Automatic Control of the Satellites' Attitude and Stored Energy using Inertial Wheels

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***Abstract* — The paper presents the design of a new automatic system for the control of the satellites' attitude and stored energy by using four inertial wheels as actuators. The design of the system's control law is based on the usage of a Lyapunov function depending on the quaternion error vector and on the orientation angular rate error of the satellite tied frame with respect to a reference frame. The control law of the inertial wheels is obtained from the stability condition of the closed loop system. During the satellite's tracking stage, the control is achieved by means of the inertial wheels; the initialization of the satellite tied frame is made by means of the satellite's thrusters. The software implementation and validation of the new control architecture is achieved by using the Matlab/Simulink environment.**

***Keywords— satellite; control; inertial wheel; attitude, energy*.**

I. Introduction

The satellites must have good rotational handling and agility to answer well to multiple tasks; to control the motion of the satellites, these must be equipped with an automatic system for the control of their attitude and, in particular cases, of their stored energy. First, the satellite must be oriented through the rapid action of its thrusters; these can produce large moments. During its motion, the satellite must track the Sun and a terrestrial station; the maneuver of the two targets' tracking means the rotation of the satellite from its initial position to a target frame; this is the initialization maneuver. Then, by means of the inertial wheels, the satellite tracks a reference frame; the control of the satellite's attitude is achieved in the same time with the control of the energy (power) stored by the inertial wheels such that the two targets are pursued.

The parameterization of the satellite's attitude is mainly described by the cosine rotation matrix which is associated to the orthonormal group $SO(3)$ [1-2]; the Euler equations associated to the satellite's evolution are sometimes difficult to use and, therefore, the attitude's dynamics must be put into a double integrator form with respect to the parameters describing the satellite's attitude [3]. In this paper, to describe and control the attitude of the satellite, we use an attitude matrix expressed in terms of quaternion in order to avoid singularities for the large values of the Euler angles and a large amount of calculation in the case of trigonometric functions' usage.

The architecture designed in this paper (for the determination and control of the satellites' attitude and stored energy) is based on a cluster consisting of four inertial wheels and uses feedback from the system's angular rates. The control law modifies the vector of exterior moments applied to the satellite, this leading to the change of the equivalent kinetic moment vector; according to the satellites' dynamics, the modification of the kinetic moment modifies the vector of angular velocities, resulting new values for the quaternions and a new attitude of the satellite. The design of the control law is based on the usage of a Lyapunov function depending on the quaternion error and on the orientation angular rate error of the satellite tied frame with respect to a reference frame. The control law of the inertial wheels is obtained from the stability condition of the closed loop system. During the satellite's tracking stage, the control is achieved by means of the inertial wheels; the initialization of the satellite tied frame is made by means of the satellite's thrusters.

The paper is organized as follows: the satellites' general missions and motions are given in Section II; the dynamics of the satellites is presented in the third section, while the design of the new automatic architecture is provided in Section IV; in Section V the new architecture has been software implemented and validated. Finally, some conclusions are presented in Section VI.

II. The Mission and the Motion of the Satellite

The motion of a satellite is achieved on an elliptical trajectory in the plane containing the center of Earth. One chooses as inertial frame – $O_iX_iY_iZ_i$ frame having its origin (O_i) in the center of Earth, the axis O_iZ_i oriented towards the geographic North pole, the axis O_iX_i – towards the Vernal Equinox's, while the axis O_iY_i completes the orthonormal frame. The orbital frame $O_0X_0Y_0Z_0$ (Fig. 1) has the following axes: O_0Z_0 – oriented towards the center of Earth (O_i), O_0Y_0 – oppositely oriented with respect to the axis which is normal to the orbit's plane and O_0X_0 – oriented such that an orthonormal frame is obtained; in the case of a circular orbit, the axis O_0X_0 is tangent to the orbit, i.e. it is collinear with the direction of the velocity vector. The attitude of a satellite is defined by using the Euler angles – θ, φ and ψ; these angles define the position of the satellite tied frame with respect to the local orbital frame. Actually, the satellite tied frame is obtained from the orbital frame through three successive rotations with the angular rates $\dot{\theta}, \dot{\varphi}$ and $\dot{\psi}$ as below [3]:

$$O_0X_0Y_0Z_0 \xrightarrow[O_0Y_0]{\dot{\theta}} O_0X_0'Y_0'Z_0' \xrightarrow[O_0X_0']{\dot{\varphi}} O_0X_0''Y_0''Z_0'' \xrightarrow[O_0Z_0'']{\dot{\psi}} O_cX_cY_cZ_c \, .$$

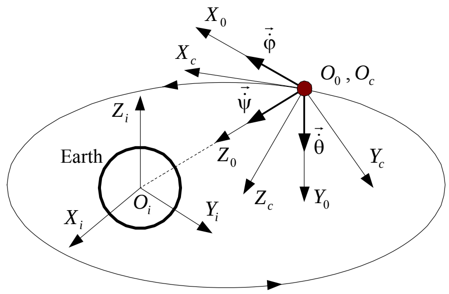

Fig. 1. Coordinate systems (frames): inertial frame ($O_iX_iY_iZ_i$), orbital frame ($O_0X_0Y_0Z_0$) and satellite tied frame ($O_cX_cY_cZ_c$)

This representation of the satellite's attitude has an important drawback: the appearance of singularities for large values of the Euler angles and a large amount of calculation (because of the trigonometric functions). Therefore, it is preferable to use an attitude matrix and quaternion vectors, the orientation of the satellite being more easily to be described.

