Miniature accelerometer precision improvement using intelligent control

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***Abstract*—The paper presents a way to improve the transient regime of a miniature accelerometer by using a fuzzy controller to close its loop. The fuzzy controller replaces an electronic block on the feedback path of the closed loop, block that, in the classical architecture, assured the system damping and, in the same time, played the role of elastic link of the accelerometer proof mass. For the proposed controller, proportional-derivative variant is chosen, and its input-output mapping is derived. The membership functions for inputs are *s*-functions, *π*-functions, respectively *z*-functions, while the output membership functions have constant values. To define the rules, a zero-order Sugeno fuzzy model is chosen. Finally, a comparative numerical study between the classical and the proposed accelerometer architectures is made.**

I. INTRODUCTION

Extremely important in the Inertial Measurement Units (IMU) of the aerospace navigation systems, the accelerometers always brought into discussion topics related to optimization techniques by which they are designed. Most of the accelerometers, because of the conferred advantages, are closed loop devices. Usually, the loop closing for such a sensor, based on the movement of a seismic mass under the influence of an inertial force, is achieved using some classical controllers of proportional-integral-derivative (PID) type. Lately, however, the intelligent control techniques have opened new opportunities to obtain controllers with high robustness, which gives simultaneously a number of advantages related to the system performance improving [1]-[4]. Moreover, miniaturization of the high-power computing systems, but also, the control technique based on the linguistic rules elaboration, come in support to the easy implementation of these control systems. In literature, already appeared several studies about the development of sensors that address this type of control, for both miniaturised and non-miniaturised sensors [5]-[7].

The here presented work was developed in a research project concerning the development of *high-precision strap-down inertial navigators, based on the connection and adaptive integration of the nano and micro inertial sensors in low cost networks, with a high degree of redundance*, financed by National Council for Scientific Research in Higher Education (CNCSIS) in Romania.

This paper presents a way to improve the accuracy of an accelerometer using a fuzzy controller to close its loop. The study starts from an optimized version of the accelerometer, based on the use of an electronic amplification and filtering block of proportional-derivative (PD) type, heaving the role to replace the elastic link of the proof mass and the damper [8].

II. MATHEMATICAL MODEL OF THE PREOPTIMISED ACCELEROMETER AND CLASSICAL LOOP CLOSING

Initial study performed on the accelerometer ([8]) led to the functional scheme in Fig. 1, showing that the accelerometer is a closed loop one, having on the direct path, a displacement transducer, and on the feedback path an amplification and filtering block and a permanent magnet whereon slides a coil. The aims of this initial study were: system dimensioning, choice of optimal configuration of the amplification and filtering block and of the optimal values of the elements in its scheme [8].

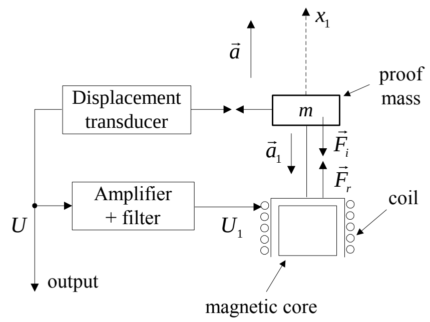

Figure 1. Functional scheme of the accelerometer

According to [8], notations in Fig. 1 are: $\vec{a}-$ carrier vehicle acceleration; $\vec{a}_1, x_1-$ acceleration and the displacement of the proof mass relative to a reference frame related to the carrier vehicle; $\vec{F}_i-$ inertial force appearing in the vehicle non-inertial reference frame; U – output voltage of the displacement transducer; U_1- coil supplying voltage; $\vec{F}_r-$ feedback force. Relative to the vehicle frame, from Fig. 1 it results the equation:

$$m\vec{a}_1 = \vec{F}_i + \vec{F}_r. \quad (1)$$

How $\vec{F}_i = -m\vec{a}$, from eq. (1), the equivalent form is obtained:

$$a(t) = -\frac{\mathrm{d}^2 x_1(t)}{\mathrm{d}t^2} + \frac{F_r(t)}{m}, \quad (2)$$

where $\mathrm{d}^2 x_1(t)/\mathrm{d}t^2 = -a_1(t)$. For the x_1 displacement, the corresponding output voltage of the displacement transducer can be derived as:

$$u(t) = k_t \cdot x_1(t), \quad (3)$$

with k_t the transducer constant.

