Adaptive Control of the Aircraft Pitch Angular Motion by using the Dynamic Inversion Principle

Romulus Lungu, Mihaela Iancu, Stefan Ispas and Mihai Lungu
University of Craiova, Faculty of Electrical Engineering, Craiova, Romania
romulus_lungu@yahoo.com, rlungu@elth.ucv.ro

Abstract— **This paper approaches the adaptive control of an aircraft longitudinal motion by using the dynamic inversion principle, the parametric estimation, and the linear dynamic compensation. The control law has two components: the former is provided by a linear dynamic compensator, with the state tracking error as input; the latter is an adaptive component which represents the change rate of the estimated state vector deviation with respect to the desired one and the change rate of the variance between the system estimated output and the desired one, respectively. The obtained adaptive control structures consist of parametric estimators, dynamic compensators and command filters. The estimated state and the estimated output are calculated with respect to the estimated vector of the aircraft parameters. The obtained control system is particularized for the adaptive control of the pitch angular rate and the pitch angle, respectively, in the case of an F-15 aircraft whose flight may be affected or not by wind shears. The theoretical results are validated by numerical simulations in the absence or in the presence of wind shears by using complex Matlab/Simulink models; the states and the command variables history for the designed optimal control systems are plotted and the functionality of the two systems is proved.**

I INTRODUCTION

The authors approach the adaptive control problem using the nonlinear and linear aircraft model of the longitudinal motion. The control law has two components: the former is provided by a linear dynamic compensator, with state tracking error as input; the latter is an adaptive component representing the change rate of the estimated state vector deviation with respect to the desired one and the change rate of the variance between the system estimated output and the desired one, respectively. The estimated state and the estimated output are calculated with respect to the estimated vector of the aircraft parameters.

The pseudo-command depends on the relative degree of the controlled state variable. Choosing as the controlled state variable the pitch angular velocity, the relative degree is 1 and, therefore, the command filter (the reference model) is a first order subsystem. If the controlled state variable is the pitch angle, the relative degree is 2 and the command filter is a second order subsystem. The system has, as disturbances, the longitudinal wind velocities.

The longitudinal motion of an F-15 aircraft flying at $6000\,\mathrm{m}$ altitude, having velocity $V_0 = 100\,\mathrm{m/s}$, is studied. For the linear model of the longitudinal motion, Simulink models are obtained and the system time characteristics are plotted; these characteristics are: the time history of the pitch angular rate, pitch angle, and the time history of the command variables, with or without the wind shears. Making a review of these time characteristics, we will remark that the wind shears have an insignificant influence upon the aircraft longitudinal dynamics.

II ADAPTIVE CONTROL SYSTEM BASED ON THE DYNAMIC INVERSION PRINCIPLE

Generally, the systems' dynamics is modeled by a differential equations' system, whose parameters are approximately known because their evolution is affected by known and unknown errors. The component of the adaptive control law is calculated such that the deviation of estimated parameters' vector $(\hat{\vartheta})$, with respect to the real one (ϑ), is compensated. We consider the system described by the state equation:

$$\dot{x} = f(t, x, \Gamma, v) \tag{1}$$

or

$$\dot{x} = \varphi(x, t, \Gamma)\vartheta + v + \Delta(t, x, \Gamma, v), \tag{2}$$

where x is the state vector, $\Gamma -$ the external disturbances vector, $\Delta -$ all disturbances' vector, $v -$ the control law, while v is:

$$v = v_s + v_a \tag{3}$$

with $v_s -$ the linear dynamic compensation component, and $v_a -$ the adaptive component. These laws can be chosen by the next forms:

$$v_s = Ke, \tag{4}$$

$$v_a = \dot{x}_r - \varphi(t, x, \Gamma)\hat{\vartheta}; \tag{5}$$

K is, in general, the gain matrix, $e -$ the error vector of the automatic control system,

$$e = x_r - x, \tag{6}$$

x_r is the desired state vector, and $\hat{\vartheta}$ is the estimation of the vector ϑ. The adaptive component v_a makes the dynamic inversion of the resulted system i.e. [2]:

$$v_a = \hat{f}^{-1}(t, \dot{x}, \Gamma), \tag{7}$$

where $\hat{f}$ is the approximation of the function f, such that, in steady regime, we get:

$$v_a \to 0, \dot{x} \to \dot{x}_r \to \dot{\hat{x}} \tag{8}$$

and

$$\tilde{\vartheta} = \hat{\vartheta} - \vartheta \to 0, \tag{9}$$

respectively.