During its motion, the satellite should track the Sun and a terrestrial station (e.g. the one placed in the point having the longitude 80.467° W and the latitude 28.467° N). The satellite's axis of symmetry O_0Z_c ($O_0X_cY_cZ_c$ is the satellite tied frame) must be permanently oriented towards the terrestrial station (the direction of the vector $\vec{l}_t$), O_0Y_c – the solar panels' axis must be perpendicular to the direction of the position vector $\vec{l}_s$ (from satellite towards the Sun), while the axis O_0X_c completes the orthonormal frame – Fig. 2 [4]. The frame $O_iX_iY_iZ_i$ is chosen as the inertial frame; thus, the satellite frame ($O_0X_cY_cZ_c$) must be permanently superposed over the reference frame (the target frame) $O_0X_RY_RZ_R$ having the axis O_0Z_R oriented towards the terrestrial station and the axis O_0Y_R perpendicular to the vector $\vec{l}_s$. The vectors $\vec{r}_t$, $\vec{r}_s$ and $\vec{r}_c$ express the positions of the terrestrial station, of the Sun and of the satellite with respect to the center of Earth (the origin of the inertial frame).

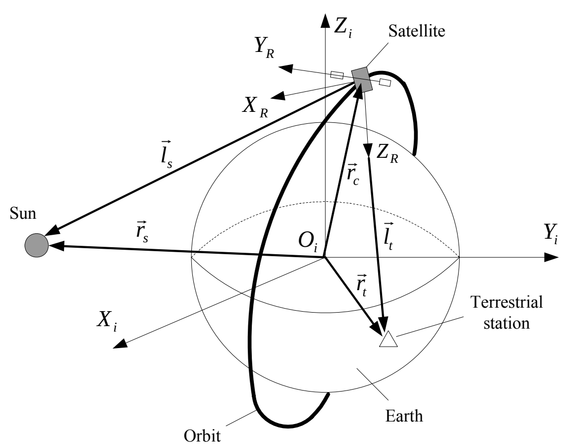

Fig. 2. The highlight of the satellite's mission

First, the satellite must be oriented through the rapid action of its thrusters; these produce large moments. The maneuver of the two targets' tracking means the rotation of the satellite from its initial position $O_0X_0Y_0Z_0$ to $O_0X_RY_RZ_R$; this is the initialization maneuver. Then, by means of the inertial wheels, the satellite tracks the reference frame ($O_0X_RY_RZ_R$); the control of the satellite's attitude is achieved in the same time with the control of the energy (power) stored by the inertial wheels such that the two targets are pursued.

III. The Dynamics of the Satellite on its Trajectory

The satellite's attitude (the Euler angles – θ, φ and ψ) can be defined by means of the quaternion vectors $\boldsymbol{q} = [q_1 \quad q_2 \quad q_3]^{\mathrm{T}}$ and $\hat{\boldsymbol{q}} = [q_1 \quad q_2 \quad q_3 \quad q_4]^{\mathrm{T}}$. The Euler angles associated to the satellite are similar to the ones expressing the attitude of an aircraft with respect to the Earth tied frame: φ is associated to the roll angle, θ – associated to the pitch angle and ψ – associated to the direction angle [5]. The differential equations of the quaternions are [6]:

$$\dot{\boldsymbol{q}} = -\frac{1}{2}\omega^{\times}\boldsymbol{q} + \frac{1}{2}q_4\omega, \dot{q}_4 = -\frac{1}{2}\omega^{\mathrm{T}}\boldsymbol{q}; \quad (1)$$

the correlation formulas between the components of the vector $\boldsymbol{q}$ and the satellite attitude angles are [6]:

$$\begin{aligned} \theta &= \operatorname{atan}\frac{2(q_1q_3 + q_2q_4)}{-q_1^2 - q_2^2 + q_3^2 + q_4^2}, \\ \varphi &= \operatorname{asin}[2(q_1q_4 - q_2q_3)], \\ \psi &= \operatorname{atan}\frac{2(q_1q_2 + q_3q_4)}{-q_1^2 + q_2^2 - q_3^2 + q_4^2}. \end{aligned} \quad (2)$$