Differential equation of the electrical current passing through the coil in the feedback path is:

$$u_1(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + R \cdot i(t), \quad (4)$$

with $L-$ coil inductance, $R-$ coil resistance, and $i-$ electrical current. The feedback force can be expressed by the relation:

$$F_r(t) = \pi BNDi(t), \quad (5)$$

where B is magnetic induction of the permanent magnet, $N-$ number of turns, and $D-$ coil diameter. If ρ is the electrical resistivity, characterizing the coil conductor, and d is conductor diameter, then L and R have the next formulas:

$$L = \frac{\mu_0 \mu_r \pi N D^4}{4d}, R = \frac{4\rho ND}{d^2}. \quad (6)$$

Analysis on the optimal configuration of the system led to the next relationship between the input and the output voltages of the amplification and filtering block

$$U_1 = -\left(1 + \frac{Y_1}{Y_2}\right)U; \quad (7)$$

$Y_1 = \mathrm{s}C_1$ and $Y_2 = 1/R_2$. As a consequence, the dynamic regime of the accelerometer is described by the equations (2)-(5) and (7).

Applying the Laplace transform to the previous mentioned relations, they become:

$$\begin{aligned} a(\mathrm{s}) &= -\mathrm{s}^2 x_1 + \frac{F_r(\mathrm{s})}{m}, \\ U(\mathrm{s}) &= k_t x_1(\mathrm{s}), \\ U_1(\mathrm{s}) &= F(\mathrm{s})U(\mathrm{s}), \\ I(\mathrm{s}) &= \frac{U_1(\mathrm{s})}{\mathrm{s}L + R}, \\ F_r(\mathrm{s}) &= \pi BNDI(\mathrm{s}). \end{aligned} \quad (8)$$

$F(\mathrm{s}) = -(\tau \mathrm{s} + 1)$ is the transfer function of the amplification and filtering block ($\tau = C_1 R_2$). Starting from the relations (8), it results the system block diagram with transfer functions in Fig. 2; the next notations were used: $k_r = \frac{\pi BND}{mR}$ and $\tau_1 = \frac{L}{R}$. The accelerometer equivalent Matlab/Simulink model results under the form presented in Fig. 3.

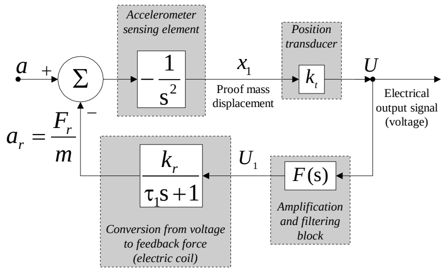

Figure 2. Block diagram of the accelerometer

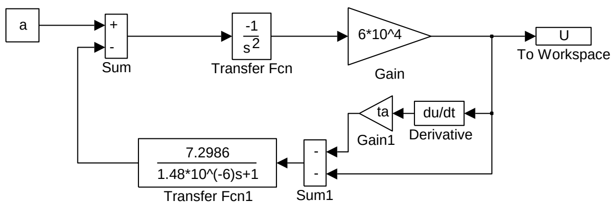

Figure 3. Accelerometer Matlab/Simulink model

Representing the amplitude-frequency characteristics ($|H(j\omega)|$ as a function of f) and the step response of the system for different values of the parameter τ (1 ms, 1,5 ms, 2 ms, 2,5 ms, 3 ms), the curves in Figs 4 and 5 were obtained; $H(j\omega)$ is the transfer function of the closed loop system, under the frequency domain form. Based on the observation (see Figs 4 and 5) that a non-monotone $|H(j\omega)|$ function in the $[0,\infty)$ frequency interval equate with the occurrence, when applying a unit step acceleration type, of unwanted oscillations of inertial mass, has achieved an optimization subroutine of τ which provided the value $\tau_{opt} = 2{,}1373266\,\mathrm{ms}$.