The estimated vector $\hat{\vartheta}$ verifies the equation [1]:

$$\dot{\hat{\vartheta}} = \gamma\varphi(t, x, \Gamma)^T e, \tag{10}$$

where γ is a positive defined matrix.

Combining the last two equations, we obtain an automatic adaptive control system, which is modeled by the block diagram in Fig. 1. Here, k is 1 or 0.

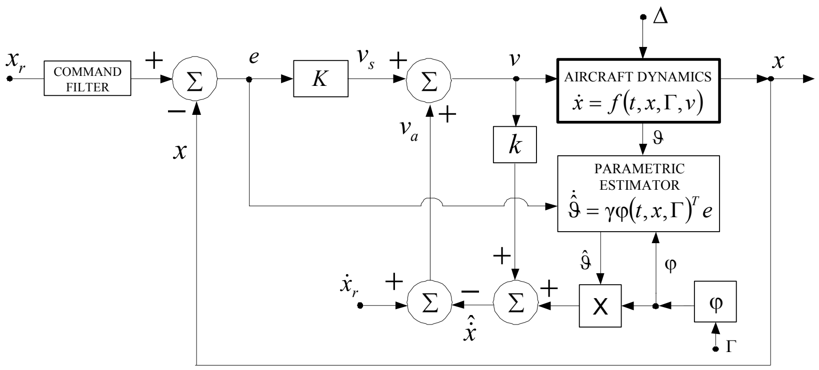

Fig. 1 Nonlinear adaptive control system based on dynamic inversion principle, with linear dynamic compensator

Theorem IV from [1] proves that equation (10) provides the Lyapunov stability of the system in Fig. 1 and the convergence toward zero of the system error $(e \to 0)$. Thus, the Lyapunov function $(V(t) > 0)$ is chosen as:

$$V(t) = \frac{1}{2} e^T e + \frac{1}{2} \tilde{\vartheta}^T \gamma^{-1} \tilde{\vartheta}. \tag{11}$$

The asymptotic stability condition $(\dot{V}(t) < 0)$ is fulfilled as follows: by derivation of $V(t)$ and taking into account the equation (2), it successively results:

$$\dot{V}(t) = e^T \dot{e} + \tilde{\vartheta}^T \gamma^{-1} \dot{\tilde{\vartheta}}, \tag{12}$$

with $\dot{e}$ having the form:

$$\dot{e} = x_r - x = -\varphi\vartheta - Ke + \varphi\ e + \varphi\ \hat{\vartheta} \tag{13}$$

or

$$\dot{e} + Ke = -\varphi\tilde{\vartheta}. \tag{14}$$

Thus, replacing (14) in (12), we obtain:

$$V(t) = -e^T Ke < 0. \tag{15}$$

Now, let us consider the case of the aircraft motion linearized model (for example the aircrafts' longitudinal motion) described by the state equations [4]:

$$\dot{x} = Ax + B\delta_p + B_v V_v, \tag{16}$$

$$y = Cx, \tag{17}$$

where x is the system state vector, while $y -$ the system output vector; these have the following forms:

$$x = \begin{bmatrix} V_x & \alpha & \omega_y & \theta & H \end{bmatrix}^T, y = \omega_y, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}; \tag{18}$$

V_x is the aircraft longitudinal velocity, $\alpha -$ the attack angle, $\omega_y -$ the pitch angular rate, $\theta -$ the pitch angle, $H -$ the aircraft flight altitude, $\delta -$ the command vector, $\delta = \begin{bmatrix} \delta_p & \delta_T \end{bmatrix}^T$, with $\delta_p -$ the rudder deflection, $\delta_T -$ the engine command, while $V_v -$ the perturbation vector (wind speed), $V_v = \begin{bmatrix} V_{vx} & V_{vz} \end{bmatrix}^T$. By using (17) we get:

$$x = C^+ y = C^+ \omega_y, \tag{19}$$

where C^+ is the pseudo-inverse of the matrix C. By means of (19), the equation (16) becomes:

$$\begin{aligned} \delta &= B^+(\dot{x} - Ax - B_v V_v) = B^+(C^+\dot{\omega}_y - C^+CAx - B_v V_v) \\ &= (CB)^+(\dot{\omega}_y - CAx) - B^+ B_v V_v. \end{aligned} \tag{20}$$