The absolute motion of the satellite is described by the equation [7]:

$$\dot{\boldsymbol{K}} + \omega^{\times}\boldsymbol{K} = \boldsymbol{u}, \boldsymbol{u} = \boldsymbol{u}_t + \boldsymbol{u}_g + \boldsymbol{u}_p, \omega^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}, \quad (3)$$

where $\omega = [\omega_1 \quad \omega_2 \quad \omega_3]^{\mathrm{T}}$ is the vector of the satellite's angular rates, $\boldsymbol{K}$– the equivalent kinetic moment, $\boldsymbol{u}$ – the vector of exterior moments applied to the satellite, $\boldsymbol{u}_t$ – the vector of command moments, $\boldsymbol{u}_g$ – the vector of gravitational gradient moments, $\boldsymbol{u}_p$ – the vector of disturbing moments.

A platform consisting of N=4 inertial wheels is usually placed on satellites; the motion of the inertial wheels is described by the equation: $\dot{\boldsymbol{K}}_a = \boldsymbol{u}_a$, with $\boldsymbol{K}_a(N \times 1)$– the vector of kinetic moments of the inertial wheels and $\boldsymbol{u}_a(N \times 1)$– the vector of axial moments applied to the inertial wheels' platform. The kinetic moment $\boldsymbol{K}$ has the formula: $\boldsymbol{K} = J\omega + B\boldsymbol{K}_a$, with J – the matrix of inertia moments and $B(3 \times N)$– the transfer matrix of the vector $\boldsymbol{K}_a$ around the axes of the satellite. The matrix J is defined with respect to the satellite's matrix of inertia moments (I), this including the matrix of the wheels' inertia moments – $I_a = [I_{a1} \quad I_{a2} \quad \ldots \quad I_{aN}]^{\mathrm{T}}$; the relationship between J and I is [8]: $J = I - BI_aB^T$. The vector of kinetic moments of the inertial wheels is expressed as follows [8]: $\boldsymbol{K_a} = I_a(\Omega + B^{\mathrm{T}}\omega) = I_a\boldsymbol{\Omega}_c$, with $\Omega(N\times 1)-$ the vector of inertial wheels' angular rates with respect to the satellite's body, while $\Omega_c=\Omega+B^{\mathrm{T}}\omega$ is the resultant angular rate. Because the norm of the vector Ω is much larger than the norm of the vector ω, we may consider that $\Omega_c\cong\Omega$. The equation (3) becomes $\dot{\boldsymbol{K}}=\boldsymbol{K}^{\times}\omega+\boldsymbol{u}$; now, replacing into this equation the kinetic moment $\boldsymbol{K}$ with its above presented expression, we obtain: $\dot{\boldsymbol{K}}=\boldsymbol{K}^{\times}J^{-1}(\boldsymbol{K}\text{-}B\boldsymbol{K}_a)+\boldsymbol{u}$.

The differential equation of the quaternion kinematics is:

$$\dot{\boldsymbol{q}}=\boldsymbol{F}(\boldsymbol{q})\omega, \tag{4}$$

with $\boldsymbol{F}(\boldsymbol{q})$ having the expression:

$$\boldsymbol{F}(\boldsymbol{q})=\frac{1}{2}\left\{I_{3\times 3}+\boldsymbol{q}^{\times}+\boldsymbol{q}\boldsymbol{q}^{\mathrm{T}}-\frac{1}{2}\left[1+\boldsymbol{q}^{\mathrm{T}}\boldsymbol{q}\right]I_{3\times 3}\right\},\boldsymbol{q}^{\times}=\begin{bmatrix}0 & -q_3 & q_2\\ q_3 & 0 & -q_1\\ -q_2 & q_1 & 0\end{bmatrix}. \tag{5}$$

IV. Design of the New Automatic Architecture for the Control of the Satellites' Attitude

The design of the new architecture's control law is mainly based on the usage of reference frame's dynamics and kinematics models – described by the equations: $\dot{\boldsymbol{K}}_R=\boldsymbol{K}_R^{\times}J^{-1}\boldsymbol{K}_R+\boldsymbol{u}_R$ and $\dot{\boldsymbol{q}}_R=\boldsymbol{F}(\boldsymbol{q}_R)\omega_R$, with ω_R – the angular rate vector of the reference frame with respect to the inertial frame and $\boldsymbol{K}_R=J\omega_R$. The angular rate error vector associated to the orientation of the satellite tied frame (frame $O_0X_cY_cZ_c$) with respect to the reference frame ($O_0X_RY_RZ_R$) is:

$$\omega_e=A(\hat{\boldsymbol{q}}_e)\omega_R-\omega, \tag{6}$$

where $A(\hat{\boldsymbol{q}}_e)$ is the rotation matrix of the reference frame with respect to satellite tied frame, $\hat{\boldsymbol{q}}_e=\begin{bmatrix}\boldsymbol{q}_e^{\mathrm{T}} & q_{4e}\end{bmatrix}^{\mathrm{T}}$ – the quaternion error vector, $A=CD^{\mathrm{T}}$, with C – the satellite's rotation matrix with respect to the inertial frame and D – the rotation matrix of the reference frame with respect to the inertial one.