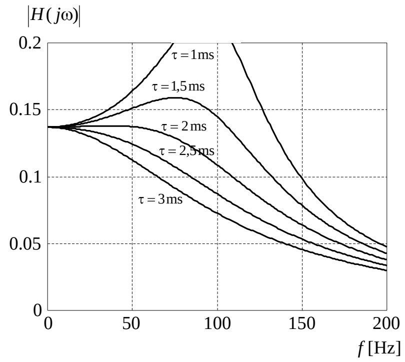

Figure 4. Amplitude-frequency characteristics

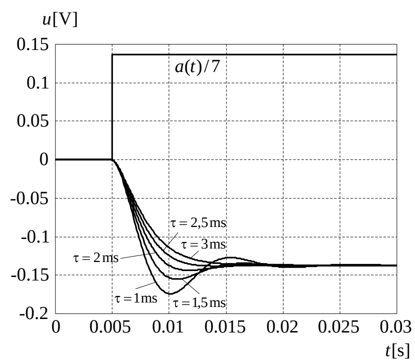

Figure 5. Step response

From the numerical values set out in [8] for the components of the accelerometer, resulted that it has a linear static characteristic with a negative scale factor $K = -0{,}137030102 \;\; \mathrm{Vs}^2/\mathrm{m}$.

III. Proposed Approach for Accelerometer Closed Loop Architecture

Considering the configuration of the accelerometer in Fig. 1, correlated with the block diagram with transfer functions in Fig. 2, one concluded that the system can be controlled by a fuzzy logic controller, placed on the direct path, after the displacement transducer, as shown in Fig. 6. As a consequence, this controller substitute the electronic amplification and filtering block (placed on the feedback path in the initial version) and controls the supply voltage of the coil. So, the new accelerometer output consists of the U_1 voltage supplying the coil which creates the feedback force (Fig. 7).

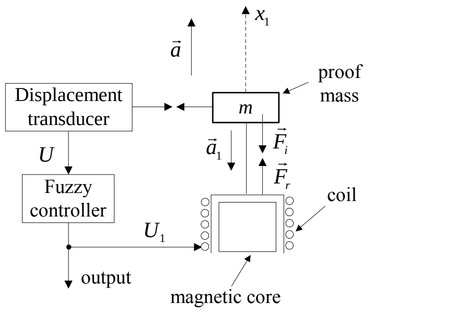

Figure 6. The new functional scheme of the accelerometer

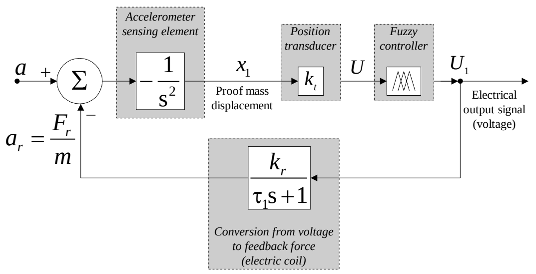

Figure 7. Accelerometer block scheme using fuzzy controller

Fuzzy logic technique, based on the fuzzy sets theory developed by L. Zadeh [9], provides a simple tool to interpret the human experience into reality, enhancing conventional system design with engineering expertise. Fuzzy logic controllers are rule-based controllers, the basic configuration of a fuzzy logic model being simply represented in next parts: the fuzzifier, the inference engine, the defuzzifier, and a knowledge base. So, the design of a fuzzy logic control system supposes the following steps: fuzzification strategy choosing; data base building; rule base elaboration; inference machine elaboration, and deffuzification strategy establishing [10].

Through analogy with the classical methods of control, the simplest fuzzy controller is the proportional controller (FP). Its input is the error and the output is the control signal. On the other way, the derivative component in a controller helps to predict the error, so, by combining the proportional and derivative actions in a controller an improvement in the closed-loop stability is obtained [11]. In this way, if the equation of a proportional-derivative (PD) controller is

$$i(t) = K_P \cdot e(t) + K_D \cdot \frac{\mathrm{d}e(t)}{\mathrm{d}t} = K_P \cdot \left[e(t) + T_D \cdot \frac{\mathrm{d}e(t)}{\mathrm{d}t} \right], \quad (9)$$

the obtained control signal is proportional to an estimate of the error T_D seconds ahead; $i(t)$ is the command variable in time, e is the operating error, K_P is the proportional gain and K_D is the derivative gain. Expressed in discrete form, the equation of a PD controller is [12]

$$i(k) = K_P \cdot e(k) + K_D \cdot \frac{[e(k) - e(k-1)]}{T_S}, \quad (10)$$

or

$$i(k) = K_P \cdot e(k) + K_D \cdot \Delta e(k). \quad (11)$$

k is the step, T_S - the sample period, and $\Delta e(k)$ - the change in error. In our case, the chosen fuzzy controller is of type PD so, the inputs to the controller are the voltage U and the derivative ΔU of the voltage U (change in voltage), while the voltage U_1 is the command variable (controller output).