Because we want $\dot{\omega}_y$ to have the same value with its desired value (the calculated value ω_{yc}), we obtain the command law:

$$\delta_p = T^+\delta = (CBT)^+(\dot{\omega}_{yc} - CAx) - (BT)^+ B_v V_v; \tag{21}$$

it expresses the longitudinal dynamic inverse model of the aircraft. If the components V_{vx} and V_{vz} can be measured, in the command law we can introduce the component $B^+ B_v V_v$ as a negative feedback; thus, $\omega_y \cong \overline{\omega}_r \cong \omega_r$. In reality, it is difficult to measure these disturbances, and, that is why, in the command law (21) the component $B^+ B_v V_v$ is missing; therefore, $\omega_y \to \overline{\omega}_r \to \omega_{r_0}$. The pseudo-command is [3]:

$$y_c^{(r)} = v, \tag{22}$$

where r represents the relative degree of the resulted system with respect to the output, and the command filter (reference model) relative degree, respectively; for the longitudinal motion (with $y = \omega_y$), it results $r = 1$. Thus,

$$\dot{y}_c = \dot{\omega}_{yc} = v. \tag{23}$$

The command vector of the aircraft longitudinal motion is expressing as follows:

$$\delta = T\delta_p, T = \begin{bmatrix} 0 & 1 \end{bmatrix}^T. \tag{24}$$

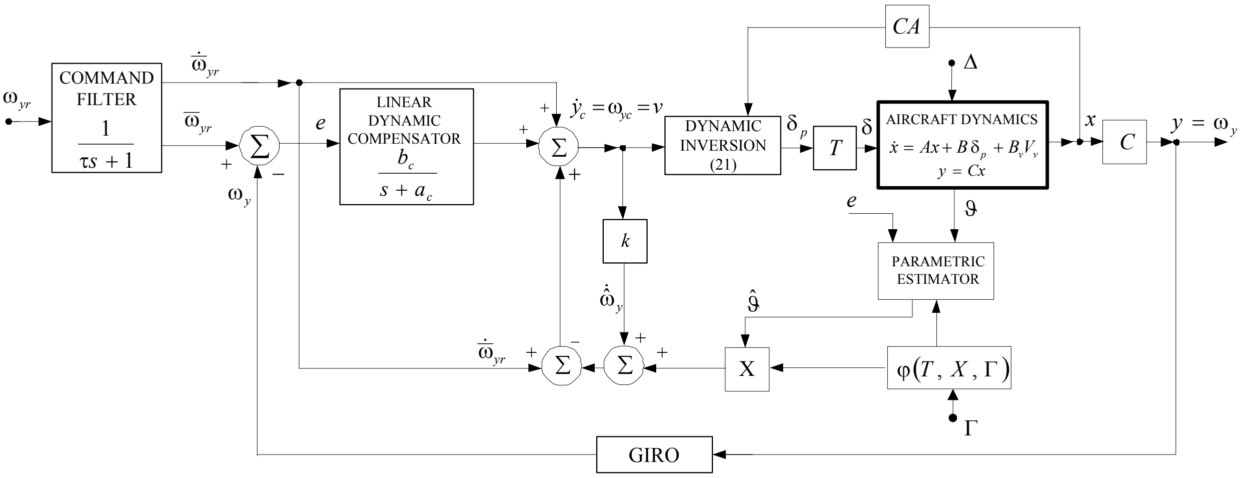

Fig. 2 Linear adaptive system based on dynamic inversion principle (with linear dynamic compensator) for the pitch angular rate control

