ -0.128 & -0.6 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.1 & 1.2 & 0 & -1.5 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ -0.04 \\ 0 \\ -12.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 \\ -0.1 \\ 0 \\ -10.8 \end{bmatrix}, D_1 = \begin{bmatrix} 0.15 & -0.15 \\ 0.15 & 0.1 \\ 0.15 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, D_2 = \begin{bmatrix} 0.1 & -0.5 \\ 0.25 & 0.2 \\ 0.25 & 0.1 \\ 0.1 & 0.1 \end{bmatrix},$$

$$G = \begin{bmatrix} 0.8 & 0.8 \\ 0.8 & 0.8 \\ 0.8 & 0.8 \\ 0.8 & 0.8 \end{bmatrix}, E = \text{diag}[2 \ 1.4 \ 2 \ 1], C = I_4;$$

the state vector of the system is $x = [\Delta u \ \Delta \alpha \ \Delta \theta \ \Delta q]^T$, with Δu – the variation of the aircraft longitudinal velocity, $\Delta \alpha$ – the variation of the attack angle, $\Delta \theta$ – the variation of the pitch angle, and Δq – the variation of the pitch angular rate; the only

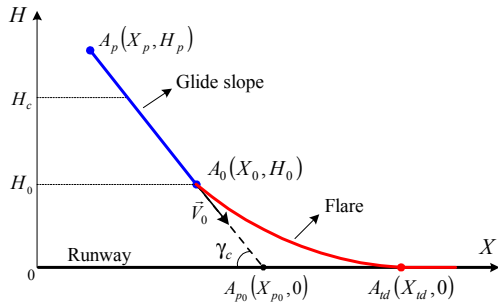


Figure 2. Aircraft landing geometry in longitudinal plane

input of the system is $u = \delta_e$ – the elevator deflection. The vector of unknown inputs has been randomly chosen; it can consist of the longitudinal or vertical wind shears, atmospheric turbulences, or errors of the sensors. The decision and activation

functions have the forms: $\xi(t) = u(t)$, $\mu_1(\xi(t)) = 1.4(1 - \tanh(\xi(t)))$, $\mu_2(\xi(t)) = 1 - \mu_1(\xi(t))$. The LMIs' solving is performed by using the Matlab/Simulink LMI tool. In Fig. 3 we present the time histories of the state estimation errors for the three multiple observers; the errors represent the differences between the real values of the four states and their estimated values, i.e. $e_i = x_i - \hat{x}_i, i = \overline{1, 4}$. We remark the cancel of the estimation errors, hence a proper functioning of the multiple observers; this is equivalent with the achievement of the state's reconstruction. Because we used the same aircraft (Boeing 747), same dynamics, and same activation functions for all the three observers, we can make a comparison between these observers from the convergence speeds' point of view. The convergence speeds of the three multiple observers are similar, with a slight advantage for the first variant (3 seconds).

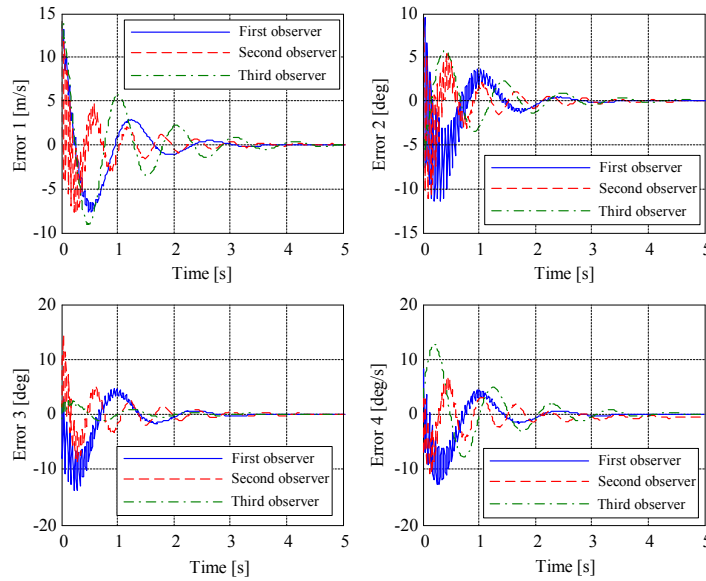


Figure 3. Estimation errors for the three multiple observers

V. CONCLUSION

The purpose of this study was to achieve the finite-time reconstruction of the system's state associated to a multiple-model (aircraft landing) using multiple observers designed for Takagi-Sugeno descriptor systems. Three multiple observers are presented; these have been software implemented and a brief comparison between them is made. The obtained results are very good, the components of the error vector tending to zero.

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