Optimal Control of Helicopter Motion

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***Abstract*— This paper presents an automatic system for the optimal control of helicopters motion. In order to control the linear velocities and the yaw angular rate we introduce 4 supplementary states as the outputs of ideal integrators; these integrates the deviations of the 4 variables (the linear velocities and of the yaw angular rate) from their desired values. To achieve the control, we calculate the gain matrix of the system by concatenation of two matrices: the former is associated to the initial state vector of the system, while the latter corresponds to the supplementary states of the system; the gain matrix of the optimal system will be calculated with respect to the solution of a Riccati algebraic equation. The theoretical results are validated by numerical simulations in the absence or in the presence of wind shears by using complex Matlab/Simulink models and a helicopter motion liniarized model; an optimal control system is designed by using a cost function. The system state has 4 supplementary states in order to control the linear velocities corresponding to the three axes of the body frame and the roll angular rate. The states and command variables history for the designed optimal control system are plotted and the system functionality is proved.**

I. Helicopter Linearized Dynamics

In the first part of the paper a linearized model of the helicopter motion is presented, pointing the state and command variables; the atmospheric disturbances are considered to be the wind shears. Then, an optimal control law is designed by means of a cost function. Using the Matlab environment, the states and command variables history for the designed optimal control system is plotted.

Helicopter dynamics is described by the state equation:

$$\dot{x} = Ax + Bu + B_v V_v, \qquad (1)$$

where x is the state vector and $u-$ the command vector, and V_v- the disturbances vector (wind shears vector); these vectors have the following forms:

$$x = \begin{bmatrix} V_x & V_y & \omega_x & \omega_y & \varphi & \theta & \delta_p & \delta_\varphi & V_z & \omega_z \end{bmatrix}^T,$$
$$u = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T = \begin{bmatrix} \delta_{zc} & \delta_{pc} & \delta_{\varphi c} & \delta_{dc} \end{bmatrix}^T, V_v = \begin{bmatrix} V_{vx} & V_{vz} \end{bmatrix}^T. \qquad (2)$$

The chosen body frame (orthogonal coordinates system) has the following three axes: $OX-$ the longitudinal axis, $OY-$ the lateral axis oriented to the right plane, $OZ-$ the axis perpendicular on the (OXY) plane and downward oriented. In equation (2) θ is the helicopter pitch angle, $\varphi-$ the helicopter roll angle, δ_p- the longitudinal flapping angle, δ_φ the lateral flapping angle, $\delta_{zc} \equiv \delta_z-$ the cyclic input, $\delta_{pc}-$ the longitudinal control of the main rotor, $\delta_{\varphi c}-$ the lateral control of the main rotor, $\delta_{dc} \equiv \delta_d-$ the tail rotor control. The forms of matrices A and B have been borrowed from [1], while B_v has the form in [2]:

$$A = \begin{bmatrix}
X_u & X_v + \omega_{ze} & X_p & X_q - V_{ze} & 0 & -g\cos(\theta_e) & X_{\delta_p} & 0 & X_w - \omega_{ye} & X_r + V_{ye} \\
Z_u - \omega_{ye} & Z_v - \omega_{xe} & Z_p - V_{ye} & Z_q + V_{xe} & -g\cos(\varphi_e)\cos(\theta_e) & -g\cos(\varphi_e)\sin(\theta_e) & Z_{\delta_p} & 0 & Z_w & Z_r \\
M_u & M_v & M_p & M_q & 0 & 0 & M_{\delta_p} & 0 & M_w & M_r \\
0 & 0 & 0 & \cos(\varphi_e) & 0 & 0 & 0 & 0 & 0 & \sin(\varphi_e) \\
0 & 0 & 0 & -1 & 0 & 0 & -\frac{1}{\tau} & 0 & 0 & 0 \\
Y_u - \omega_{ze} & Y_v & Y_p + V_{ze} & Y_q & Y_\varphi + g\cos(\varphi_e)\cos(\theta_e) & Y_\theta - g\sin(\varphi_e)\sin(\theta_e) & Y_{\delta_p} & Y_{\delta_\varphi} & Y_w + \omega_{xe} & Y_r - V_{xe} \\
L_u & L_v & L_p & L_q & L_\varphi & L_\theta & L_{\delta_p} & L_{\delta_\varphi} & L_w & L_r \\
0 & 0 & 1 & \sin(\varphi_e)\tan(\theta_e) & 0 & 0 & 0 & 0 & 0 & \cos(\varphi_e)\tan(\theta_e) \\
N_u & N_v & N_p & N_q & N_\varphi & N_\theta & N_{\delta_p} & N_{\delta_\varphi} & N_w & N_r \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{1}{\tau} & 0 & 0
\end{bmatrix}, \qquad (3)$$