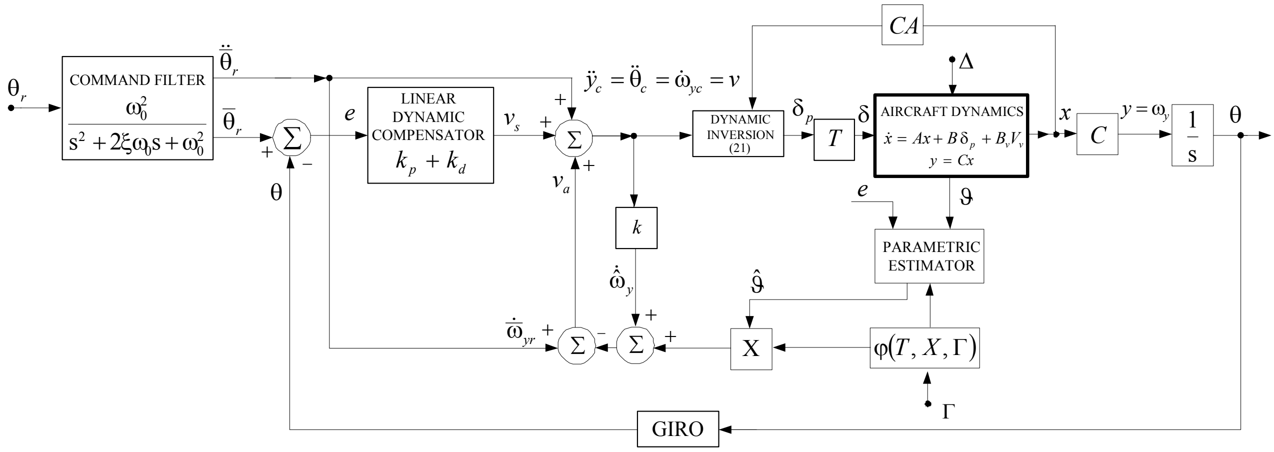

Fig. 3 Linear adaptive system based on dynamic inversion principle (with linear dynamic compensator) for the pitch angle control

Linear adaptive control system based on the dynamic inversion principle is presented in Fig. 2. For the adaptive control of the pitch angle, we use a similar structure – the one in Fig. 3. In this case, the pseudo-inverse has the form (23), with $y_c = \theta_c$ and $r = 2$. The command filter is a second order filter, while the dynamic compensator is proportional - derivative type.

III Simulation Results

Let us consider the linear model of an aircraft F-15 longitudinal motion, which flies at the altitude $H = 6000$ m with the speed $V_0 = 100$ m/s.

The system matrices A, B, and B_v are [1]:

$$A = \begin{bmatrix} -0.0112 & -0.0365 & 0 & -0.5601 & 0.0001 \\ -0.0065 & -1.1182 & 1 & 0.0001 & 0.0001 \\ 0.0015 & 8.3089 & -0.9412 & 0 & -0.0001 \\ 0 & 0 & 1 & 0 & 0 \\ -0.0017 & -13.5803 & 0 & 13.5803 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.0811 & -0.0811 \\ -0.0688 & -0.0688 \\ -5.9799 & -5.9799 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, B_v = -\begin{bmatrix} a_{11} & a_{12}/V_0 \\ a_{21} & a_{22}/V_0 \\ a_{31} & a_{32}/V_0 \\ a_{41} & a_{42}/V_0 \\ a_{51} & a_{52}/V_0 \end{bmatrix}. \quad (25)$$

The elements from first column of matrix B_v are the same elements of the matrix A but with changed sign, and the second column of matrix B_v is the second of matrix A divided by $(-V_0)$.

For the control system in Fig. 2, with initial state $x_0 = x(0) = [100\ 7\ 1\ 20\ 6000]^T$ and $\varphi = [0\ 0\ 1\ 0\ 0]^T$, $a_c = b_c = 1, \tau = 0.1\,\text{s}, \omega_{yr} = 1\,\text{deg/s}$.

In Matlab environment, we built the Matlab/Simulink model in Fig. 3. The subsystem that models the wind shears is described by the equations from [6]:

$$V_{vx} = -V_{vx0}\sin\frac{2\pi}{T_0}t, V_{vz} = -V_{vz0}\left(1-\cos\left(\frac{2\pi}{T_0}\right)t\right), \quad (26)$$

with T_0 – the time period when the aircraft is affected by the wind shears, $V_{vx0} = 15$ m/s, $V_{vz0} = 7$ m/s.

By using the Matlab/Simulink model in Fig. 4, the states and the command variables history for the designed optimal control system is plotted (Fig. 5).

For the system in Fig. 3 the following values have been chosen: $k_p = 10[1/\text{s}], k_d = 10, \theta_r = 25[\text{deg}], \varphi = [0\ 0\ 1\ 0\ 0]^T$, $x_0 = x(0) = [100\ 7\ 1\ 20\ 6000]^T$; by using the Matlab/Simulink model in Fig. 6 we obtained the time history in Fig. 7.