For the design of the control law, we choose the Lyapunov function [9]:

$$V=\frac{1}{2}\omega_e^{\mathrm{T}}J^{-1}\omega_e+2k_p\ln\left(1+\boldsymbol{q}_e^{\mathrm{T}}\boldsymbol{q}_e\right),k_p>0; \tag{7}$$

the control law must assure the convergences $\omega_e\to 0$ and $\boldsymbol{q}_e\to 0$ when $t\to\infty$. We calculate the derivative of the Lyapunov function; the stability condition $\dot{V}=-k_d\omega_e^{\mathrm{T}}\omega_e\le 0$ is obtained if and only if the following condition is fulfilled:

$$B\boldsymbol{u}_a=\boldsymbol{K}^{\times}J^{-1}(\boldsymbol{K}-B\boldsymbol{K}_a)+\boldsymbol{u}_t+\boldsymbol{u}_g+J\omega^{\times}\omega_e+k_p\boldsymbol{q}_e+k_d\omega_e. \tag{8}$$

For the stage of attitude's initialization, we choose: $B\boldsymbol{u}_a=0$, $\boldsymbol{u}_t=-\boldsymbol{K}^{\times}J^{-1}(\boldsymbol{K}-B\boldsymbol{K}_a)-\boldsymbol{u}_g+J\omega^{\times}\omega_e+\dot{\boldsymbol{K}}_R-k_p\boldsymbol{q}_e-k_d\omega_e$; $\boldsymbol{u}_t$ is the vector of command moments achieved by means of the satellite's thrusters. For the tracking stage, the control is obtained by using the inertial wheels which integrate the vector $B\boldsymbol{u}_a$ – expressed with (8), where [4]: $\boldsymbol{u}_t=-k\cdot B\boldsymbol{u}_a$, $k>0$.

The equation (8) can be written under the simplified form:

$$B\boldsymbol{u}_a=f,\boldsymbol{u}_a=\boldsymbol{u}_g+\boldsymbol{u}_n, \tag{9}$$

where f is the sum of the right side terms, $\boldsymbol{u}_g$ is the component (the couple) which achieves the attitude's control during the continuous tracking stage (stage II), while $\boldsymbol{u}_n-$ the component for the energy control; $\boldsymbol{u}_n$ does not contribute to the stabilization of the satellite's attitude; therefore, the attitude's control and the energy's control are made simultaneously but independently.

Because $\boldsymbol{u}_n$ does not produce the rotation of the satellite, the equation (9) is equivalent with the next ones:

$$B\boldsymbol{u}_g=f,B\boldsymbol{u}_n=0;\boldsymbol{u}_g=B^{+}f,B^{+}=\left(B^{\mathrm{T}}B\right)^{-1}B^{\mathrm{T}}; \tag{10}$$

B^{+} is the pseudo-inverse of the matrix B.

The eclipse period is about half the period of Sun exposure; during the Sun exposure, the solar panels produce enough energy to supply all the onboard satellite equipment; the inertial wheels are accelerated to absorb and store extra energy. Thus, in this stage, the angular rate of the inertial wheels Ω increases, $\boldsymbol{K}_a$ increases and, therefore, the reaction couple $\boldsymbol{u}_a$ increases; more precisely, the component $\boldsymbol{u}_g$ increases $(\boldsymbol{u}_n=0)$. During Sun exposure, the control of energy is less important. The kinetic energy of the inertial wheels is $E_c=\frac{1}{2}\Omega^{\mathrm{T}}I_a\Omega$, while the received power is: $P=\mathrm{d}E_c/\mathrm{d}t=$ $=\Omega^{\mathrm{T}}I_a\dot{\Omega}=\Omega^{\mathrm{T}}\dot{\boldsymbol{K}}_a=\Omega^{\mathrm{T}}\boldsymbol{u}_a$. Taking into account equations (9) and (10), the expression of the received power becomes:

$$P=\Omega^{\mathrm{T}}\boldsymbol{u}_a=\Omega^{\mathrm{T}}(\boldsymbol{u}_g+\boldsymbol{u}_n)=\Omega^{\mathrm{T}}B^{+}f+\Omega^{\mathrm{T}}\boldsymbol{u}_n=P_r+P_m, \tag{11}$$

with P_r- the consumed power for the inertial wheels' acceleration and $P_m=\Omega^{\mathrm{T}}\boldsymbol{u}_n=P-\Omega^{\mathrm{T}}B^{+}f-$ the stored power.