$$B^T = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{\delta_{dc}} & Y_{\delta_{dc}} \\
0 & 0 & 0 & 0 & 0 & 0 & A_{\delta_{pc}} & C_{\delta_{pc}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & D_{\delta_{\varphi c}} & B_{\delta_{\varphi c}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_{\delta_{zc}}
\end{bmatrix}, B_v^T = -\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{1,10} \\
a_{91} & a_{92} & a_{93} & a_{94} & a_{95} & a_{96} & a_{97} & a_{98} & a_{99} & a_{9,10}
\end{bmatrix}.$$

a_{11} ... $a_{1,10}$ are the first line elements of the matrix A and a_{91} ... $a_{9,10}$ are the ninth line elements of the matrix A.

For the wind shears modeling we use the equations [3]:

$$V_{vx} = -V_{vx0}\sin\frac{2\pi}{T_0}t, V_{vz} = -V_{vz0}\left(1-\cos\left(\frac{2\pi}{T_0}\right)t\right), \qquad (4)$$

with T_0- the flight time period inside the wind shear; the aircraft faces head wind and rear wind combined with vertical wind.

II. Optimal Control of the Helicopter

We build a new state [1]:

$$\xi = \begin{bmatrix} \dot{x} & e \end{bmatrix}^T, e = \begin{bmatrix} e_{11} & e_{12} & e_{21} & e_{22} \end{bmatrix}^T, \qquad (5)$$

where e_{11}, e_{12}, e_{21} , and e_{22} are the tracking errors for the longitudinal velocity $\overline{V}_x$, lift velocity $\overline{\dot{H}}$, lateral velocity $\overline{V}_y$, and angular rate $\overline{\omega}_z$, respectively (Fig. 1).

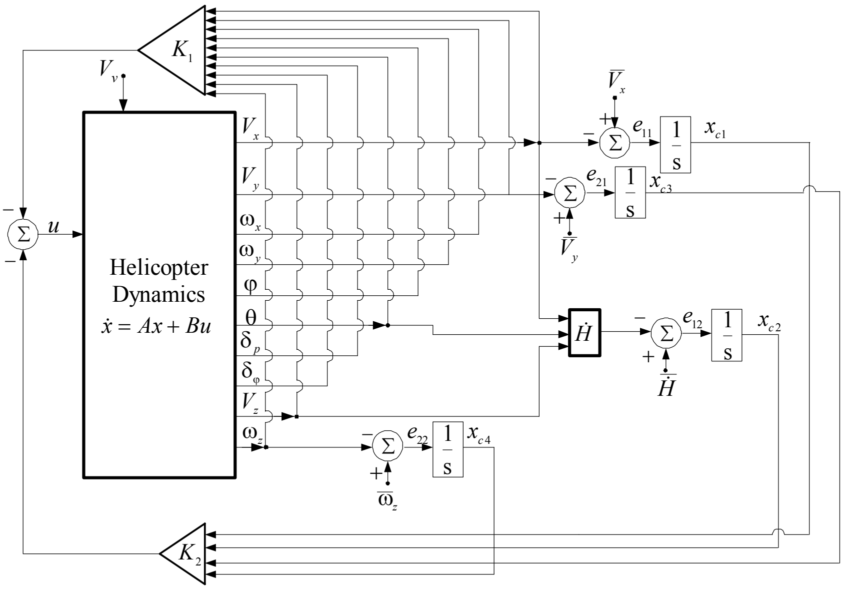

Fig. 1. The block diagram of the automatic system for the control of the variables V_x , V_y , $\dot{H}$, ω_z .