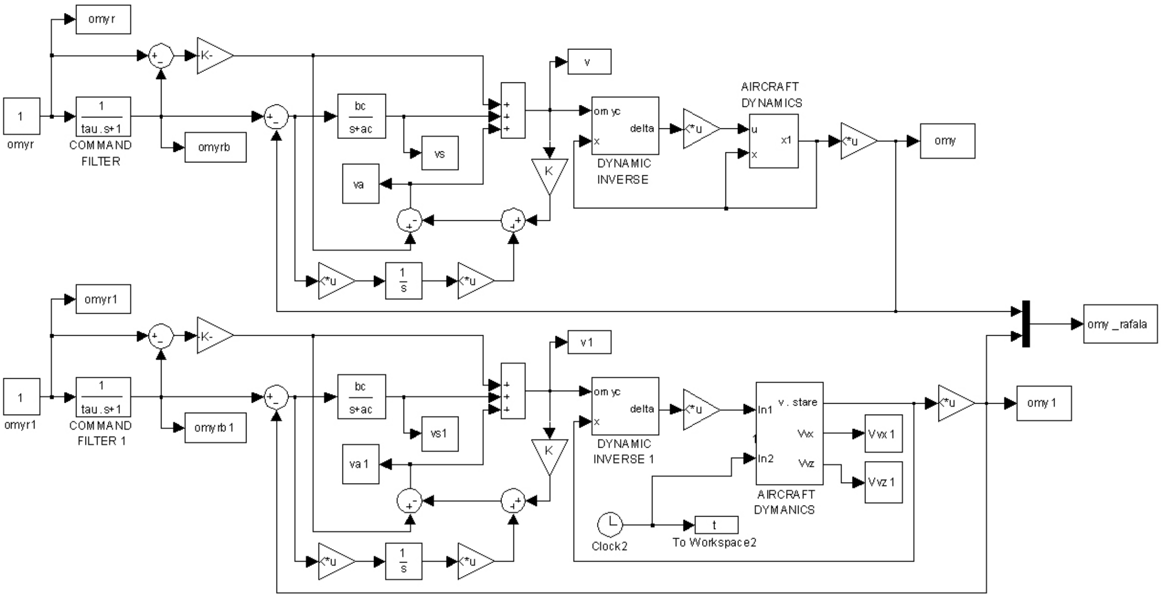

Fig. 4 Matlab/Simulink model of the system from Fig. 2.

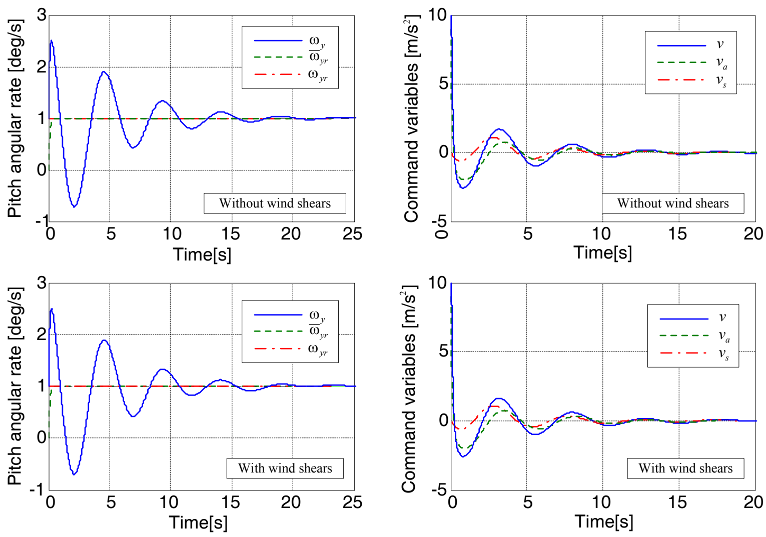

Fig. 5 Time characteristics of the system from Fig. 2.