During eclipse, the wheels are decelerated, Ω and $\boldsymbol{K}_a$ decrease and, accordingly, $\boldsymbol{u}_a$ changes its sign; more precisely, $\boldsymbol{u}_g$ changes its sign, while $\boldsymbol{u}_n$ contributes to the decrease of the stored power (P_m). Downloading the inertial wheels during eclipse is critical because the energy must be economically used.

In Fig. 3 we present the block diagram of the automatic system for the control of the satellite's attitude and stored energy. The satellite is rotated such that it is simultaneously oriented towards two targets (Sun and the terrestrial station). The control law associated to the quaternion error vector $\boldsymbol{q}_e=\begin{bmatrix}q_{1e} & q_{2e} & q_{3e}\end{bmatrix}^{\mathrm{T}}$ is proportional-derivative (P.D.) type and has the coefficients k_p and k_d. During the first stage, the control of the satellite is achieved by means of the thrusters, while, in the second stage (permanent tracking), by means of the inertial wheels.

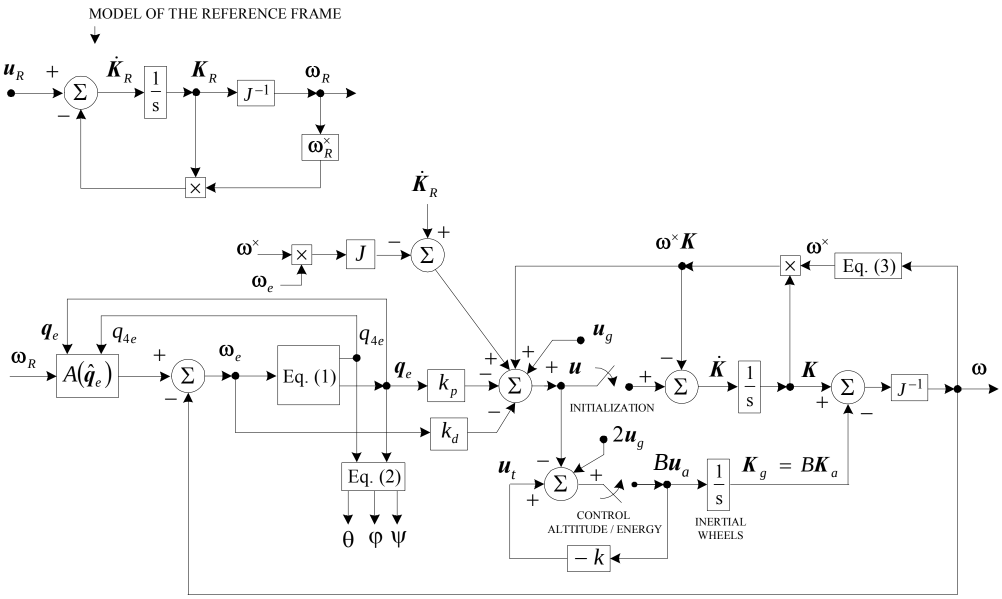

Fig. 3. Block diagram of the automatic system for the control of the satellite's attitude and stored energy

To obtain the rotation angular rate (ω_R) of the reference frame $O_0X_RY_RZ_R$ and its angular acceleration $(\dot{\omega}_R)$, we use the vectors $\vec{l}_t, \ddot{\vec{l}}_t, \vec{l}_s, \dot{\vec{l}}_s$ and $\ddot{\vec{l}}_s$ from Fig. 2 [10]. Then, by using (6), we can determine the command vector $\boldsymbol{u}_R = J\dot{\omega}_R + \omega_R^{\times} J\omega_R$. For the system presented in Fig. 3, the vectors $\boldsymbol{q}_e$ and q_{4e} are the solutions $\boldsymbol{q}$ and q_4 of the system (1), where ω is ω_e. By using these quaternions, we form the matrix: $A = A(\boldsymbol{q}, q_4) =$

$$= \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix};$$

the four components of the quaternion $\hat{\boldsymbol{q}} = [q_1 \quad q_2 \quad q_3 \quad q_4]^{\mathrm{T}}$ are actually $q_{1e}, q_{2e}, q_{3e}, q_{4e}$ – the four components of the quaternion error vector $\hat{\boldsymbol{q}}_e = [q_{1e} \quad q_{2e} \quad q_{3e} \quad q_{4e}]^{\mathrm{T}}$.