The equation of the new state is now:

$$\dot{\xi} = \overline{A}\xi + \overline{B}v. \tag{6}$$

In order to calculate the matrices $\overline{A}$ and $\overline{B}$, by using the time derivative of equation (1), equation (5), and considering $V_v = 0$, we obtain:

$$\dot{\xi} = \begin{bmatrix} \ddot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A\dot{x} + B\dot{u} \\ A_e\dot{x} \end{bmatrix}, \tag{7}$$

where A_e is calculated below; in order to cancel the errors in Fig.1 we added four ideal integrators;

$$\dot{e}_{11} = -\dot{\overline{V}}_x - \dot{V}_x = -\dot{V}_x = [-1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]\dot{x}, \tag{8}$$

$$\begin{aligned} \dot{e}_{12} &= -\ddot{\overline{H}} - \ddot{H} = -\dot{V}_x \sin\theta_0 + \dot{V}_z \cos\theta_0 = \\ &= [-\sin\theta_0\ \ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \cos\theta_0\ \ 0]\dot{x}, \end{aligned} \tag{9}$$

$$\dot{e}_{21} = \dot{\overline{V}}_y - \dot{V}_y = -\dot{V}_y = [0\ -1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]\dot{x}, \tag{10}$$

$$\dot{e}_{22} = \dot{\overline{\omega}}_z - \dot{\omega}_z = -\dot{\omega}_z = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ -1]\dot{x}. \tag{11}$$

Having in mind the above equations, we get:

$$\dot{e} = \begin{bmatrix} \dot{e}_{11} \\ \dot{e}_{12} \\ \dot{e}_{21} \\ \dot{e}_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0\,0\,0\,0\,0\,0\,0 & 0 & 0 \\ -\sin\theta_0 & 0 & 0\,0\,0\,0\,0\,0\,0 & \cos\theta_0 & 0 \\ 0 & -1 & 0\,0\,0\,0\,0\,0\,0 & 0 & 0 \\ 0 & 0 & 0\,0\,0\,0\,0\,0\,0 & 0 & -1 \end{bmatrix} \dot{x} = A_e\dot{x}. \tag{12}$$

The equation (7) can be written as following:

$$\dot{\xi} = \begin{bmatrix} \ddot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & \vdots & B \\ \cdots & \vdots & \cdots \\ A_e & \vdots & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ e \end{bmatrix} + \begin{bmatrix} B \\ \cdots \\ 0 \end{bmatrix} \dot{u}\ ;\ \dot{u} = v, \tag{13}$$

or

$$\dot{\xi} = \overline{A}\xi + \overline{B}v,\ v = \dot{u}, \tag{14}$$

where

$$\overline{A} = \begin{bmatrix} A & \vdots & B \\ \cdots & \vdots & \cdots \\ A_e & \vdots & 0 \end{bmatrix},\ \overline{B} = \begin{bmatrix} B \\ \cdots \\ 0 \end{bmatrix}. \tag{15}$$

The optimal control law can be written under one of the following forms [4]:

$$\dot{u} = v = -K\xi = [K_1\ \ K_2][\dot{x}\ \ e]^T = -K_1\dot{x} - K_2 e, \tag{16}$$

or

$$u = -K_1 x - K_2 \int_0^t e\,\mathrm{dt} = -K_1 x - K_2 x_c, \tag{17}$$

with

$$x_c = [x_{c1}\ \ x_{c2}\ \ x_{c3}\ \ x_{c4}]^T = \left[\int e_{11}\ \int e_{12}\ \int e_{21}\ \int e_{22}\right]^T. \tag{18}$$

The gain matrix K is:

$$K = -R^{-1}\overline{B}^T P, \tag{19}$$

with $P-$ the solution of Riccati algebraic equation:

$$\overline{A}^T P + P\overline{A} + Q - P\overline{B}R^{-1}\overline{B}^T P = 0; \tag{20}$$

R and Q are the desired weight matrices for which the cost function:

$$J = \frac{1}{2}\int_0^{\infty}\left(x^T Q x + u^T R u\right)\mathrm{d}t, \tag{21}$$

has the minimum value.

III. SIMULATION RESULTS

The helicopter motion is studied, considering that the initial state is $x_0 = [0 \;\; 0 \;\; 0 \;\; 0.5 \;\; 0 \;\; 0.174 \;\; 0 \;\; 0 \;\; 0 \;\; 0.2]^T$ and $\overline{V}_x = 10\,\mathrm{m/s}, \overline{V}_y = 0\,\mathrm{m/s}, \overline{\omega}_z = 0\,\mathrm{deg/s}, \dot{\overline{H}} = 5\,\mathrm{m/s}$. The matrices A and B are from [5, 6]:

$$A = \begin{bmatrix} -0.1778 & 0 & 0 & 0 & 0 & -9.7807 & -9.7807 & 0 & 0 & 0 \\ 0 & -0.3104 & 0 & 0 & 9.7807 & 0 & 0 & 9.7807 & 0 & 0 \\ 0.3326 & -0.5353 & 0 & 0 & 0 & 0 & 75.764 & 343.86 & 0 & 0 \\ 0.1903 & -0.294 & 0 & 0 & 0 & 0 & 172.62 & -59.958 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -8.1222 & 4.6535 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -0.0921 & -8.1222 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 17.168 & 7.1018 & -0.6821 & -0.107 \\ 0 & 0 & -0.2834 & 0 & 0 & 0 & 0 & 0 & -0.1446 & -5.5561 \end{bmatrix},$$

$$B^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19.925 & 2.0816 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3.339 & 0.2216 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0632 & 3.1739 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -74.364 \end{bmatrix}.$$