For our system, the [-1, 1] interval was chosen like universe for all of the input and output signals. Also, we had opted for a number of three membership functions for each of the two inputs, and five membership functions for the output. The linguistic terms for inputs are N (negative), Z (zero) and P (positive), while for output are NB (negative big), NS (negative small), Z (zero), PS (positive small), and PB (positive big). The membership functions for inputs are *s*-functions, *π*-functions, respectively *z*-functions. To define the rules, a Sugeno fuzzy model was chosen, model proposed by Takagi, Sugeno and Kang [13]. In this way, for a two input - single output system the fuzzy rule is of the form:

$$\text{“if } (x_1 \text{ is } A) \text{ and } (x_2 \text{ is } B) \text{ then } \; y = f(x_1, x_2)\text{”}, \quad (12)$$

where A and B are fuzzy sets in the antecedent, and $y=f(x_1, x_2)$ is a crisp function in the consequent; $f(x_1, x_2)$ is a polynomial function. For the output membership functions constant values were chosen (NB=-1, NS=-0.5, Z=0, PS=0.5, PB=1), while the parameters of the inputs' membership functions are presented in Table 1.

TABLE I.
Parameters of the Inputs' Membership Functions

	mf type	*mf* parameters			
		x_{left}	x_{m1}	x_{m2}	x_{right}
*mf*1 (A_1^1 and A_2^1)	*z*-function	-1	-	-	0
*mf*2 (A_1^2 and A_2^2)	π -function	-1	0	0	1
*mf*3 (A_1^3 and A_2^3)	*s*-function	0	-	-	1

$A_q^i \; (q = \overline{1,2},\; i = \overline{1,3})$ are the associated individual antecedent fuzzy sets of each input variable, x is the independent variable on the universe, x_{left} is the left breakpoint, x_{right} is the right breakpoint, and $[x_{m1}, x_{m2}]$ is middle interval in that the peak flat – characterising the *s*-function, *z*-function, and π-function shapes.

Starting from the inputs' and output's membership functions, a set of nine inference rules were derived:

$$
\begin{aligned}
&\text{Rule1: If } U \text{ is } A_1^1 \text{ and } \Delta U \text{ is } A_2^1, \text{ then } y^1(U,\Delta U) = -1,\\
&\text{Rule2: If } U \text{ is } A_1^1 \text{ and } \Delta U \text{ is } A_2^2, \text{ then } y^2(U,\Delta U) = -0.5,\\
&\text{Rule3: If } U \text{ is } A_1^1 \text{ and } \Delta U \text{ is } A_2^3, \text{ then } y^3(U,\Delta U) = 0,\\
&\text{Rule4: If } U \text{ is } A_1^2 \text{ and } \Delta U \text{ is } A_2^1, \text{ then } y^4(U,\Delta U) = -0.5,\\
&\text{Rule5: If } U \text{ is } A_1^2 \text{ and } \Delta U \text{ is } A_2^2, \text{ then } y^5(U,\Delta U) = 0,\\
&\text{Rule6: If } U \text{ is } A_1^2 \text{ and } \Delta U \text{ is } A_2^3, \text{ then } y^6(U,\Delta U) = 0.5,\\
&\text{Rule7: If } U \text{ is } A_1^3 \text{ and } \Delta U \text{ is } A_2^1, \text{ then } y^7(U,\Delta U) = 0,\\
&\text{Rule8: If } U \text{ is } A_1^3 \text{ and } \Delta U \text{ is } A_2^2, \text{ then } y^8(U,\Delta U) = 0.5,\\
&\text{Rule9: If } U \text{ is } A_1^3 \text{ and } \Delta U \text{ is } A_2^3, \text{ then } y^9(U,\Delta U) = 1.
\end{aligned}
\tag{13}
$$

The rule-based inference chosen for each consequent is also presented in Fig. 8, and the resulted fuzzy control surface in Fig. 9.