V. Numerical Simulation Results

In this section, the satellite's attitude and energy control system (Fig. 3) using inertial wheels is software implemented and validated in Matlab/Simulink environment for the case of a mini-satellite. The attitude of the satellite (the angles θ, φ and ψ) will be controlled by means of $\boldsymbol{q}_e$ and q_{4e}. Every iteration, the change of $\boldsymbol{q}_e$ and q_{4e} modifies the vector of exterior moments applied to the satellite ($\boldsymbol{u}$), this leading to the change of the equivalent kinetic moment vector ($\boldsymbol{K}$); according to equation (3) the modification of $\boldsymbol{K}$ changes the vector of angular velocities (ω) and the matrix $A(\hat{\boldsymbol{q}}_e)$, resulting new values for the quaternions and a new attitude of the satellite. The matrix of inertia moments (J) is calculated by means of the equation $J = I - BI_aB^{\mathrm{T}}$, where $I = \mathrm{diag}[200 \quad 200 \quad 175] \mathrm{kg\,m}^2, I_a = \boldsymbol{K}_a\Omega_c^+ \cong \boldsymbol{K}_a\Omega^+$, with Ω^+ – the pseudo-inverse of the matrix (vector) consisting of the inertial wheels' angular rate vector $\Omega = [\Omega_1 \quad \Omega_2 \quad \Omega_3 \quad \Omega_4]^{\mathrm{T}}, \Omega_i = 22000 \mathrm{rot/min} = 22000 \cdot 2\pi \mathrm{rad}/60\mathrm{s} \cong$ $\cong 2200 \mathrm{rad/s}$. We have chosen: $\omega(0) = 10^{-4}[-0.3 \quad -0.2 \quad 0.4]^{\mathrm{T}}$, $\boldsymbol{K}_a(0) = [0.1 \quad 0.1 \quad 0.1 \quad 0.1]^{\mathrm{T}}$. Because $I_a = \boldsymbol{K}_a\Omega^+$, we calculate first $I_a(0)$ and then, with the equations $\boldsymbol{K} = J\omega + B\boldsymbol{K}_a$ and $J = I - BI_aB^T$, we obtain $J(0)$ and $\boldsymbol{K}(0)$. We neglected the component $\boldsymbol{u}_g$, the vector $\dot{\omega}_R$ and, as a consequence, $\dot{\boldsymbol{K}}_R = J\dot{\omega}_R$ is also neglected; we considered $\hat{\boldsymbol{q}}_e = [0.45 \quad 0.3 \quad -0.5 \quad 0.6764]^{\mathrm{T}}$, the angular rate of the reference model – $\omega_R = 10^{-4}[-0.1 \quad -0.1 \quad 0.2]^{\mathrm{T}}$ rad/s and the following values for the control law's coefficients: $k_p = 25, k_d = 30, k = 5$; these have been obtained using the Ziegler-Nichols tuning algorithms.

We consider that the initialization of the satellite has been made by means of its thrusters; the contact afferent to $\boldsymbol{u}$ is close during this first stage, while the contact before $B\boldsymbol{u}_a$ is open; for the second phase (permanent tracking), the contact afferent to $\boldsymbol{u}$ is open, while the other one is closed, the satellite control being achieved by means of the inertial wheels.

The Matlab/Simulink model used in the automatic control system's validation is presented in Fig. 4. In Fig. 5 we present the time histories of the satellite's angular velocities $(\omega_1, \omega_2, \omega_3)$, the components of the exterior moments applied to the satellite (u_1, u_2, u_3), the components of the kinetic moment vector (K_1, K_2, K_3), the components of $\boldsymbol{K}_g$ (K_{g1}, K_{g2}, K_{g3}), and the deviation of the attitude angles $(\Delta\theta, \Delta\varphi, \Delta\psi)$. In Fig. 6 we present the time histories associated to the four components of the quaternion error vector $\hat{\boldsymbol{q}}_e = [q_{1e} \quad q_{2e} \quad q_{3e} \quad q_{4e}]^T$. The quality indicators for the system in Fig. 3 can be modified choosing other values for the coefficients k_p, k_d, and k.