The Matlab/Simulink model for the system, without wind shears $(V_v = 0)$, in Fig.1 is presented in Fig.2, while the Simulink model for the system with wind shears in Fig. 1, characterized by $\overline{V}_{x0} = 10\ \mathrm{m/s}$, $\overline{V}_{y0} = 0\ \mathrm{m/s}$, $\overline{\omega}_{z0} = 0\ \mathrm{deg/s}$, $\dot{\overline{H}}_0 = 5\ \mathrm{m/s}$, $\theta_0 = 10\ \mathrm{deg}$ is presented in Fig. 3.

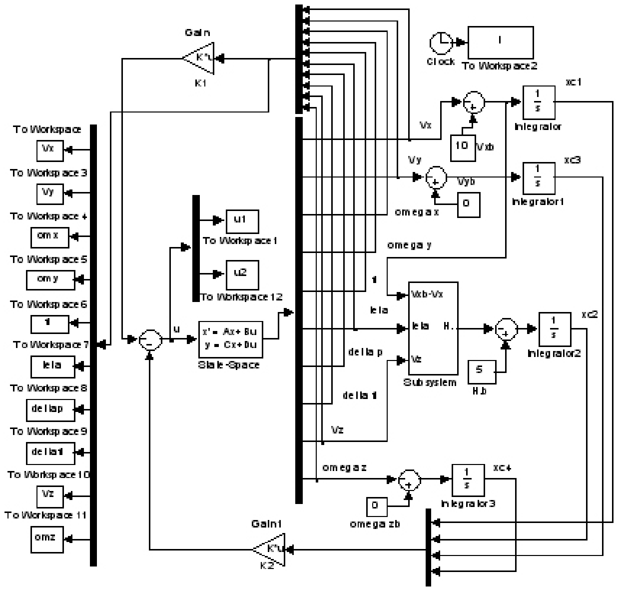

Fig. 2. Matlab/Simulink model of the system in Fig.1 without wind shears

In equation (20) the positive define matrices Q and R have the following forms: $Q = I_{14}$, $R = I_4$, respectively; the matrices K_1, K_2, and P are obtained by solving the equation (20) in Matlab environment. We obtained:

$$P = 10^{-4}\begin{bmatrix} 75 & 0 & 1 & -5 & 2 & -174 & -41 & 10 & 0 & 0 & -60 & -4 & 0 & 0 \\ 0 & 69 & 4 & 1 & 158 & 1 & 6 & 40 & 0 & 0 & 0 & 0 & -58 & 0 \\ 1 & 4 & 4 & 0 & 18 & -1 & 6 & 27 & 0 & 0 & 0 & 0 & -3 & 0 \\ -5 & 1 & 0 & 6 & 2 & 25 & 28 & -9 & 0 & 0 & 3 & 0 & -1 & 0 \\ 2 & 158 & 18 & 2 & 647 & -2 & 27 & 173 & 0 & 0 & -1 & 0 & -121 & 0 \\ -174 & 1 & -1 & 25 & -2 & 703 & 176 & -46 & -1 & 0 & 126 & 8 & -1 & 0 \\ -41 & 6 & 6 & 28 & 27 & 176 & 258 & 14 & 1 & 0 & 30 & 1 & -5 & 0 \\ 10 & 40 & 27 & -9 & 173 & -46 & 14 & 378 & 0 & 0 & -8 & -1 & -31 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 5 & 0 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ -60 & 0 & 0 & 3 & -1 & 126 & 30 & -8 & 0 & 0 & 151 & -6 & 0 & 0 \\ -4 & 0 & 0 & 0 & 0 & 8 & 1 & -1 & -5 & 0 & -6 & 106 & 0 & 0 \\ 0 & 58 & -3 & -1 & -121 & -1 & -5 & -31 & 0 & 0 & 0 & 0 & 149 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 102 \end{bmatrix},$$