Rule table of the FLC

ΔU/(U)	**N**	**Z**	**P**
N	NB (-1)	NS (-0.5)	Z (0)
Z	NS (-0.5)	Z (0)	PS (0.5)
P	Z (0)	PS (0.5)	PB (1)

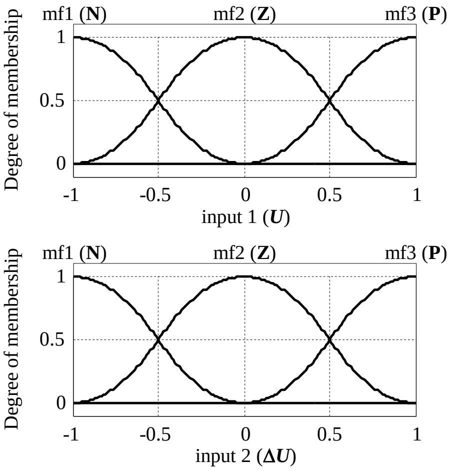

Figure 8. Membership functions and rule-based inference

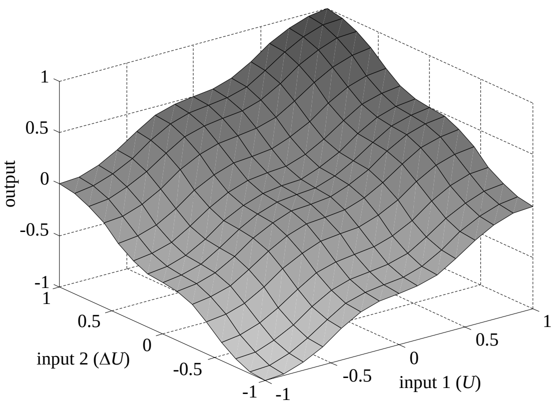

Figure 9. The fuzzy control surface

The new Matlab/Simulink model of the accelerometer results by the form in Fig. 10.

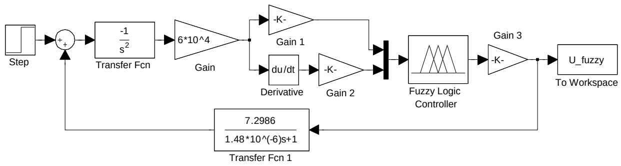

Figure 10. Matlab/Simulink model of the accelerometer using fuzzy controller

Representing on the same chart the step response of the accelerometer for initial variant (for $\tau = 1.5\,\text{ms}$, $\tau = \tau_{opt}$ and $\tau = 3\,\text{ms}$) and for new variant (using fuzzy controller), the characteristics in Fig. 11 were obtained. Fig. 12 is a zoom of the characteristics in Fig. 11, which shows that by using fuzzy controller to close the loop of the accelerometer, its transient regime was reduced from 12.25 ms (obtained for $\tau = \tau_{opt}$ in the classic version - Fig. 12 b) to 0.08 ms (Fig. 12 a), that is 153.125 times.

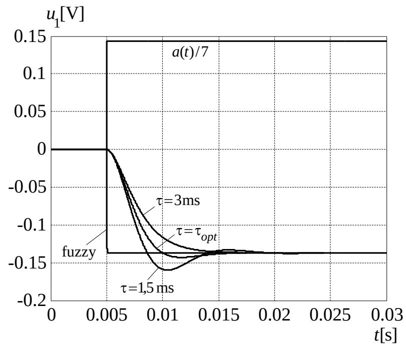

Figure 11. Step response: fuzzy vs. classical

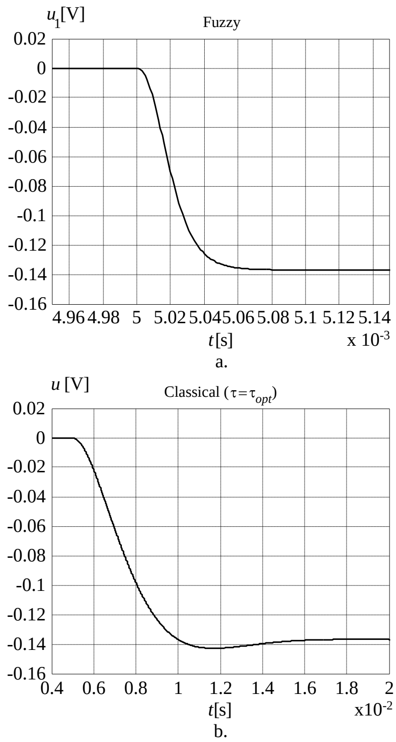

Figure 12. Transient response: fuzzy vs. classical ($\tau = \tau_{opt}$)