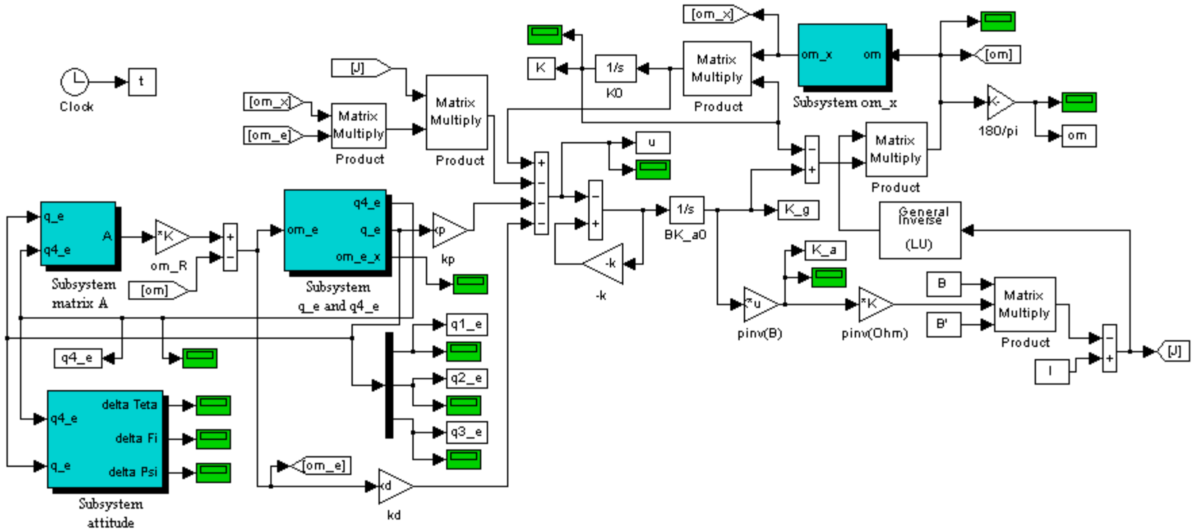

Fig. 4. Matlab/Simulink model of the automatic system for the control of the satellite's attitude and stored energy

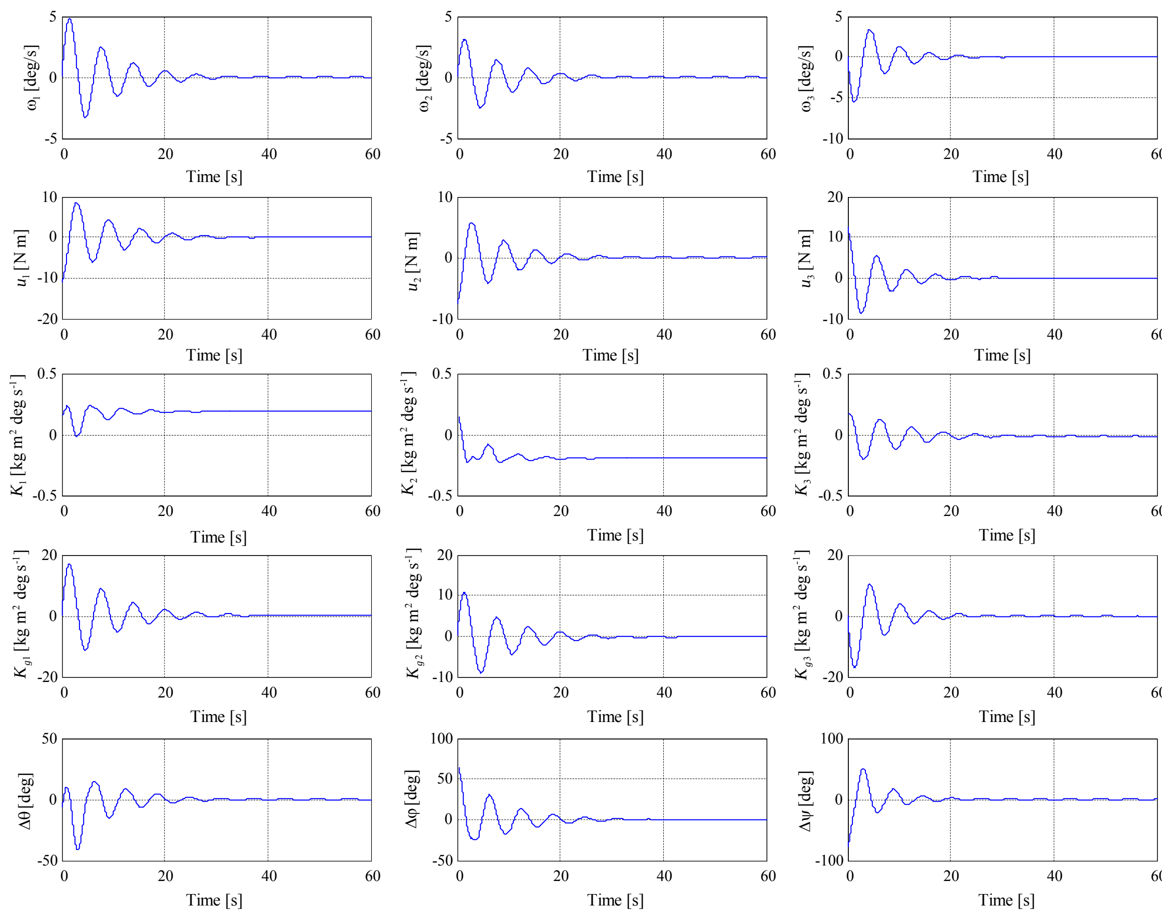

Fig. 5. Time histories of the main variables associated to the automatic system for the control of the satellite's attitude and stored energy