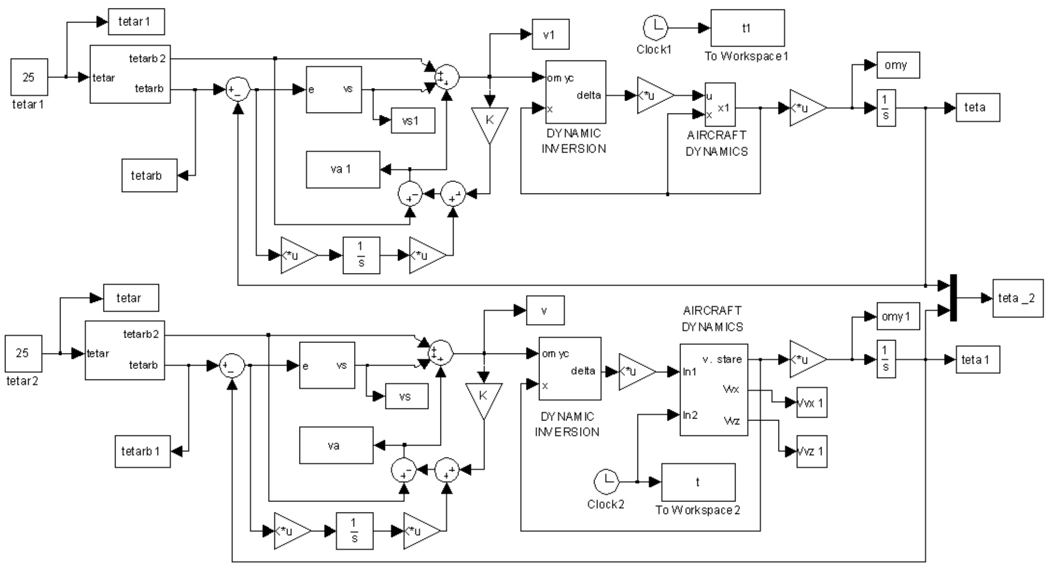

Fig. 6 Matlab/Simulink model of the system from Fig. 3.

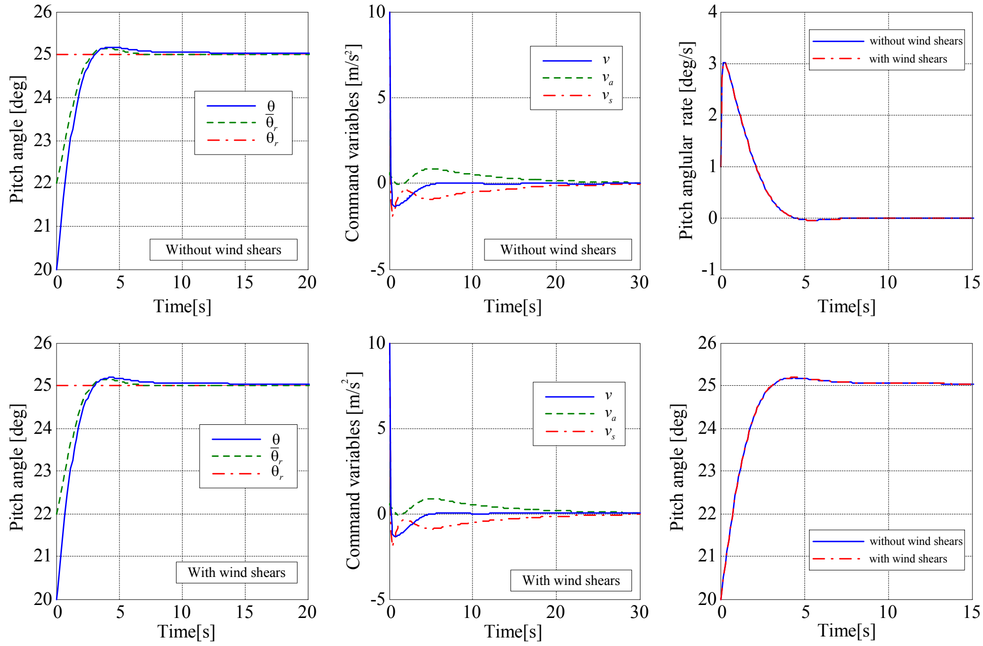

Fig. 7 Time characteristics of the system from Fig. 3

IV CONCLUSION

An adaptive law for the control of an aircraft longitudinal motion is designed by using the dynamic inversion principle, the parametric estimation, and the linear dynamic compensation. The obtained control system is particularized for the adaptive control of the pitch angular rate and the pitch angle, respectively, in the case of an F-15 aircraft whose flight may be or may be not affected by wind shears.

REFERENCES

[1] R.F. James, Aircraft Control Using Nonlinear Dynamic Inversion in Conjunction with Adaptive Robust Control, Master Thesis, 2004.

[2] A.J. Calise, N. Hovakymyan, and M. Idan, “Adaptive Output Control of Nonlinear Systems Using Neural Networks,” Automatica, August 2001.

[3] A.J. Calise, H. Lee, and N. Kim, “High Bandwhite Adaptive Flight Control,” AIAA Giudance, Navigation and Control Conference, August 2000.

[4] M. Lungu, Sisteme de conducere a zborului, Sitech Publisher, 2008.

[5] R. Lungu, Automatizarea aparatelor de bord, Universitaria Publisher, Craiova, 2000.

[6] J. Che and D. Chen, “Automatic Landing Control using H^{∞} control and Stable Inversion,” Proceedings of the 40th Conference on Decision and Control, Florida, USA, pp. 241–246, 2001.