The assessment of the accelerometer precision, with design in the classical variant, made in [8], led to the conclusion that if it is used in an inertial navigation system, the resulted positioning errors are due in most part of the transient regime of the accelerometer. To achieve a qualitative test of the accelerometer new architecture one suppose that it is boarded on a vehicle which moves straight and is subject to an acceleration signal by the form of repeated steps as in Fig. 13; the same input signal is applied to the scheme which models the classical variant. Accelerometer response in the two variants is shown in Fig. 14: Fig. 14 a - fuzzy controller, Fig. 14 b – classical variant.

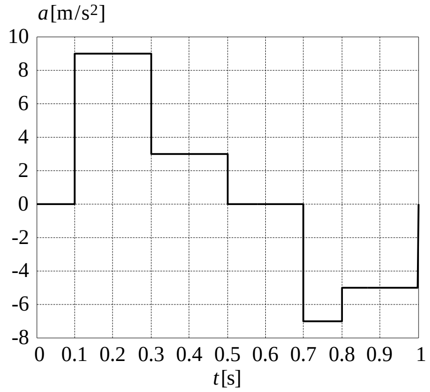

Figure 13. Repeated step applied acceleration

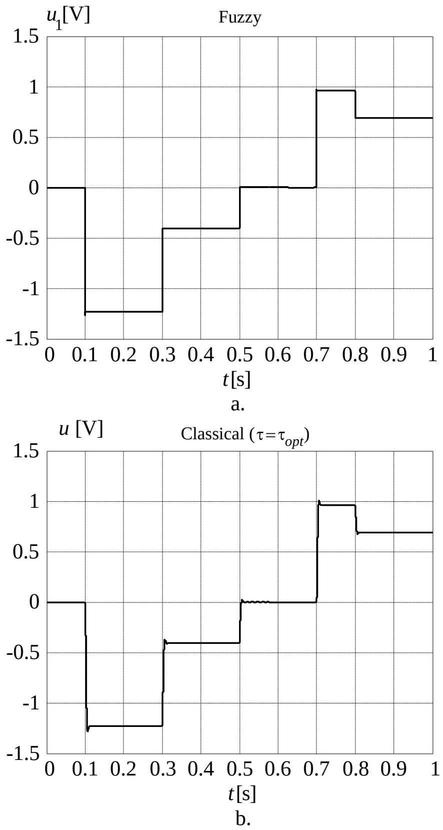

Figure 14. Accelerometer response for repeated step input

Calculating the distance covered by the vehicle by using two methods, a theoretical one and another one based on the $u_1(t)$ information (respectively on the $u(t)$) obtained through numerical simulation at the system output, the next results are highlighted: 1) vehicle coordinate from the theoretical method, $x = 1.525$ m; 2) vehicle coordinate from the numerical simulation of the architecture including fuzzy controller, $x_{fuzzy} = 1.5248065$ m; 3) vehicle coordinate from the numerical simulation of the classical architecture, $x_{classical} = 1.5232985$ m. The relative positioning errors appearing in the two cases are $\varepsilon_{fuzzy} = 1.268 \cdot 10^{-2}\,\%$, respectively $\varepsilon_{classical} = 0.1115\%$; the values show that the improvement of the transient regime by the fuzzy controller produces a reduction, with approximate one order of magnitude, of the relative positioning error, when the accelerometer is used in a mono-dimensional inertial navigator.

IV. Conclusions

The paper presented a new approach in terms of closing the loop of an accelerometer, by substituting an electronic amplification and filtering block (having the role of damper and elastic link for the accelerometer proof mass) with a proportional-derivative fuzzy controller placed unconventionally at the end of the direct path of the loop. The fuzzy controller aim was to control the voltage applied to an electrical coil which provided the feedback force. Comparatively with the classical architecture, the new approach reducing the transient time of the accelerometer of approximately 150 times, improving in this way the positioning performance of the accelerometer if an inertial navigation system included it. A comparative test proved that the positioning relative error was reduced with approximately one order of magnitude if the fuzzy logic closed loop architecture is used beside the classical architecture.

Acknowledgment

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