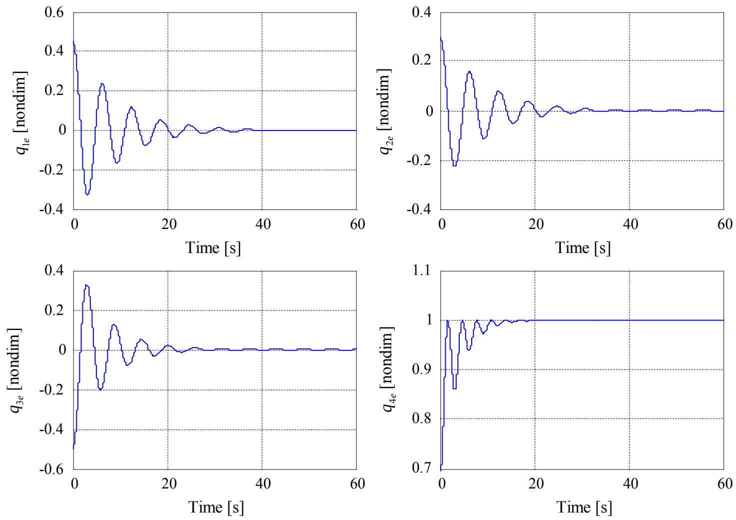

Fig. 6. Time histories associated to the four components of the quaternion $\hat{\boldsymbol{q}}_e$

According to Figs. 6 and 7, the control of the satellite's attitude is achieved by modifying the angular velocity vector (ω) and the quaternion $\hat{\boldsymbol{q}}_e$; the closed loop control system is characterized by good convergence, global asymptotically stability and $\boldsymbol{q}(t) \rightarrow 0, \omega(t) \rightarrow 0$. When the components of the vectors $\boldsymbol{q}(t)$ and $\omega(t)$ become zero, other variables become null: the components of the exterior moments applied to the satellite $\boldsymbol{u}(u_1, u_2, u_3)$, the components of the kinetic moment vector $\boldsymbol{K}(K_1, K_2, K_3)$ and the components of $\boldsymbol{K}_g(K_{g1}, K_{g2}, K_{g3})$; on the other hand, taking into account that the deviation of the attitude angles are null in steady regime ($\Delta\theta \rightarrow 0, \Delta\varphi \rightarrow 0$, $\Delta\psi \rightarrow 0$), it can be remarked the convergence of the three Euler angles to their desired values.

VI. Conclusion

The purpose of this study was to design a new architecture for the control of the satellites' attitude and stored energy, by using four inertial wheels as actuators. The motion of the satellite is considered to be characterized by an elliptical trajectory in the plane containing the center of Earth; during its motion, the satellite must track the Sun and a terrestrial station. The control law of the inertial wheels has been obtained from the stability condition of the closed loop system. The design of the new architecture's control law is also based on the usage of reference frame's dynamics and kinematics. All the steps of the new architecture's design procedure have been software implemented and validated in Matlab/Simulink environment; to validate the new automatic control system, a mini-satellite motion has been chosen. The obtained results are very good; the closed loop control system has been proved to be characterized by convergence and global asymptotically stability.

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References

[1] R. Bayadi and R. N. Banavar, "Almost global attitude stabilization of a rigid body for both internal and external actuation schemes," European Journal of Control, vol. 20, pp. 45-54, 2014.

[2] T. Lee, "Robust adaptive attitude tracking on SO(3) with an application to a quadrotor UAV," IEEE Transactions on Control Systems Technology, vol. 21, no. 5, pp. 1924-1930, 2013.

[3] K. Kreutz, "Manipulator control by exact linearization," IEEE Transactions on Automatic Control, vol. 34, no. 7, pp. 763-767, 1989.

[4] P. Tsiostras, H. Shen, and C. Hall, "Satellite Attitude Control and Power Tracking with Energy/Momentml Wheels," Journal of Guidance, Control and Dynamics, vol. 24, no. 1, pp. 23-34, 2001.

[5] C. Heiberg, D., Bailey, and B. Wie, "Precision Spacecraft Pointing using Single-Gimbal Control Moment Gyroscopes with Disturbances," Journal of Guidance, Control, and Dynamics, vol. 23, no. 1, pp. 77-85, 2000.

[6] J.T. Wen and K.K. Delgado, "The Attitude Control Problem," IEEE Transactions on Automatic Control, vol. 36, no. 10, pp. 1372-1379, 1991.

[7] L. Benziane, A. Benallegue, and A. Tayebi, "Attitude stabilization without angular velocity measurements," in IEEE International Conference on Robotics & Automation (ICRA), pp. 3116-3121, 2014.

[8] N. Chaturvedi, A. Sanyal, and N. McClamroch, "Rigid-body attitude control," IEEE Control Systems Magazine, vol. 31, no. 3, pp. 30-51, 2011.

[9] H. Yoon and P. Tsiotras, "Spacecraft Adaptive Attitude and Power Tracking with Variable Speed Control Moment Gyroscopes," Journal of Guidance, Control and Dynamics, vol. 25, no. 6, pp. 1081-1090, 2002.

[10] H. Hahlani, "Design of a Payload Control System for Tracking Moving Objects," Journal of Guidance, Control and Dynamics, vol. 12, no. 3, pp. 365-374, 1989.