$$K_1 = 10^{-4}\begin{bmatrix} 751 & -157 & -188 & -399 & -792 & -2315 & 1382 & 449 & 10140 & 192 \\ -13551 & 2854 & 2680 & 9299 & 12798 & 57658 & 86367 & 13195 & 237 & -3 \\ 3065 & 12667 & 8641 & -2575 & 54955 & -13487 & 6217 & 11994 & 76 & -4 \\ 21 & -1 & 25 & -13 & -50 & -70 & 55 & 98 & 266 & -9412 \end{bmatrix},$$

$$K_2 = 10^{-4}\begin{bmatrix} 325 & -9991 & 127 & -243 \\ 9739 & 288 & -2251 & 3 \\ -2246 & -197 & -9743 & 4 \\ 6 & -243 & 8 & 9997 \end{bmatrix}.$$

The system time characteristics are presented in Fig. 4 (time characteristics of the system in Fig.1 without wind shears) and in Fig. 5 (time characteristics of the system in Fig.1 with wind shears), respectively.

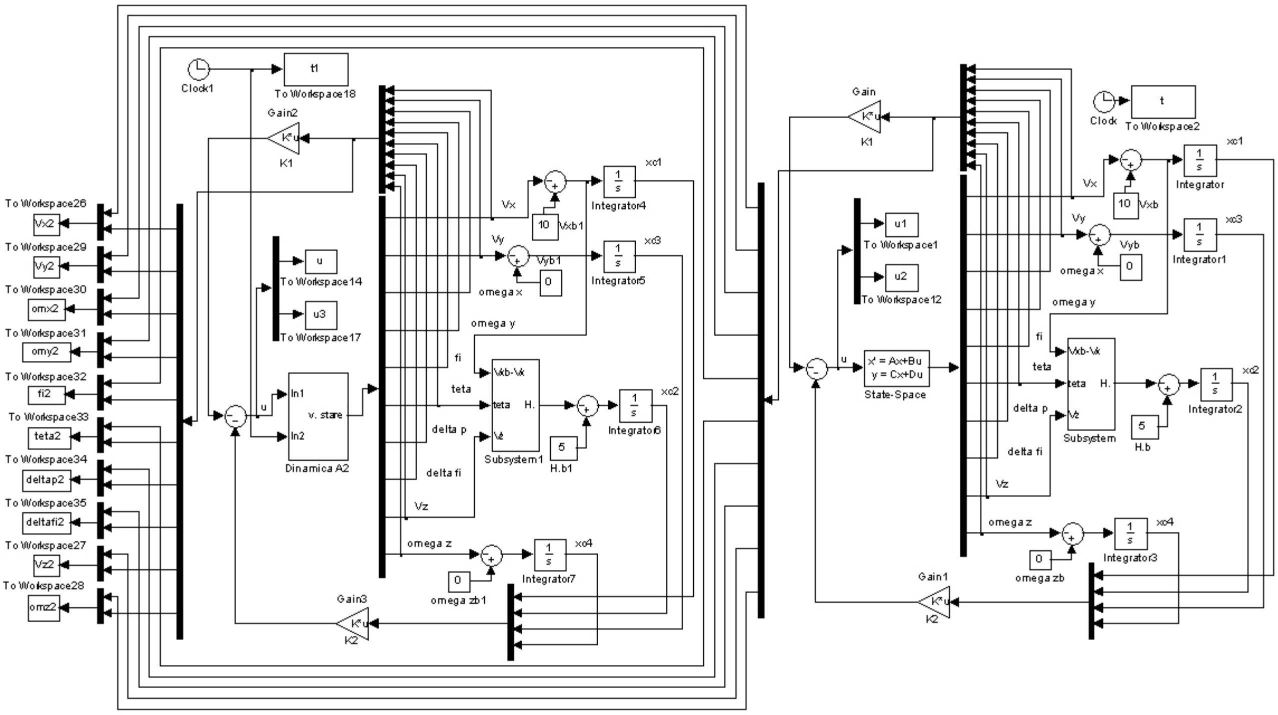

Fig. 3. Matlab/Simulink model of the system in Fig.1 with wind shears.

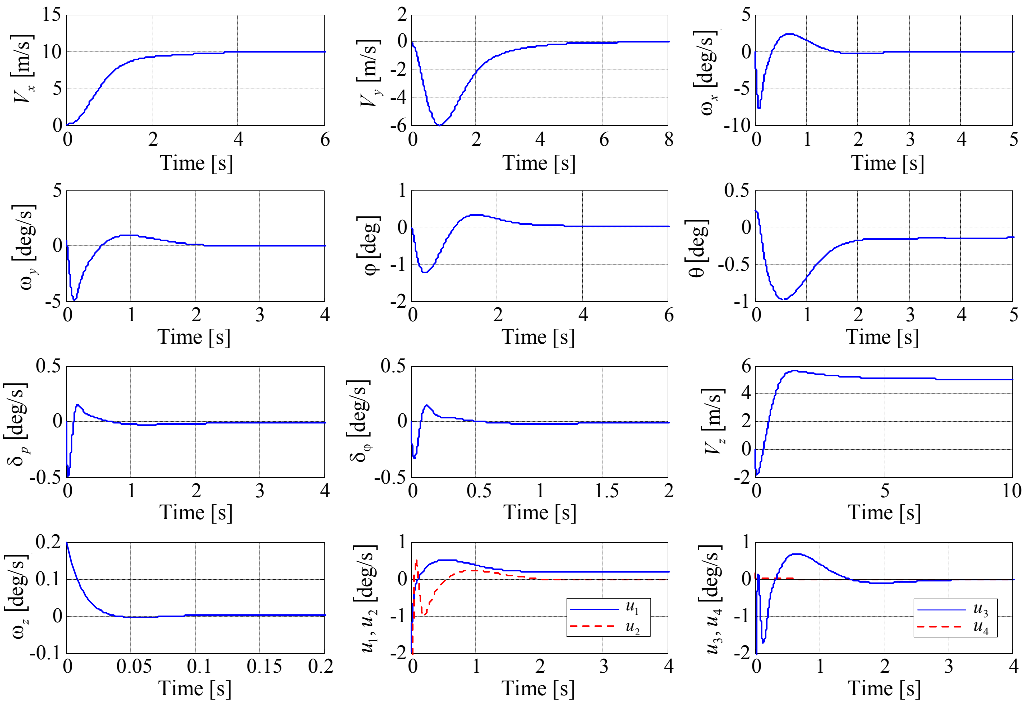

Fig. 4. Time characteristics of the system in Fig.1 without wind shears.

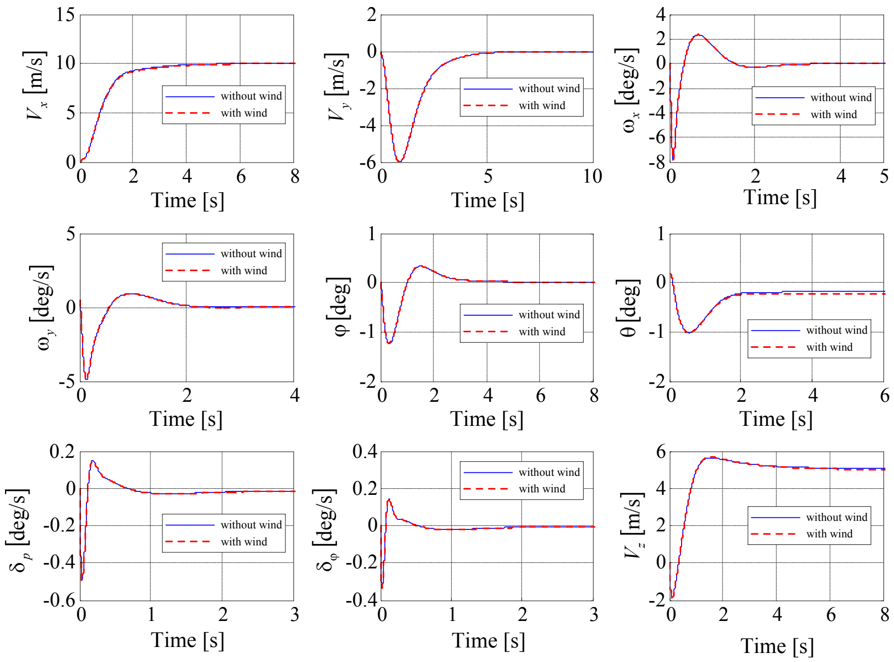

Fig. 5. Time characteristics of the system in Fig.1 with wind shears

IV. CONCLUSION

Using a helicopter motion liniarized model, an optimal control system is designed by using a cost function. The system state has 4 supplementary states in order to control the linear velocities corresponding to the three axes of the body frame and the roll angular rate. By means of the system Matlab/Simulink models, with and without considering the wind shears, the optimal control system dynamic characteristics are built.

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